

# Over-modulation Algorithm of PMSM Control System under Flux-weakening Operation

T. T. Liu\*, G. J. Chen, Z. H. Li, & Q. H. Meng

School of Mechanical Engineering, Hangzhou Dianzi University, Hangzhou 310018, China

\*Email: liutingting817@163.com

**ABSTRACT:** In this paper, an over-modulation algorithm based on space vector pulse width modulation (SVPWM) is proposed. For purpose of improving the utilization ratio of inverter's DC bus voltage, modulation region is judged by the action time of zero voltage vector. This over-modulation algorithm is applied in flux-weakening operation region of permanent magnet synchronous motor (PMSM) control system, which extends the operation region of flux-weakening control and increases the output torque and power of motor. The control strategy is verified by simulation experiments of PMSM control system, with over-modulation algorithm, the running effect of PMSM under flux-weakening operation is discussed.

**KEYWORDS:** PMSM; SVPWM; Over-modulation; Flux weakening operation.

## INTRODUCTION

Permanent magnet synchronous motor (PMSM) with its advantages of high power density, high efficiency, and maintainability, has gained a wide range of applications in aerospace, CNC systems, electric vehicles and other fields [1, 2]. With accelerated development of modern industry, requirements of PMSM control system are no longer confined to rated operating conditions, but gradually extended to constant power operation region. Over-modulation strategy and flux-weakening control is an effective way to achieve the above requirements [3].

Rotor magnetic field of PMSM is generated by the permanent magnet, and it cannot be directly weakened. The flux-weakening realization is through  $d$  axis armature reaction, so as to achieve equivalent effect of weakening the magnetic field. Six-step voltage method can maximize the use of DC bus voltage, but the algorithm is more sensitive to motor parameters and load changes [4]; Control method with current regulator has good stability, but it is difficult to choose optimal  $d$  axis voltage when with variable speed and torque [5]; A continuous over modulation method is proposed in [6], according to different modulation coefficient, the over-modulation region is divided into two modes: in mode I, vector amplitude is changed, and in mode II, amplitude and phase angle changed simultaneously; The two modes are combined into a single one, which is easy to be processed by computer [7]; The precise relationship between fundamental voltage and modulation coefficient is studied in [8], reference angle and holding angle in different output voltage are calculated, the linear control can be obtained in whole over-modulation region, but the algorithm is complex; A  $dq$  axis currents feed forward compensation method is proposed in [9], the difference between actual speed and rated speed determines the starting point of flux-weakening control, increase  $d$ -axis current to widen the speed range of flux-weakening control; A flux-weakening algorithm based on voltage loop feedback compensation is proposed, the difference between DC bus voltage and output voltage of PI controller as the input of flux-weakening controller, but this method has large system harmonics [10, 11].

Based on the research progress above, it can be seen that over-modulation control is an effective way to extend the flux-weakening operating region of PMSM, the bottleneck of flux-weakening control is low utilization ratio of DC bus voltage. In this paper, aiming at improving the voltage utilization ratio of DC bus, combine the over-modulation strategy and the flux-weakening strategy, an over-modulation algorithm is designed, with the action time of zero voltage vector as the judgment of over-modulation starting point, which increases the output torque and power of PMSM and extends flux-weakening operation region.

## FLUX-WEAKENING CONTROL PRINCIPLE

In PMSM vector control system, to ensure that the output voltage not exceeding inverter’s maximum value, current PI regulator generally contains saturated module. With the increase of stator’s terminal voltage, the output value of PI regulator approaches saturation gradually, the regulating ability of PI regulator decreased and the motor current is out of control. To ensure the normal operation, it is necessary to add a certain module to prevent current PI regulator failure. The flux-weakening control principle of PMSM: When the motor stator current remain constant, by increasing the negative component at  $d$ -axis, while reducing the positive component in  $q$ -axis, that is, the stator current vector in  $dq$  coordinate system is turned to a certain angle  $\beta$ , as shown in Figure 1. The air gap flux is reduced by the magnetic effect of  $d$  axis current component, motor terminal voltage is not exceeding the limit value, and current PI regulator is prevented from saturation [12].

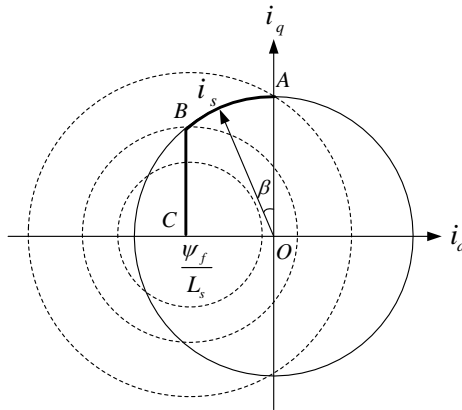


Figure 1. Current vector in flux-weakening operation.

In flux-weakening operation, through voltage close loop controller to adjust the current component at  $d$  axis to ensure the smooth transition between constant torque and constant power area. Based on this, block diagram of PMSM control system with flux-weakening module is shown in Figure 2.

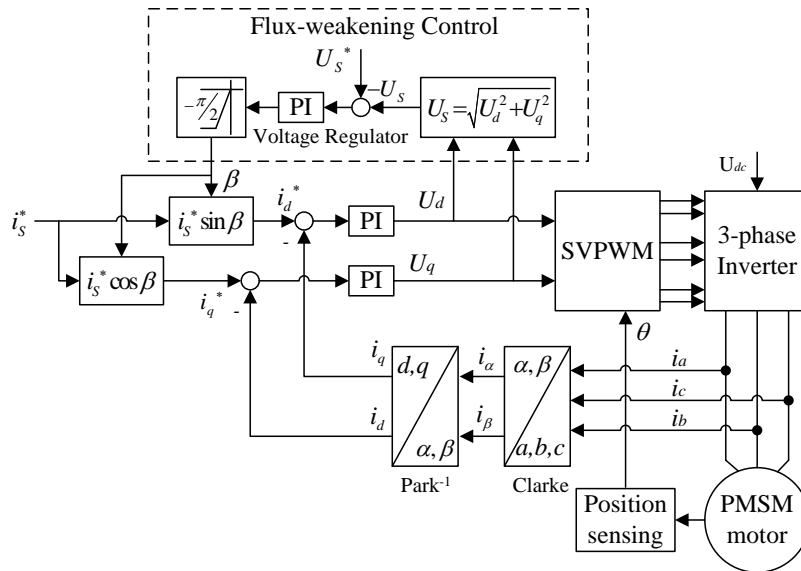


Figure 2. Block diagram of PMSM flux-weakening control system.

In PMSM control system shown in Figure 2, when terminal voltage is low, voltage PI regulator is in saturation state, the output value  $\beta$  is zero, which is equivalent to  $i_d = 0$  control, motor running in constant torque region; When terminal voltage reaches to limit,  $U_s^* > U_s$ , voltage PI regulator exit from saturation, through voltage PI regulator to control the angle  $\beta$  between stator current vector and  $q$  axis, which produce  $d$  axis current magnetic component, PMSM enter into constant power region (flux-weakening operating region). Inverter output voltage is adjusted by SVPWM module based on voltage vector amplitude  $U_s$  and position angle  $\theta$  of rotor magnetic field, the over-

modulation algorithm is added in this module, which increases output voltage of inverter and extends the flux-weakening operation region.

### OVER-MODULATION BASED ON SVPWM

The principle of SVPWM is time average equivalence principle, in a PWM switching period, combine adjacent basic voltage vectors to make sure the average voltage of inverter is equal to the reference one. For one output voltage vector  $\vec{U}$ , the adjacent basic voltage vectors are  $\vec{U}_i$  and  $\vec{U}_{i+1}$ ,  $\vec{U}$  can be get through a combination of  $\vec{U}_i$  and  $\vec{U}_{i+1}$ .  $T_i$  and  $T_{i+1}$  are reaction time of  $\vec{U}_i$  and  $\vec{U}_{i+1}$ , reaction time of zero vector is  $T_0 = T_s - T_i - T_{i+1}$ ,  $T_s$  is switching period of inverter. In Sinusoidal modulation, reaction time of each vector can be written as follows [13]:

$$\begin{cases} T_i = \frac{\sqrt{3}T_s |\vec{U}_r| \sin(\frac{\pi}{3} - \theta)}{U_{dc}} \\ T_{i+1} = \frac{\sqrt{3}T_s |\vec{U}_r| \sin \theta}{U_{dc}} \\ T_0 = T_s - T_i - T_{i+1} \end{cases} \quad (1)$$

Where,  $\vec{U}_r$  is reference voltage vector,  $\theta$  is angle between  $\vec{U}_r$  and  $\vec{U}_i$ ,  $U_{dc}$  is DC bus voltage.

In sinusoidal modulation, output voltage vector is rotating in a circular orbit with a uniform velocity, the output of inverter is three-phase sinusoidal voltage. According to vector synthesis principle, any voltage vector must be located in a regular hexagon,  $\vec{U}_1 \sim \vec{U}_6$  are vertexes of this regular hexagon. The maximum amplitude of output voltage vector  $\vec{U}$  is  $\frac{U_{dc}}{\sqrt{3}}$ , the maximum effective value is  $0.707 U_{dc}$ . Without over-modulation, trajectory of reference voltage vector  $\vec{U}_r$  is a circular,  $\vec{U}$  is equal to  $\vec{U}_r$ . With over-modulation,  $\vec{U}_r$  is still a circular, but inverter can not provide such a large voltage vector with a circular trajectory, at this time  $\vec{U}$  and  $\vec{U}_r$  are different [14].

Define modulation ratio:  $M = \frac{|\vec{U}_r|}{\frac{2}{\pi} U_{dc}}$ , when the magnitude of a reference voltage vector is inscribed circle within regular hexagon, which is  $|\vec{U}_r| = \frac{U_{dc}}{\sqrt{3}}$ , the magnitude of output voltage vector is equal to the radius of inscribed circle, the output voltage of inverter reaches maximum, at this point  $M = 0.906$ . When  $|\vec{U}_r| = \frac{2U_{dc}}{\pi}$ ,  $M = 1$ . Inverter voltage utilization  $m$  is defined as: the ratio of valid values of inverter line voltage and DC bus voltage. We can derive that in sinusoidal modulation:  $m = \sqrt{\frac{3}{2}} \cdot \frac{|\vec{U}_r|}{U_{dc}} = \frac{\sqrt{6}}{\pi} M$ .

When  $M > 0.906$ , that is SVPWM over-modulation. In over-modulation mode, magnitude of reference voltage vector  $\vec{U}_r$  exceeds the radius of inscribed circle, the actual output voltage vector  $\vec{U}$  is different with  $\vec{U}_r$ , we must select a output voltage vector to replace the original one, amplitude and phase changes as small as possible. According to the modulation ratio, region of SVPWM can be divided into three intervals [3].

#### Linearization Region ( $0 \leq M \leq 0.906$ )

In linearization region,  $\vec{U}_r$  is within regular hexagon,  $\vec{U}$  is synthesized by two adjacent voltage vectors, the acting time is  $T_i, T_{i+1}$ ,  $T_i, T_{i+1}$  can be obtained by formula (1). When  $|\vec{U}_r| = \frac{U_{dc}}{\sqrt{3}}$ , the tracks of  $\vec{U}$  slides along a regular hexagon inscribed circle,  $\vec{U}_r = \vec{U}$ .

Over-modulation Region I ( $0.906 \leq M \leq 0.952$ )

The inscribed circle of regular hexagon is defined  $a$  and circum-circle is defined  $b$ . As is shown in Figure 3, when  $\bar{U}_{r1}$  is between  $a$  and  $b$ , that is over-modulation region I. When  $\bar{U}_{r1}$  exceeds the regular hexagon, reduce amplitude of  $\bar{U}_{r1}$  to make the adjusted voltage vector on the edge of regular hexagon, the adjusted voltage vector is defined  $\bar{U}_{p1}$ . For one sector, track of  $\bar{U}_{p1}$  is GCDH. When  $\bar{U}_{r1}$  located on the edge of hexagon, which is in OC position, angle between  $\bar{U}_{r1}$  and  $\bar{U}_i$  (or  $\bar{U}_{i+1}$ ) is  $\alpha_1$ .

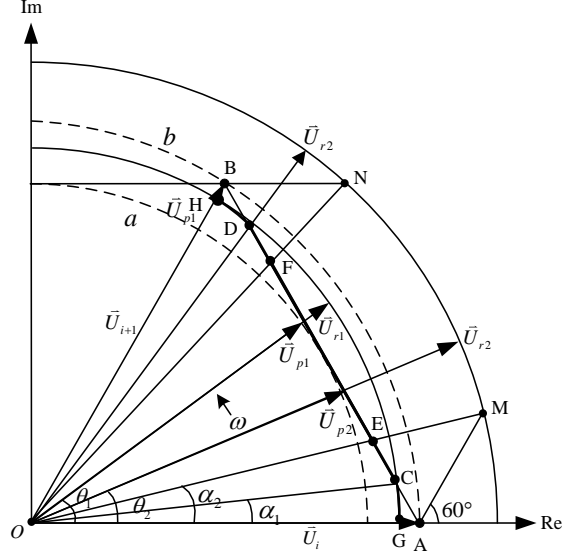


Figure 3. Vectors in over-modulation algorithm.

In over-modulation region I, it can be calculated by Fourier series, expression equation of  $|\bar{U}_r|$  and  $\alpha_1$  is as follows [12]:

$$F(\alpha_1) = \frac{2\sqrt{3} \cdot U_{dc}}{\pi} \cdot \left[ \frac{\alpha_1}{\sin\left(\frac{\pi}{3} + \alpha_1\right)} + \frac{1}{2} \cdot \ln \frac{1 + \sin\left(\frac{\pi}{6} - \alpha_1\right)}{1 - \sin\left(\frac{\pi}{6} - \alpha_1\right)} \right] = |\bar{U}_r| \tag{2}$$

Equation of  $M$  and  $\alpha_1$  can be obtained:

$$M = \sqrt{3} \cdot \left[ \frac{\alpha_1}{\sin\left(\frac{\pi}{3} + \alpha_1\right)} + \frac{1}{2} \cdot \ln \frac{1 + \sin\left(\frac{\pi}{6} - \alpha_1\right)}{1 - \sin\left(\frac{\pi}{6} - \alpha_1\right)} \right], \quad 0 \leq \alpha_1 \leq \frac{\pi}{6} \tag{3}$$

When  $\alpha_1 = \frac{\pi}{6}$ ,  $M = 0.906$ , this is the critical point transiting from sinusoidal modulation to over-modulation. When

$\alpha_1 = 0$ ,  $M = \frac{\sqrt{3}}{2} \ln 3 = 0.952$ ,  $M$  reaches maximum.

In over-modulation region I, according to (1), the calculated  $T_0$  must be negative, makes  $T_0 = 0$ , reduce values of  $T_i, T_{i+1}$  at the same time. In order to maintain the phase constant, reduce  $T_i$  and  $T_{i+1}$  in proportion to obtain the new  $T_i$  and  $T_{i+1}$ , the equation is as follows:

$$\begin{cases} T_i' = T_s \cdot \frac{T_i}{T_i + T_{i+1}} \\ T_{i+1}' = T_s \cdot \frac{T_{i+1}}{T_i + T_{i+1}} \end{cases} \quad (4)$$

Then we get the adjusted acting time of each basic voltage vectors, the over-modulation algorithm in region I is achieved.

Over-modulation Region II ( $0.952 \leq M \leq 1$ )

When reference voltage vector  $\bar{U}_{r2}$  located beyond the hexagon circum-circle  $b$ , that is over-modulation region II. The calculated  $T_0$  is negative, when  $\bar{U}_{r2}$  located beyond AMNB, if  $T_i > T_s$ , make  $T_i' = T_s, T_{i+1}' = 0$ ; if  $T_{i+1} > T_s$ , make  $T_{i+1}' = T_s, T_i' = 0$ ; when  $\bar{U}_{r2}$  located within AMNB,  $T_i'$  and  $T_{i+1}'$  are calculated according to equation (4), the adjusted voltage vector locates in EF. When  $\bar{U}_{r2}$  located on the edge of hexagon, which is in OM position, angle between  $\bar{U}_{r2}$  and  $\bar{U}_i$  (or  $\bar{U}_{i+1}$ ) is  $\alpha_2$ . When amplitude of  $\bar{U}_{r2}$  reach the maximum  $|\bar{U}_{r2}| = \frac{4U_{dc}}{3\sqrt{3}}$ , the output of inverter is square wave.

According to Fourier analysis, in over-modulation region II, expression equation of  $|\bar{U}_r|$  and  $\alpha_2$  is as follows:

$$F(\alpha_2) = \frac{U_{dc}}{\sqrt{3} \cdot \pi} \cdot \left[ 4\sqrt{3} \cdot \sin \alpha_2 + 3 \cdot \ln \frac{1 + \sin\left(\frac{\pi}{6} - \alpha_2\right)}{1 - \sin\left(\frac{\pi}{6} - \alpha_2\right)} \right] = |\bar{U}_r| \quad (5)$$

Equation of  $M$  and  $\alpha_2$  can be obtained:

$$M = 2 \sin \alpha_2 + \frac{\sqrt{3}}{2} \cdot \ln \frac{1 + \sin\left(\frac{\pi}{6} - \alpha_2\right)}{1 - \sin\left(\frac{\pi}{6} - \alpha_2\right)}, \quad 0 \leq \alpha_2 \leq \frac{\pi}{6} \quad (6)$$

Based on the above analysis, the SVPWM over-modulation control strategy can be concluded, each modulation region shown in Figure 3: SVPWM strategy can be divided into three intervals, when  $\bar{U}_r$  located on circle  $a$ ,  $M = 0.906$ , effective value of output line voltage of inverter is  $0.707U_{dc}$ ; when on circle  $b$ ,  $M = \frac{\pi}{3} = 1.05$ , effective value of output line voltage of inverter is  $0.742U_{dc}$ ; when  $|\bar{U}_{r2}| = \frac{4U_{dc}}{3\sqrt{3}}$ ,  $M = 1.21$ , effective value of output line voltage of inverter is  $0.779U_{dc}$ , the output of inverter is square wave. Obviously in the over-modulation region, the inverter output voltage and modulation ratio  $M$  is nonlinear.

In over-modulation region I:

$$\frac{|\bar{U}_r|}{\sin \frac{\pi}{3}} = \frac{\frac{2}{3}U_{dc}}{\sin\left(\frac{\pi}{3} + \alpha_1\right)}, \quad 0 \leq \alpha_1 \leq \frac{\pi}{6} \quad (7)$$

According to equation (7), it can be obtained that:

$$M = \frac{\pi}{2\sqrt{3} \sin\left(\frac{\pi}{3} + \alpha_1\right)}, \quad 0 \leq \alpha_1 \leq \frac{\pi}{6} \quad (8)$$

Expression equation of  $|\bar{U}|$  and  $\alpha_1$  is as follows:

$$\frac{|\bar{U}|}{\frac{2}{\pi} U_{dc}} = \sqrt{3} \cdot \left[ \frac{\alpha_r}{\sin\left(\frac{\pi}{3} + \alpha_r\right)} + \frac{1}{2} \cdot \ln \frac{1 + \sin\left(\frac{\pi}{6} - \alpha_r\right)}{1 - \sin\left(\frac{\pi}{6} - \alpha_r\right)} \right], \quad 0 \leq \alpha_1 \leq \frac{\pi}{6} \quad (9)$$

$\alpha_1$  as the intermediate variable, equation of  $|\bar{U}|$  and  $M$  is:

$$\frac{|\bar{U}|}{\frac{2}{\pi} U_{dc}} = \sqrt{3} \cdot \left[ M \cdot \left( \sin^{-1} \frac{1}{M} - \frac{\pi}{3} \right) + \frac{1}{2} \cdot \ln \frac{M + \sqrt{M^2 - 1}}{M - \sqrt{M^2 - 1}} \right], \quad \frac{\pi}{2\sqrt{3}} \leq M \leq \frac{\pi}{3} \quad (10)$$

In over-modulation region II, expression equation of  $|\bar{U}|$ ,  $M$  and  $\alpha_1$  is as follows:

$$M = \frac{\pi}{2\sqrt{3} \sin\left(\frac{\pi}{3} - \alpha_2\right)}, \quad 0 \leq \alpha_2 \leq \frac{\pi}{6} \quad (11)$$

$$\frac{|\bar{U}|}{\frac{2}{\pi} U_{dc}} = 2 \sin \alpha_h + \frac{\sqrt{3}}{2} \cdot \ln \frac{1 + \sin\left(\frac{\pi}{6} - \alpha_h\right)}{1 - \sin\left(\frac{\pi}{6} - \alpha_h\right)}, \quad 0 \leq \alpha_2 \leq \frac{\pi}{6} \quad (12)$$

Equation of  $|\bar{U}|$  and  $M$  is:

$$\frac{|\bar{U}|}{\frac{2}{\pi} U_{dc}} = 2 \sin \left( \frac{\pi}{3} - \sin^{-1} \frac{1}{M} \right) + \frac{\sqrt{3}}{2} \cdot \ln \frac{M + \frac{\sqrt{3}}{2} - \frac{1}{2} \sqrt{M^2 - 1}}{M - \frac{\sqrt{3}}{2} + \frac{1}{2} \sqrt{M^2 - 1}}, \quad \frac{\pi}{3} \leq M \leq \frac{\pi}{\sqrt{3}} \quad (13)$$

Analyze equation (10) and (13), it can be obtained that: with the increase of modulation ratio  $M$ , the nonlinear relationship between  $|\bar{U}|$  and  $M$  becomes more serious, the nonlinear influence caused by over-modulation is the decrease of inverter output voltage. For PMSM control system, if  $M$  is small (over-modulation region I), the decrease of PI parameters has no effect on steady-state performance of PMSM; if  $M$  increases (over-modulation region II), with the excessive decrease of PI parameters, performance of PI regulator declines seriously, which resulting in instability and oscillation of stator currents. Therefore, determine modulation ratio according to equations (10) and (13) to get linearized control of inverter output voltage.

## SIMULATION EXPERIMENTS OF FLUX-WEAKENING OPERATION

A simulation model with flux-weakening and over-modulation control strategy is established under MATLAB/Simulink environment, to verify the correctness of over-modulation algorithm and flux-weakening control. PMSM parameters used in simulation experiments: rated power  $P=2.5\text{kW}$ ; stator resistance  $R_s=2.875\ \Omega$ ; inductances in  $dq$  axis:  $L_d=0.0085\ \text{H}$ ,  $L_q=0.0085\ \text{H}$ ; moment of inertia  $J=0.0008\ \text{kg}\cdot\text{m}^2$ ; DC bus voltage  $U_{dc} = 100\text{V}$ . The stator current value is given, set different utilization ratio of DC bus voltage  $m$ , characteristic curves of PMSM are obtained.

Torque-speed curves, power-speed curves, stator voltage-speed curves of PMSM at different stator currents are shown in Figures 4 to 6.

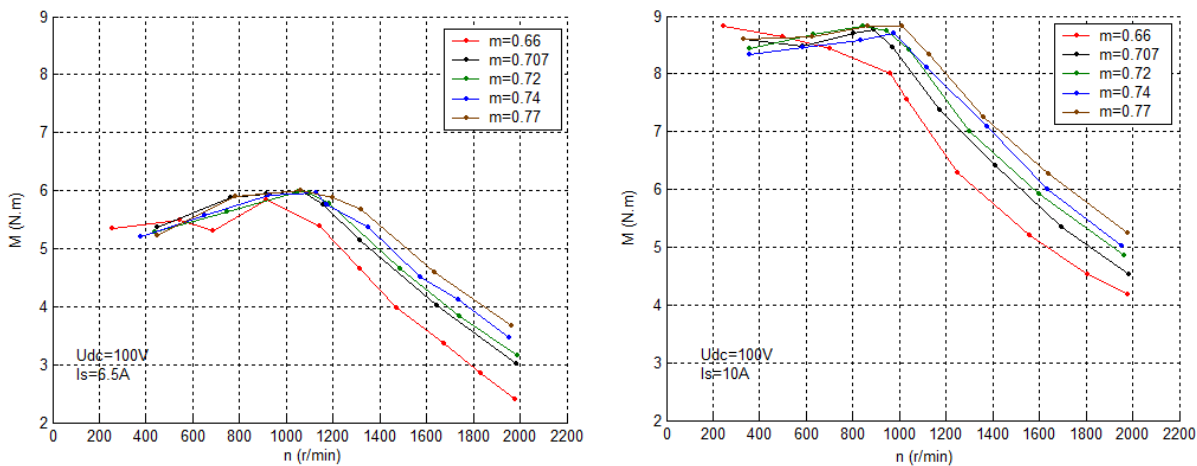


Figure 4. Torque-speed curves.

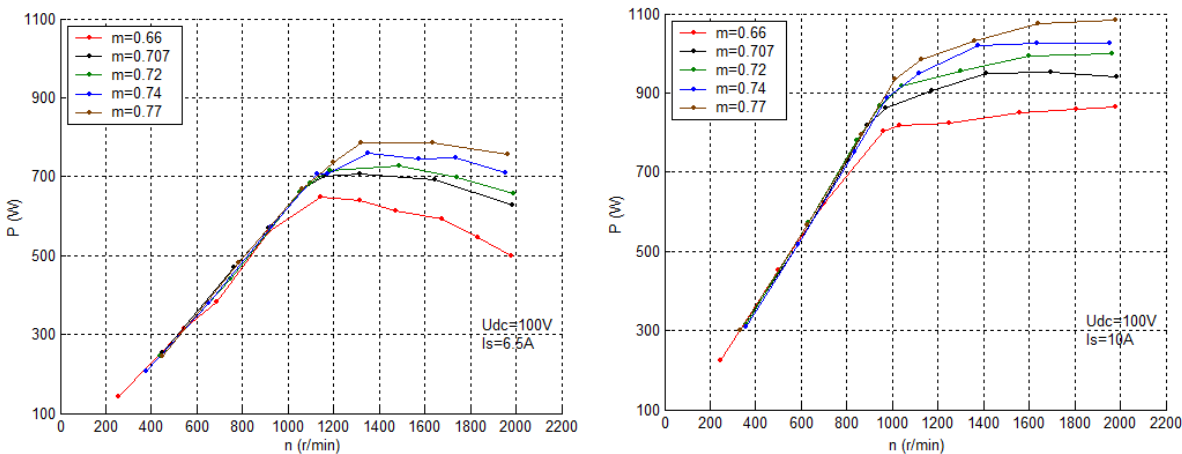


Figure 5. Power-speed curves.

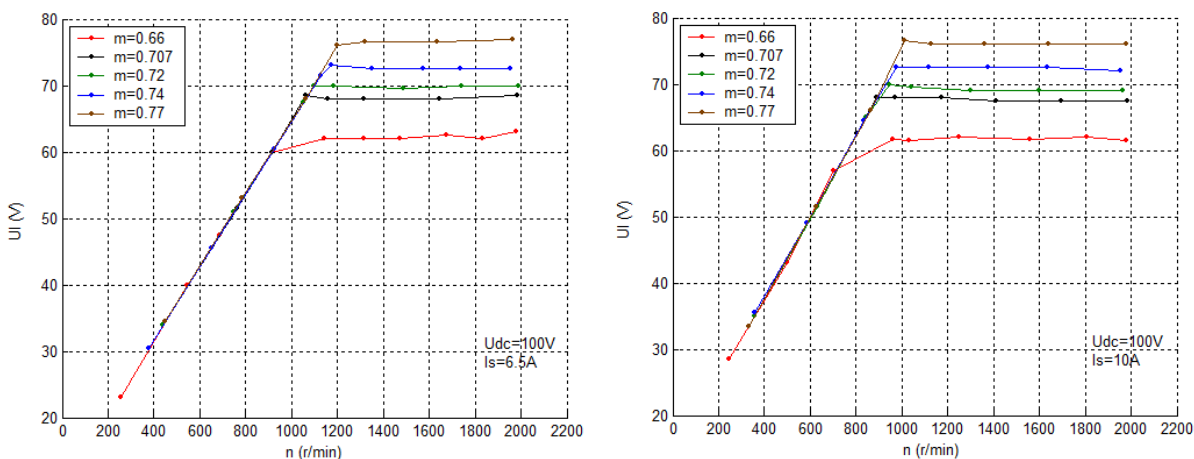


Figure 6. Stator voltage-speed curves.

As can be seen from Figures 4 to 6, characteristic curve  $m = 0.66$  as an example, at the turning speed, output torque of the motor is divided into two distinct regions. With over-modulation algorithm, utilization ratio of DC bus voltage  $m$  increases from 0.707 to 0.77, experiment results show that the turning speed of motor is improved, the output torque, output power and stator voltage in flux-weakening operation are larger than that in sinusoidal modulation. We can

conclude that, the flux-weakening control strategy is correct, with over-modulation control, PMSM operation region has been expanded, torque-speed curves, power-speed curves and stator voltage-speed curves of PMSM can well reflect the effect of over-modulation control in flux-weakening operation.

## CONCLUSIONS

In this paper, for the purpose of improving utilization ratio of DC bus voltage, the starting point of over-modulation is judged by the action time of zero voltage vector, an over-modulation algorithm based on SVPWM is proposed. Apply this over-modulation algorithm in PMSM control system, the flux-weakening operation region is extended and the output torque and power of the motor are increased. Simulation experiments show the correctness of over-modulation algorithm, it has obtained satisfactory control effect in PMSM flux-weakening control system.

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