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Chaotic Analysis of Rectangular Thin Plate Based on Fourth-order Runge-Kutta Method

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ABSTRACT: In practical applications, rectangular thin plate plays a significant role, especially in the case of a variety of magnetic interaction; therefore, its safety is of great importance to concern. This paper studies the nonlinear magneto-elastic vibration and chaotic analysis of clamped rectangular thin plate under different circumstances (including force, magnetic coupling and thermal, force, and magnetic coupling two cases). Firstly, a magnetoelasticity vibration equations of force and magneto-elastic coupling magnetic field is established, with the application of chaotic theory to study it, with fourth-order Runge-Kutta method for simulation analysis, and system Lyapunov exponents diagram and Poincare sectional view are drawn; and then the impact of changes in temperature field on the system is considered, also the equation is established for simulation analysis to identify the influence of parameters on motion characteristics, and then summed up the law, which will play a guiding role for practical applications.

KEYWORDS: Rectangular thin plate; Magnetoelasticity; Chaotic analysis.

INTRODUCTION

Rectangular thin plate is a basic component that is often encountered in engineering structures, and there is a symmetrical plane between the upper and lower surfaces, typically it is operated in the electromagnetic field, mechanical field, temperature field and other multiple action environment, and its dynamic characteristics has an important influence to the structure security of the system. Therefore, the study on multi-field magnetic coupling dynamics of elastic thin plate has guiding significance and practical value. This paper studies the nonlinear magneto-elastic vibration and chaotic analysis of clamped rectangular thin plate under the influence of force, magnetic coupling and thermal, force, and magnetic coupling two cases. On the basis of the nonlinear magneto-elastic plate and chaotic theory, the nonlinear vibration equation of clamped rectangular thin plate is established, its chaotic problems have an in-depth study, and its number is solved by programming differential equations based on fourth-order Runge-Kutta method.

About the contents of a rectangular thin plate, many scholars have put forward views in many aspects, and summarize the research orientation of past research scholars, it can be divided into the following aspects: free vibration problem of rectangular thin plate; nonlinear dynamics research; random bifurcation problem of rectangular thin plate vibration; bending problem of thin plate; study under transverse concentrated harmonic load, and so on. Zhong Yang [1] et al. solved the free vibration problem of clamped rectangular thin plate, finally obtained free vibration frequency equation and the frequency, and compared the frequency of different situations and has been verified; Gao Yongyi [2, 3] et al. put forward dynamics of thin plate on nonlinear elastic foundation, established nonlinear vibration model under thin plate transverse harmonic excitation, and obtained the premise of ignoring plate nonlinear component; Ge Gen [4] et al. firstly established a dynamics equations of random excitation in four edges simply supported plate, then translated the complex process that represents system energy changes into simple one-dimensional dispersion process with proposed non-integrable system stochastic averaging method, and finally analyzed the overall stability and bifurcation situation based on theory; Cheng Xuansheng [5-7] et al. successively studied the thermal bending problem of trilateral clamped and one edge free plate, and the object is concrete, changing the solution process to obtain a better solution, which provided a theoretical basis for practical engineering calculations; Tan Fei [8, 9] et al. solved the plate thermal bending problem with the use of hybrid boundary method to improve the efficiency and precision in the solution process; Zhang Wei [10] et al. verified nonlinear vibration response of the four edges simply supported plate and added transverse concentrated harmonic load effect, and found that under the effect of excitation amplitude the system will appear complex nonlinear phenomena. In addition, the application of fourth-order Runge-Kutta method is also very extensive [11-13].

NONLINEAR MAGNETO-ELASTIC VIBRATION EQUATIONS AND CHAOTIC THEORY OF THIN PLATE

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Nonlinear Magneto-elastic Vibration Equations of Thin Plate

According to the basic assumptions of Kirchhoff, a combination of mechanical field and electromagnetic field for interaction may obtain the nonlinear magnetic elastic vibration equation of thin plate as:

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}+F_x+P_x=\rho h\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}+F_y+P_y=\rho h\frac{\partial^2 v}{\partial t^2} \tag{2}$$

$$\begin{aligned}&\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x}+N_{xy}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(N_y\frac{\partial w}{\partial y}+N_{xy}\frac{\partial w}{\partial x}\right)+\\&\frac{\partial m_x}{\partial x}+\frac{\partial m_y}{\partial y}+F_z+P_z=\rho h\frac{\partial^2 w}{\partial t^2}-\frac{\rho h^3}{12}\frac{\partial^2\left(\nabla^2 w\right)}{\partial t^2}\end{aligned} \tag{3}$$

In above formula, N_x, N_y and N_{xy} are corresponding in-plane internal forces; M_x, M_y and M_{xy} are bending internal forces; p_x, p_y, and P_z are mechanical loads; ρ is material density; operator $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$.

Chaotic Theory

Chaotic theory aims at the behavior that cannot explain with interrupted and separate data in dynamic systems, and it is a research method combines qualitative thinking and quantitative analysis. As one of the chaotic theory researchers, the famous scholar Lorenz proposed that the chaotic theory has three characteristics: seemingly random, degree of sensitivity dependent on preconditions, and sensitive of internal change system dependent on preconditions.

A complex chaotic behavior usually makes people think its equation of motion must be equally complex, but in fact, a very simple system can also produce chaotic phenomena. Summarize the existing research results, there are usually three ways to chaos: (1) Period doubling bifurcation, which is a common way that is to translate a fixed point into period two, and then into an infinite times circle until into the chaotic state. (2) Quasi-periodic KAM torus fracture, namely from the equilibrium state to periodic motion and then to the quasi-periodic motion. (3) Intermittent way, which is a process that the system translate from intermittent chaos to continuity chaos, with variability.

Currently, numerical analysis is the most commonly used method to solve the chaotic problem. It includes a lot of methods, and this paper describes Lyapunov exponent method in details. It is described as follows:

Suppose F is n-dimensional mapping of $R^n\to R^n$, generating n-dimensional discrete dynamical system:

$$x_{n+1}=F\left(x_n\right)$$

Regard an infinitesimal n-dimensional ball as the initial condition of the system, and as the spontaneous change in evolution, the ball will become oval. Arrange its entirety in order depending on the length of the ellipsoidal axis, so according to the length of the m-th axis, the increased rate of m-th Lyapunov exponent $P_m(n)$ can be defined as:

$$\sigma_m=\lim_{n\to\infty}\frac{1}{n}\ln\left[\frac{P_m(n)}{P_m(0)}\right],m=1,2,\cdots,n$$

Therefore, Lyapunov exponent is associated with the properties of trajectories contraction or expansion of phase space, namely it is determined by the former m exponents of Lyapunov exponent and by the long-term equilibrium rate of m-dimensional volume exponential growth defined by the former m axis.

CHAOTIC ANALYSIS OF RECTANGULAR THIN PLATE UNDER THE ACTION OF FORCE AND MAGNETIC COUPLING

Equation of Motion

Suppose clamped rectangular thin plate is placed in a transverse magnetic field $\vec{B}(0,0,B_z)$, and also it is influenced by load $\vec{P}(0,0,P_z)$ distributed in the same direction, as shown in Figure 1.

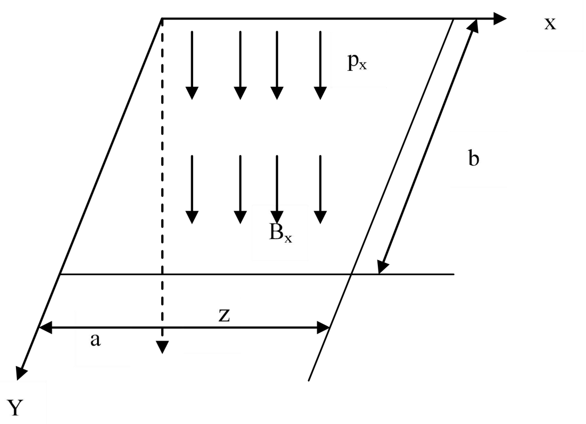


Figure 1. Schematic Diagram of the Model.

Without considering the impact of thermal effect, its magneto-elastic coupling vibration equation is:

$$w(x,y,t) = X(t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right) \tag{4}$$

Here, the boundary condition is: when x=0, a, $w=\frac{\partial w}{\partial x}=0$; when y=0, b, $w=\frac{\partial w}{\partial y}=0$.

Suppose the form of the solution is:

$$W(x,y,t) = \phi(t)w(x,y) \tag{5}$$

First-order vibration mode function is:

$$w(x,y) = \left[1-\cos\left(\frac{2\pi}{a}x\right)\right]\left[1-\cos\left(\frac{2\pi}{b}y\right)\right] \tag{6}$$

Mechanical load includes the following two forms:

$$P_z = p\cos(wt) \tag{7a}$$

$$P_z = p\left[1-\cos\left(\frac{2\pi}{a}x\right)\right]\left[1-\cos\left(\frac{2\pi}{b}y\right)\right]\cos(wt) \tag{7b}$$

Wherein, p is the mechanical load amplitude.

Combine the formula (5), (7) and formula (4), and then integrating the results, the following formula can be calculated:

$$\ddot{\phi} + \mu\dot{\phi} - \alpha\phi + \beta\phi^3 - F\cos(\omega t) = 0$$

Using dimensionless, let: $x = \sqrt{\frac{\beta}{\alpha}}\phi, \tau = \sqrt{\alpha t}, \omega_0 = \frac{\omega}{\sqrt{\alpha}}, \gamma = \frac{\mu}{\alpha}, f = \frac{\sqrt{\beta}}{\sqrt{\alpha}}F$, write τ into t, simplify the above formula into:

$$\ddot{x} + \gamma\dot{x} - x + x^3 = f\cos(\omega_0 t)$$

Numerical Analysis

Parameters of given plate: thickness of the plate is $h = 0.6\times10^{-3}m$, the density is $\rho = 2900kg/m^3$, the modulus of elasticity is E=72GPa, Poisson's ratio is v = 0.33, the conductivity is $\sigma = 3.6\times10^7(\Omega\bullet m)^{-1}$, side length takes a=0.4m, b=0.8m, $\omega_0 = 0.8$. Substitute these parameters into coupling vibration equations for numerical values with fourth-order Runge-Kutta method, and then obtain simulation results.

In numerical calculation, in order to ensure the dynamic response of the system is in a steady state, the displacement, external load, time, and other values are read so as to generate Lyapunov exponent diagram.

When the frequency ratio is $\omega_0 = 0.8$, electromagnetic field strength is $B_z = 1.5T$, by changing the mechanical load p (170-190 N/m^2), Lyapunov exponents diagram can be obtained, as shown in Figure 2. From this figure, it can be concluded that with the mechanical load p becomes larger, the system alternates between periodic and chaotic states. Figure 3 and Figure 4 show a Poincare sectional view of the system when mechanical load p changes. When p=172 N/m^2 , the system is in chaotic motion situation, with the aggravation of the mechanical load p, such as when p=176 N/m^2 , the system becomes periodic motion, and with load p continues to increase the system is in the chaotic motion again. In the chaotic state, it is also accompanied by some periodic windows, which is consistent with the typical characteristics of nonlinear chaotic motion.

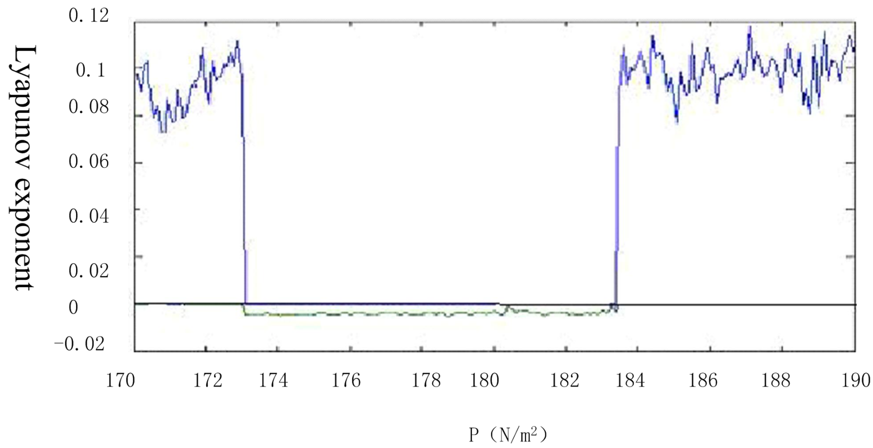


Figure 2. Lyapunov exponent.

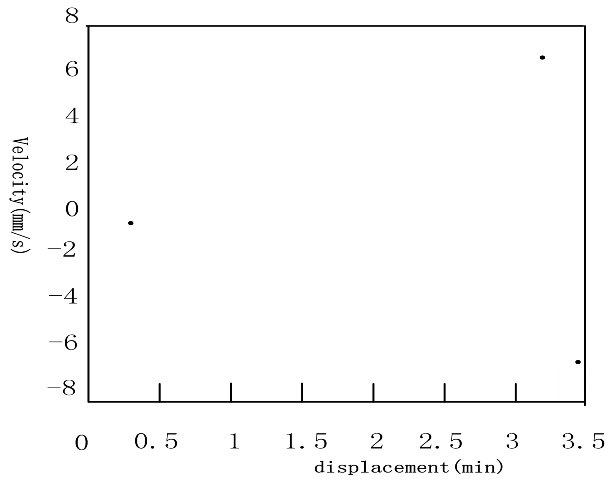


Figure 3. Sectional view when p = 172 N/m^2.
Figure 4. Sectional view when p=176 N/m^2.

CHAOTIC ANALYSIS OF RECTANGULAR THIN PLATE UNDER THE ACTION OF THERMAL, FORCE AND MAGNETIC COUPLING

On the basis of the second part, considering the impact of thermal effects, magneto-elastic coupling vibration equation of the plate will change correspondingly. When the elastic body is placed in a non-uniform temperature field $T(x,y,z)$, its inside will produce thermal stress, and at this time the stress-strain relation will use the following forms:

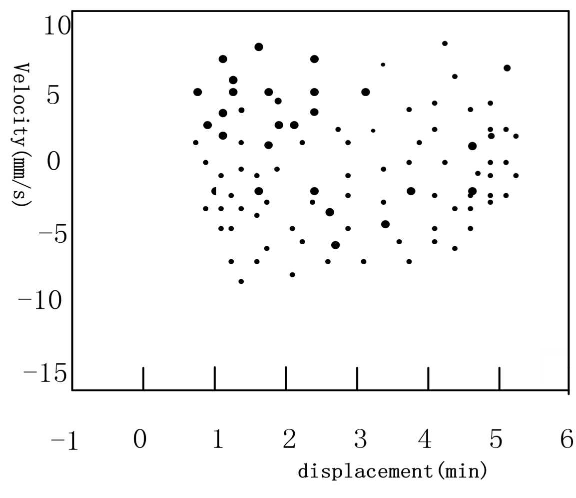
$$e_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right] + \alpha_T T, e_{xy} = \frac{1}{G}\tau_{xy}$$

$$e_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_z + \sigma_x\right)\right] + \alpha_T T, e_{yz} = \frac{1}{G}\tau_{yz}$$

$$e_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right] + \alpha_T T, e_{zx} = \frac{1}{G}\tau_{zx}$$

In the above formulas, $G = \frac{E}{2(1+v)}$ is shear modulus of the material, α_T is thermal expansion coefficient, T is temperature.

After considering the effect of temperature field, magnetic elastic vibration equation becomes:



$$D_M \nabla^4 w + \frac{D_N}{2}[3\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial y^2} + 3\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2 +$$
$$4\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2(1-v)\varepsilon_T \nabla^2 w] + \frac{\sigma h^3}{12}B_z^2 \frac{\partial(\nabla^2 w)}{\partial t} - \rho h \frac{\partial^2 w}{\partial t^2} + P^3 = 0 \tag{8}$$

In this formula, $D_N = \frac{Eh}{1-v^2}$ is tensile stiffness, $D_M = \frac{Eh^3}{12(1-v^2)}$ is Bending stiffness, and ε_T is integral eigenvalues of temperature field.

Equation of Motion

Under the condition of transverse magnetic field and transverse distributed load, supposing that the temperature T is a function of z and is independent of x and y, so the functional form is as following:

$$T(x,y,z) = T_0\left(1 + z + z^2\right) \tag{9}$$

In this formula, T_0 is the initial temperature. Thereby the temperature integral features values can be obtained:

$$\varepsilon_T = \frac{1}{h}\int_{-h/2}^{h/2} \alpha_T T(x,y,z)\,dz = \alpha_T T_0\left(1 + h^2/12\right) \tag{10}$$

In this formula, α_T is the thermal expansion coefficient of the material.

Combine the formula (5), (6), (7), (10) with the formula (8), and then use integration method and simplify it:

$$\ddot{\phi} + a_1\dot{\phi} - a_2\phi - F\cos(\omega t) = 0 \tag{11}$$

Using dimensionless, and let $x = \sqrt{\frac{a_3}{a_2}}\phi$, $\tau = \sqrt{a_2}t$, $\omega_0 = \frac{\omega}{\sqrt{a_2}}$, $\mu = \frac{a_1}{\sqrt{a_2}}$, and $f = \frac{\sqrt{a_3}}{a_2\sqrt{a_2}}F$, then simplified formula of formula(11)can be obtained

$$\ddot{x} + \mu\dot{x} - x + x^3 = f\cos(\omega_0\tau) \tag{12}$$

Numerical Analysis

Suppose thickness of the plate is $h = 1.5\times10^{-3}m$, the density is $\rho = 2780kg/\mathrm{m}^3$, the modulus of elasticity is E=71 GPa, Poisson's ratio is v=0. 33, the conductivity is $\sigma = 3.6\times10^7(\Omega\bullet m)^{-1}$, the thermal expansion coefficient is $\alpha_T = 26\times10^{-6}\left({}^oC\right)^{-1}$, the mechanical force is P=102 N/m^2, side length takes a=0.5 m, b=0.4m, and $\omega_0 = 0.8$, substitute these parameters into vibration equations for numerical values with fourth-order Runge-Kutta method, and then obtain the results.

In numerical analysis, in order to better analyze the chaotic motion of the thin plate, determined mechanical load is given in this paper, in time response calculated by integral, begin to read the data when the mechanical load repeated several times to ensure that the dynamic response of the system is in steady state, and then read the relevant data, so as to generate a corresponding Poincare sectional view and Lyapunov exponent diagram.

Under the condition of $\omega_0 = 0.8$, $B_z = 1.5T$ and P=102 N/m^2, Lyapunov exponents diagram is obtained by controlling the temperature (10 degrees to 60 degrees). Figure 5 it can be concluded that in the process of the temperature continues to raise, the curve transfers between periodic and chaotic states.

From Figures 6 and 7, it can be concluded that with the temperature continues rising, the system will be into chaotic state, followed by periodic status, along with the simulation process, two states occur alternately.

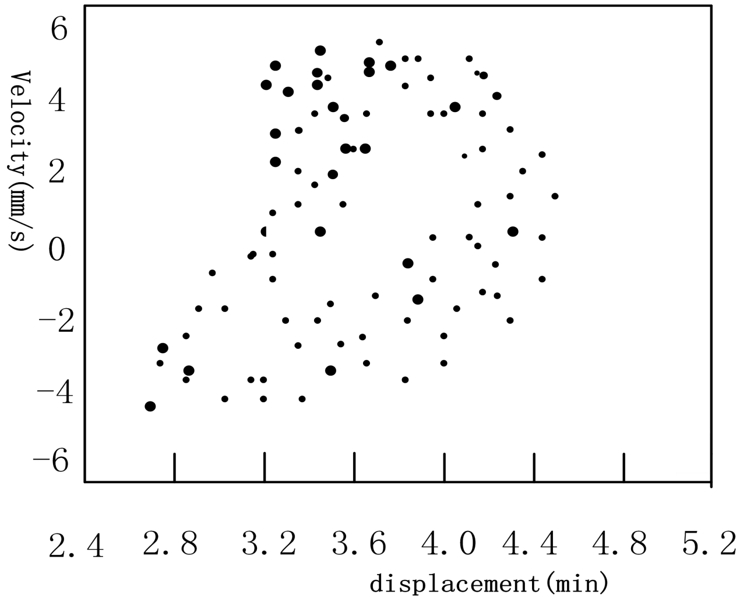


Figure 5. Lyapunov exponent.

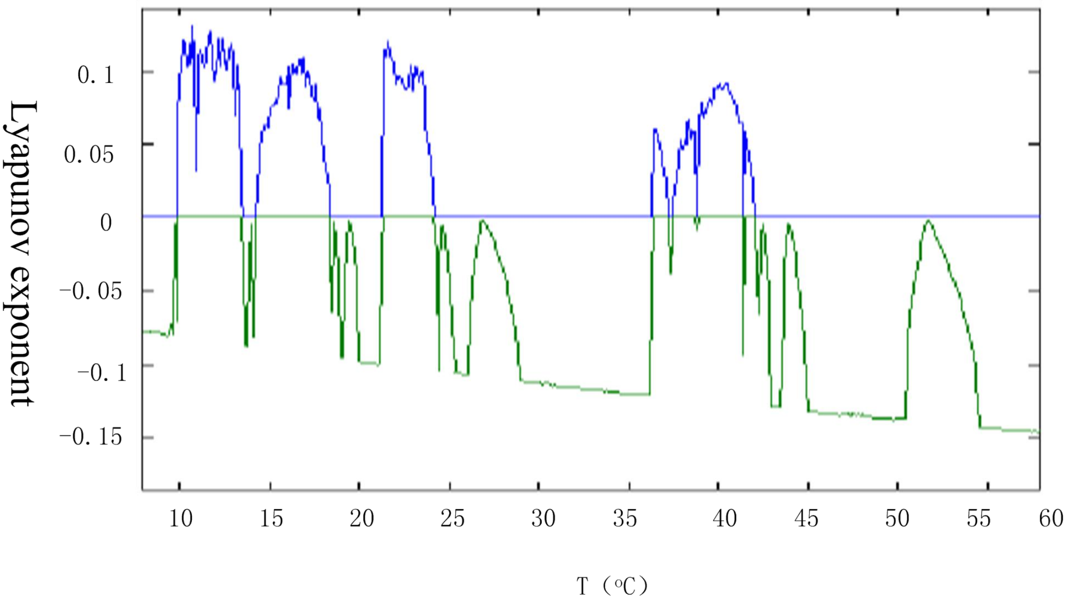


Figure 6. Sectional view when T=10.

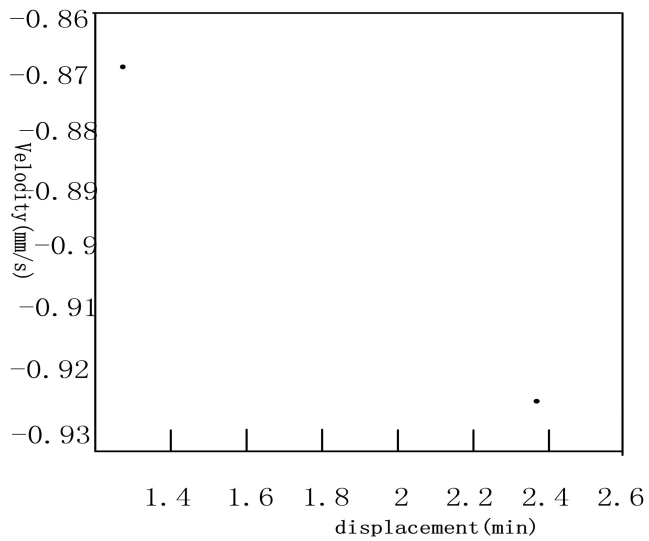


Figure 7. Sectional view when T=25.

CONCLUSIONS

Through simulation studies, this paper studies the rectangular plate and makes two conclusions:

(1) A magnetic vibration equation of rectangular thin plate is established, and its differential form is derived. Programming simulation study on vibration situation under the effect of force and magnetic coupling, and summing up a phenomenon: One of the external conditions of the system into the chaotic motion is by changing the external loads and the associated parameters to the control vibration characteristics of the system.

(2) In the condition of adding temperature field, the nonlinear magneto-elastic coupling vibration equation of rectangular thin plate is derived. The vibration equation of rectangular thin plate system affected by multi-environment coupling exhibits obvious nonlinear, its motion feature is quite complicated, and the state converses mutually between periodic and chaotic motion. By changing the temperature, mechanical load and associated parameters can make the system gradually in chaotic motion, or will control the occurrence of chaotic motion, so as to ensure the security in practical application.

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