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Research on Simulation of Mixed Wave

Y. F. Zan, D. F. Han*, & L. H. Yuan

College of Shipbuilding Engineering, Harbin Engineering University, Harbin 150001, China

*Email: duanfenghan@hrbeu.edu.cn

ABSTRACT: Based on double peak wave spectrum model, this paper studied three types of mixed waves of wind and swell including wind-dominated mixed wave, mixed wave of wind and swell in equal strength, and swell and swell dominated mixed wave, analyzed the spectrum morphological characteristics of the three types of mixed waves, established the random wave model, and simulated two-dimensional wave and three-dimensional wave, respectively. Moreover, the effect of water depth on waveform was researched, and the results showed that the wavelength was significantly reduced as the water depth was reduced to some extend. This paper concluded that the random wave model based on double peak wave spectrum can effectively simulate mixed wave with different ratios of wind and swell.

KEYWORDS: Double Peak Wave Spectrum; Three-dimensional Wave; Water Depth; Simulation.

INTRODUCTION

As the most significant external load for marine resource exploitation and utilization, the wave directly effects the movement of ships, ocean platform and other ocean structures. Simulation of wave not only plays a role as important application to fields of computer graphics, virtual reality, military simulation, development of games and marine monitoring, but also of practical significance to hydrodynamics, fluid mechanics and oceanography.

Su Yumin *et al.* [1] developed simulation system based on Gerstner ocean wave model, and performed numerical simulation of it using Pierson-Moscowitz ocean wave spectrum. In addition, they disccused 3D wave simulation method from the three aspects including establishment and optimization, texture mapping, and lighting effect of mesh model. Xu Jingbo *et al.* [2-3] selected ITTC dual parameter spectrum as harmonic frequency, which was applied with frequency division processing through equi-energy divided method, and thus obtained the gravity frequcenty of divided spectrum as harmonic frequency. Subsequently, the time domain of each harmonic was synthesized based on long peak wave model, resulting in the ocean wave simulation algorithm and thus obtaining simulation waveform in time-domain and space-domain. Shi Wei *et al.* [4-6] conducted numerical simulation of 3D random wave based on Pierson-Moscowitz ocean wave spectrum and obtained directional spectrum and 3D spatial model of 3D random wave. MITCHELL et al [7] established 3D wave using Fourier transform and GPU technology and rendered deep water wave. Løset T.K. [8] performed simulation research on real-time wave, conducting CPU calculation and GPU calculation, respectively. Results show that GPU calculation efficiency is much higher than CPU calculation efficiency under the condition of big ocean surface. Grindstad T.C. [9], using fast Fourier transform, conducted simulation on deep ocean wave based on JONSWAP spectrum.

Above researches focus on single peak wave spectrum, however since the wave phenomenon is caused by complex reasons and impacted by numerous factors, in most situations, it seldom appears wind wave alone, but appears mixed wave, of which the spectrum structure is double peak or multi-peak. Therefore, single peak wave spectrum can only better describe the form of mixed wave, in this paper, based on double peak wave spectrum, simuation research was conducted for 2D and 3D mixed wave, respectively, and waves with different morphologies were simulated, so that the morphologic characteristics of different types of mixed waves were compared.

2 2D WAVEFORM ANALYSIS

Double peak wave spectrum

In this paper, we conducted research of wave based on Ochi-Hubble [10-11] double peak wave spectrum, finding that the whole wave spectrum is composed by low frequency component and high frequency component, and wach component is dertermined by three parameters including significant wave height H_s , spectrum peak frequency ω_m , and shape parameter λ .

$$S(\omega)=\frac{1}{4}\sum_{j=1}^{2}\frac{\frac{4\lambda_j+1}{4}\left(\omega_m^4\right)^{\lambda_j}H_{s,j}^2}{\Gamma(\lambda_j)\omega^{4\lambda_j+1}}\exp\left[-\frac{4\lambda_j+1}{4}\left(\frac{\omega_{m,j}}{\omega}\right)^4\right] \tag{1}$$

Where, Γ is gamma function, $j=1,2$ represents low frequency component and high frequency component, respectively; $H_{s,1}$ and $H_{s,2}$ represents the significant wave height under low frequency and high frequency, respectively; $\omega_{m,1}$ and $\omega_{m,2}$ represents spectrum peak frequency under low frequency and high frequency, respectively; λ_1 and λ_2 represents shape parameter under low frequency and high frequency, respectively

Spectrum peak period can be expressed as:

$$T_{m,j}=\frac{2\pi}{\omega_{m,j}} \qquad j=1,2 \tag{2}$$

The significant wave height of mixed wave can be obtained based on energy supposition theory according to Rice theory:

$$H_s=\sqrt{H_{s,1}^2+H_{s,2}^2} \tag{3}$$

Therefore, the wave energy of double peak wave spectrum equals the superposed energy of low frequency component and high frequency component:

$$E=\frac{1}{4}\rho g\left(H_{s,1}^2+H_{s,2}^2\right) \tag{4}$$

Although double peak spectrum pattern varies a lot with the parameters, the classic double peak spectrum patterns can be categorized according to the spectrum energy ratio of low-medium frequency component and high frequency component into three major types including wind-dominated mixed wave, mixed wave of wind and swell in equal strength, and swell dominated mixed wave. Table 1 prensents the parameter definitions of the three classic mixed wave spectrums.

Table 1. Parameters of mixed wave spectrum.

Type	Significant wave height of low frequency component $H_{s,1}$	Spectrum peak period of low grequency component $T_{p,1}$	Shape parameter of low frequency component λ_1	Significant wave height of high frequency component $H_{s,2}$	Spectrum peak period of high grequency component $T_{p,2}$	Shape parameter of high frequency component λ_2
1	1.5	20	3	0.8	6	6
2	0.8	11	2.1	1	6	2.5
3	0.8	12	3	1.5	6	6

Wind-dominated mixed wave

Based on the parameters of type 1 of Table 1. it can obtain the wave spectrum form shown in Figure 1, of which the wave spectrum energy concentrate at high frequency peak with middle frequency of 1.046 *rad/s*, while only few energy is distributed at low frequency peak with frequency of 0.523 *rad/s*.

Mixed wave of wind and swell in equal strength

According to parameters of type 2 of Table 1, it can obtain the wave spectrum form shown in Figure 2, of which most energy is approximately equally distributed at both high frequency peak (1.040 *rad/s*) and low frequency peak (0.571 *rad/s*).

Swell-dominated mixed wave

According to parameters of type 3 of Table 1, it can obtain the wave spectrum form shown in Figure 3, of which most wave spectrum energy concentrate at low frequency peak with frequency of 0.313 *rad/s*, while relatively less energy concentrate at high frequency peak with frequency of 1.046 *rad/s*.

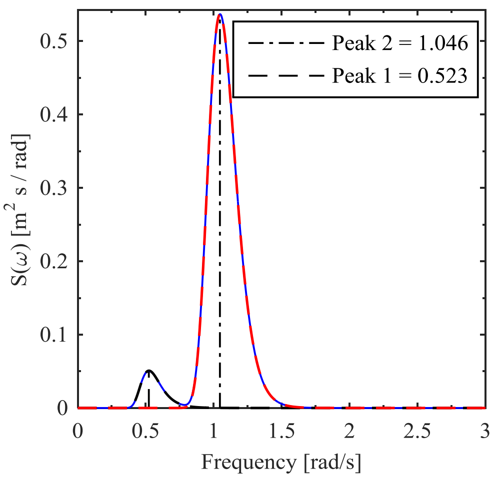

Figure 1. Double peak wave spectrum form of wind-dominated mixed wave.

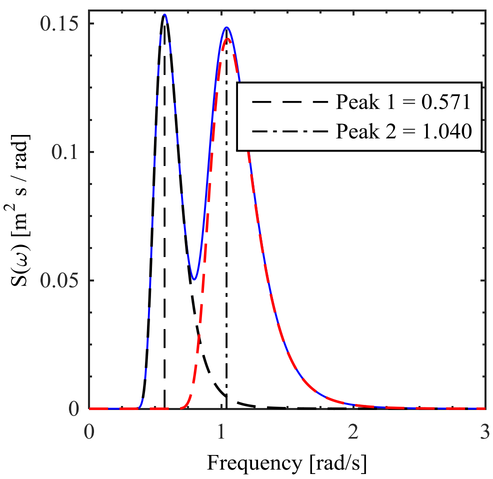

Figure 2. Double peak wave spectrum form of mixed wave of wind and swell in equal strength.

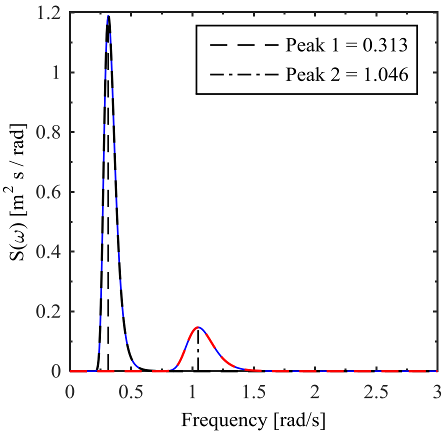

Figure 3. Double peak wave spectrum form of swell-dominated mixed wave.

Establishment of wave form model

2D random wave can be regarded as the superposition of waves with different frequencies, phases and heights, therefore in time-domain and space-domain, the wave height at position of x at t can be expressed as:

$$z(x,t) = z_0 + \sum_{j=1}^{m} A_j \cos\left(k_j x - \omega_j t + \varphi_j\right) \tag{5}$$

Where, z_0 represents the height of sea level, φ_j represents the random phase, and A_j represents the amplitudes of waves of different frequencies, which can be expressed as:

$$A_j = \sqrt{2S\left(\omega_j\right)\Delta\omega} \tag{6}$$

where, $S\left(\omega_j\right)$ is the wave spectrum value at ω_j, k is wave number, therefore the function of wave circular frequency and water depth h under the condition of finite water depth can be expressed as:

$$\omega^2 = gk \tanh hk \tag{7}$$

In the equation, g is the acceleration of gravity

Wave number k at infinite water depth is:

$$k = \frac{\omega^2}{g} \tag{8}$$

Simulation of waveform

Simulations are performed for three classic mixed waves, respectively. It can be seen from Figures 4-6, which show the time-domain curves of mixed waves within 200 s at $x = 0$. Figure 4 show the time-domain curve of wind-dominated mixed wave, which is of steep peak and short period; Figure 5 shows the mixed wave of wind and swell in equal strength, of which the peak probably tends to be flat and the period tends to be longer; Figure 6 shows the swell-dominated mixed wave, of which the peak point significantly become less, while the period becomes longer.

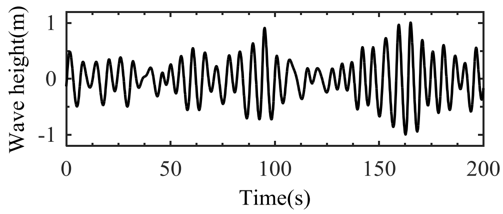

Figure 4. Time-domain curve of wind-dominated mixed wave.

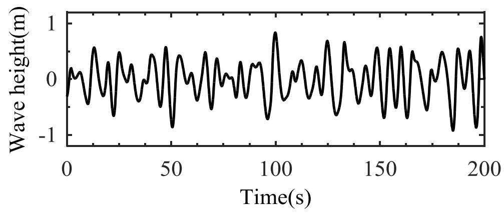

Figure 5. Time-domain curve of mixed wave of wind and swell in equal strength.

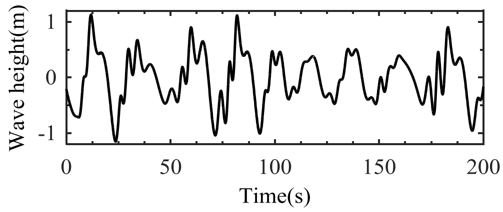

Figure 6. Time-domain curve of swell-dominated mixed wave.

Effect of water depth on wave

Since water depth is a function of waveform parameters, hereby the effect of water depth on wave is studied by taking wind-dominated mixed wave as example.

Figures 7-10 show the waveform of wind-dominated mixed wave under different water depths at simulation t =1000 *s*. Figure 7 and Figure 8 show the waveform at infinite water depth and depth of 100 *m*, respectively, which are of regular pattern and relatively flat peak. It can be seen that in Table 2 that the 1/3 of significant wave heights are 1.681 *m* and 1.612 *m*, respectively; while the spectrum peak wavelength are 44.221 *m* and 48.211 *m*, which is relatively close to each other. As shown in Figure 9, it can be found that when water depth decreases to 5 m, the peak becomes sharp, the significant wave height is 1.716 *m* and the spectrum peak wavelength decreases to 32.989 *m*. While when water depth changes into 2 m, the waveform becomes significantly steep with significant wave height of 1.517 *m* and spectrum peak wavelength of 25.928 *m*.

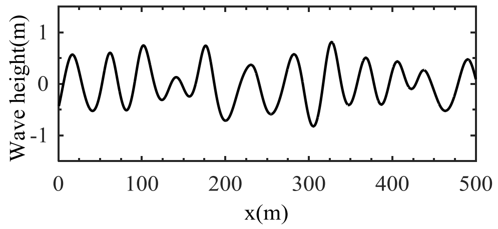

Figure 7. Waveform at infinity water depth.

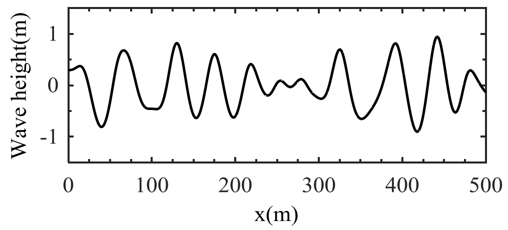

Figure 8. Waveform at water depth of 100 *m*.

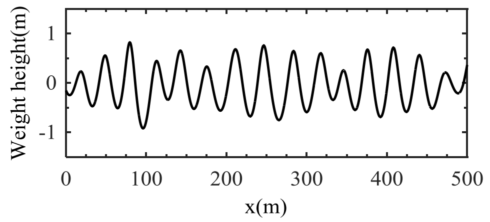

Figure 9. Waveform at water depth of 5 *m*.

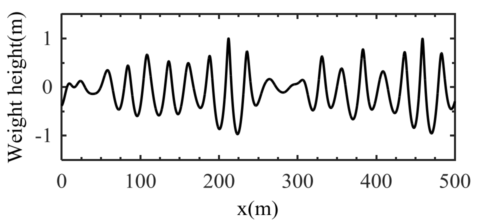

Figure 10. Waveform at water depth of 2 *m*.

Table 2. Wave height and wavelength at different water depths.

Water depth (*m*)	1/3 of significant wave height $H_{1/3}$	Spectrum peak wavelength $\lambda(m)$
Infinity	1.681	44.221
100	1.612	48.211
5	1.716	32.989
2	1.517	25.928

SIMULATION OF 3D WAVE

Directional spectrum

Since wave spectrum is one-dimensional, while actual ocean wave is three-dimensional, of which the energy is distributed within wide frequencies and directions. To characterize that ocean wave is related to directions, ITTC suggests employ directional spectrum function, which can be expressed as:

$$D(\theta)=\frac{2}{\pi}\cos^2\left(\frac{\theta}{2}\right) \quad |\theta|\le\frac{\pi}{2} \tag{9}$$

If the frequency distribution and direction distribution of wave energe can be regarded linear and unrelated with each other, then the energy distribution of ocean wave can be expressed by the product of spectrum function and directional function:

$$S(\omega,\theta)=S(\omega)D(\theta) \tag{10}$$

Figures 11-13 show the directional spectrum form of wind-dominated mixed wave, mixed wave of wind and swell in equal strength, and swell-dominated mixed wave. Similar to the energy concentration area of wave spectrum, it can be seen that the wave energy is extended in each direction.

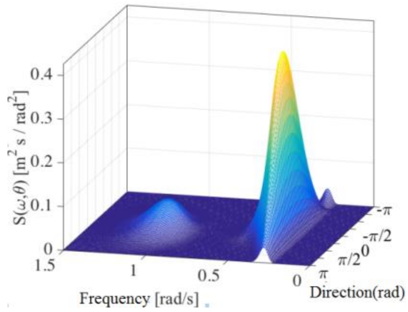

Figure 11. Directional spectrum form of wind-dominated mixed wave.

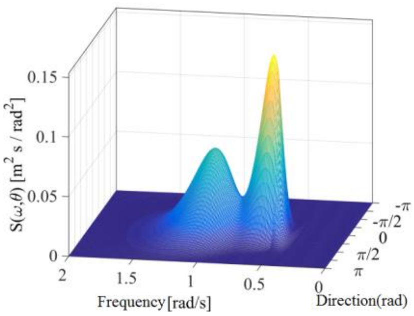

Figure 12. Directional spectrum form of mixed wave of wind and swell in equal strength.

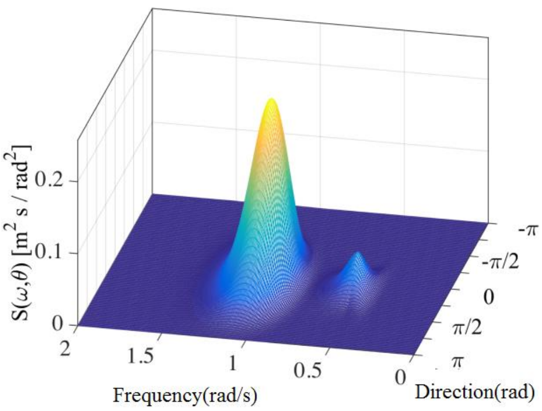

Figure 13. Directional spectrum form of swell-dominated mixed wave.

Establishment of wave

In the research of 2D random wave, it only considers the wave change in *x*-*z* plane within time domain, however if it comes to the simulation of 3D plane, the random wave will be the superposed wave with different angles between *x*-*y* plane and *x* axis, which means the wave is the superposition of waves with different frequencies, wave heights, phases in different propagation directions:

$$z(x,y,t) = z_0 + \sum_{i=1}^{n}\sum_{j=1}^{m} A_{i,j}\cos(k_i x\cos\theta_j + k_i x\sin\theta_j - \omega_i t + \varphi) \tag{11}$$

Where k_i is wave number, φ is random phase. $A_{i,j}$ is the amplitudes of waves with different frequencies, which can be expressed as:

$$A_{i,j} = \sqrt{2S(\omega,\theta)\Delta\omega\Delta\theta} \tag{12}$$

Simulation of 3D wave

Figures 14-16 show 3D waveforms of three classic mixed waves at infinite water depth and main propagation direction of $30°$ Wave spectrum parameters are shown in Table 1. The three figures intuitively demonstrate that the wave propagation directions are uniformly $30°$, wherein Figure 14 show the waveform of wind-dominated mixed wave, it can be seen the wave peak is steep which is similar to that of 2D wave; Figure 15 show the waveform of mixed wave of wind and swell in equal strength, and wave peak is relatively flat; Figure 16 show the waveform of swell-dominated mixed wave, which is of larger wavelength and flat peak.

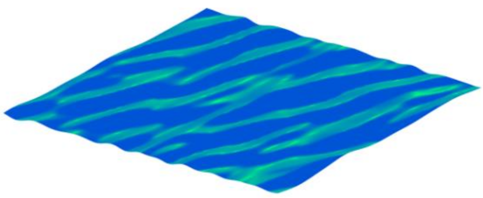

Figure 14. Waveform of wind-dominated mixed wave.

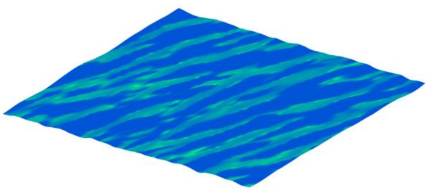

Figure 15. Waveform of mixed wave of wind and swell in equal strength.

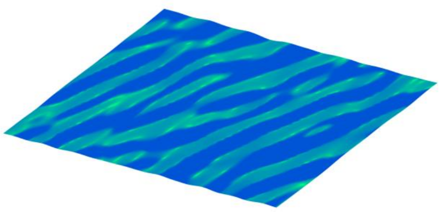

Figure 16. Waveform of swell-dominated mixed wave.

CONCLUSIONS

This paper presented simulation of complex mixed type waves, and analyzed the 2D and 3D waves of three classic mixed waves such as wind-dominated mixed wave, mixed wave of wind and swell in equal strength, and swell-dominated wave. Through research it can find that when wind is in dominate position, the wave peak is steep and wave period is short; for mixed wave of wind and swell in equal strength, the peak probably tends to be flat and wave period tends to be longer; With regard to swell-dominated mixed wave, the wave peak points significantly become less and the period becomes longer; In addition, water depth plays a significant role to the waveform. When at shallow water depth, the waveform significantly becomes steep as well as wavelength becoming shorter.

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