

Flexible Flow Shop Scheduling with Learning and Forgetting Effects

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ABSTRACT: This paper presents a new mathematical model for scheduling flexible flow shop problem with sequence dependent setup times in which learning and forgetting effects of workers consider in setup times. Manpower have significant rule for doing setup on machines and their skill will be increase when they do similarity jobs and it is possible for them to prepare machines to process of jobs with higher speed (Learning effect). On other hand, when workers do number of new jobs on different machines therefore they will be forget the setup on before machines (Forgetting effect).

KEYWORDS: Scheduling flexible flow shop problem; Learning effect; Forgetting effect; Manpower.

INTRODUCTION

Flexible flow shops (FFS) are common manufacturing environments in which a set of n jobs are to be processed in a series of m stages optimizing a given objective function. There are a number of variants, all of which have most of the following characteristics in common:

1. The number of processing stages m is at least 2.
2. Each stage k has $M_k \geq 1$ machines in parallel and in at least one of the stage $M_k > 1$.
3. All jobs are processed following the same production flow: stage 1, stage 2, ..., stage m . A job might skip any number of stages provided it is processed in at least one of them.
4. Each job j requires a processing time P_k in stage k . We shall refer to the processing of job j in stage k as operation O_{kj} [1].

FFS is found in all kinds of real world scenarios including the electronics [2-5], paper [6] and textile [7], industries. Examples are also found in the production of concrete [8], the manufacturing of photographic film [9, 10], and others [11, 12]. We also find examples in non-manufacturing areas like civil engineering [13], internet service architectures [14] and container handling systems [15, 16].

Based on studies that are done on scheduling of automation systems, the processing times cannot change by number of jobs and their sequence. But we can consider sequence dependent setup times for reaching the real condition on the shop floor. In these conditions, rule of manpower is very important and the skill of workers increase when process similar jobs iteratively (it is called learning effect in the literature of scheduling). On the other hand, forgetting effect occurs on workers when they do un-similar jobs on different machines. Manpower have key rule in the scheduling problems therefore the number of activities that are related them are very big and it seem that considering learning and forgetting effects in the scheduling problem is suitable.

In this research, flexible flow shop problem and manpower with learning and forgetting effects will be considered. The objective is to find the sequence of jobs on machines and the sequence of workers for doing the setup of machines to minimize the weighted sum of makespan and maximum tardiness.

First study on learning effect in scheduling was introduced by Biskup [17]. Moshio [18] demonstrated the minimization of makespan on single machine with considering learning effect will be solved as polynomial. Moshio and Sidney [19] studied on single machine when the learning effect was not identical for all of jobs. Lee et al [20] studied bi-criteria optimization on single machine and presented a heuristic approach for solving them to find good solutions. Lee and Wu [21] proposed a meta-heuristic algorithm for minimizing the maximum completion time in the two machine problem by learning effect that is dependent to jobs. Ern and Guner [22] studied single machine problem based on bi-criteria to minimize the sum of completion time and maximum tardiness. Another studies are done on

single machine with learning effects such as: Cheng and Wang [23], Kuo and Yang [24], Lee [25], Biskup and Simons [26], Moshio and Sidney [27]. A little studies were done on forgetting effect in the literature. Yang and Chand [28] considered learning and forgetting effects on single machine with group setup time. Their objective was to minimize the maximum completion time of jobs. Nembhard and Uzumeri [29] considered forgetting effect of an organization that applied new technologies and approaches on period of time. Jaber et al [30] called learning and forgetting effects to be consistent. Globerson [31], Shtub et al [32] called power models were suitable for considering forgetting effects.

PROBLEM DEFINITION AND PROPOSED MATHEMATICAL MODEL

This paper considers flexible flow shop problem with manpower. Each worker has learning and forgetting coefficient for preparing machines to process of jobs. The processing of jobs is done automatically by machines. In this study, we determine the sequence of jobs on each machine, assign workers to each machine and determine the route of each worker to prepare of machines for processing of jobs based on learning and forgetting effects of workers. The objective is to minimize the weighted sum of maximum completion time and maximum tardiness. The structure of problem graphically illustrated in Figure 1.

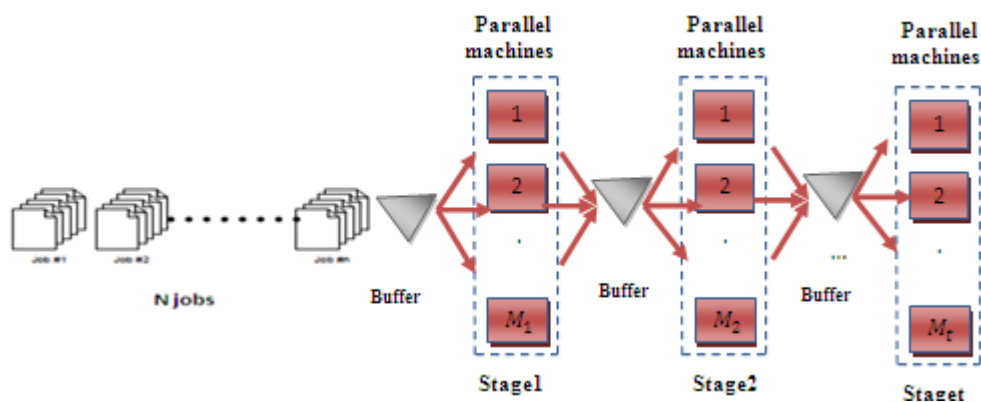


Figure 1. Structure of flexible flow shop problem.

The assumptions, indices, parameters and decision variables are introduced before developing the proposed mathematical model.

Assumptions

- Machines are available any time on the horizon.
- Jobs are available any time on the horizon.
- The processing time of jobs is pre-known.
- The due date for each job is pre-known.
- Preemption is allowed.
- There are no precedence constraints among of the jobs.
- Each machine can processed only one job at the time.
- Each job processed only by one machine at the time.
- Each worker can be set only one machine at time.
- The process of jobs done automatically by machines and workers prepare only machines for processing of jobs.
- Workers have different skills for setting machines to process jobs (it is related to expert; age; degree ;...).
- Learning and forgetting effects of workers are based on iteration.
- Setup times of jobs on machines by workers are pre-defined.
- Each job has a pre-known due date.
- Transportation times between stages are negligible.
- Capacities of buffers between stages are unlimited.

- All of data in the problem are deterministic.

Indices

| | |
|--|----------------------------|
| i, j, k : Indices of jobs | $i, j, k=(1, 2, \dots, n)$ |
| s, s' : Indices of sequences | $s, s'=(1, 2, \dots, n)$ |
| l, l' : Indices of number of jobs that are done by each worker | $l, l'=(1, 2, \dots, n)$ |
| w, w' : Indices of workers | $w, w'=(1, 2, \dots, nw)$ |
| m, m' : Indices of machines | $m, m'=(1, 2, \dots, M_t)$ |
| t, t' : Indices of stages | $t, t'=(1, 2, \dots, ns)$ |

Parameters

P_{imt} : Processing time of job i on machine m at stage t

Se_{wimt} : Setup time of machine m at stage t for processing job i by worker w

$Setup_{ijwmt}$: Setup time of machine m at stage t between jobs i and j by worker w

a_{iw} : Learning coefficient of job i by worker w

b_{iw} : Forgetting coefficient of job i by worker w

d_i : Due date of job i

M_t : Number of parallel identical machines at stage t

nw : Number of workers at shop floor

ns : Number of stages

n : Number of jobs for processing

M : A positive big number

Decision Variables

$x_{ismtwt} = \begin{cases} 1 & \text{if job } i \text{ that is } sth \text{ job on machine } m \text{ at stage } t, lth \text{ job that is done by worker } w \\ 0 & \end{cases}$

S_{it} : Starting time of worker when machine should be prepare for processing of job i at stage t

F_{it} : Leaving time of worker when machine is prepared for processing of job i at stage t

C_{it} : Completion time of job i at stage t

C_{max} : Maximum of completion time (Makespan)

r_{it} : Number of setup on the same machine that are done before of job i at stage t by the same worker

h_{it} : Number of setup on the different machine that are done before of job i at stage t by the same worker

T_i : Tardiness of job i

$Tmax$: Maximum of tardiness

According to the above mentioned parameters and variables, the mathematical model is suggested as a mixed integer formulation as follows:

Mathematical Model

$$\text{Min}z = \alpha T_{\max} + (1 - \alpha)C_{\max} \quad (1)$$

$$\sum_{s=1}^n \sum_{m=1}^{Mt} \sum_{l=1}^n \sum_{w=1}^{nw} x_{ismlwt} = 1 \quad \forall i, t \quad (2)$$

$$\sum_{i=1}^n \sum_{l=1}^n \sum_{w=1}^{nw} x_{ismlwt} \leq 1 \quad \forall m, s, t \quad (3)$$

$$\sum_{i=1}^n \sum_{l=1}^n \sum_{m=1}^{Mt} x_{ismlwt} \leq 1 \quad \forall w, l, t \quad (4)$$

$$\sum_{i=1}^n \sum_{l=1}^n \sum_{w=1}^{nw} x_{ismlwt} \leq \sum_{i=1}^n \sum_{l=1}^n \sum_{w=1}^{nw} x_{is-1mlwt} \quad \forall s > 1, m, t \quad (5)$$

$$\sum_{i=1}^n \sum_{s=1}^n \sum_{m=1}^{Mt} x_{ismlwt} \leq \sum_{i=1}^n \sum_{s=1}^n \sum_{m=1}^{Mt} x_{isml-1wt} \quad \forall l > 1, w, t \quad (6)$$

$$x_{ismlwt} \geq -M \cdot x_{js'm'l'wt} \quad \forall w, m, i, j, t, s' > s, l' < l \quad (7)$$

$$x_{ismlwt} \leq -M \cdot (1 - x_{js'm'l'wt}) \quad \forall w, m, i, j, t, s' > s, l' < l \quad (8)$$

$$S_{it} \geq \sum_{j=1, j \neq i}^n \sum_{m=1}^{Mt} \sum_{m'=1}^{Mt} \sum_{l=2}^n \sum_{s=1}^n \sum_{s'=1}^n \sum_{w=1}^{nw} (F_{jt} \cdot x_{js'm'l-1wt} \cdot x_{ismlwt}) \quad \forall i, t \quad (9)$$

$$S_{it} \geq \sum_{j=1, j \neq i}^n \sum_{m=1}^{Mt} \sum_{l'=1}^n \sum_{l=1}^n \sum_{s=2}^n \sum_{w'=1}^{nw} \sum_{w=1}^{nw} (C_{jt} \cdot x_{js-1m'l'w't} \cdot x_{ismlwt}) \quad \forall i, t \quad (10)$$

$$S_{it} \geq \sum_{m=1}^{Mt} \sum_{m'=1}^{Mt} \sum_{l'=1}^n \sum_{l=1}^n \sum_{w'=1}^{nw} \sum_{w=1}^{nw} \sum_{s'=1}^n \sum_{s=1}^n (C_{it-1} \cdot x_{ismlwt} \cdot x_{is'm'l'w't}) \quad \forall i, t > 1 \quad (11)$$

$$S_{it} \geq \sum_{m=1}^{Mt} \sum_{m'=1}^{Mt} \sum_{s'=1}^n \sum_{s=1}^n \sum_{l'=1}^n \sum_{w=1}^{nw} (F_{jt'} \cdot x_{js'm'l'wt'} \cdot x_{ismlwt}) \quad \forall i, j, t' < t, t > 1 \quad (12)$$

$$r_{it} = \sum_{s=1}^n \sum_{m=1}^{Mt} \sum_{w=1}^{nw} \sum_{l=1}^n \left(\sum_{j=1, j \neq i}^n \sum_{s'=1}^s \sum_{l'=1}^l x_{js'm'l'wt} \right) \cdot x_{ismlwt} \quad \forall i, t \quad (13)$$

$$h_{it} = \max \left\{ \sum_{s=1}^n \max_{s'=1:s} (s' \cdot \left(\sum_{m=1}^{Mt} \sum_{l=1}^n \sum_{l'=1}^l \sum_{w=1}^{nw} \sum_{j=1, j \neq i}^n x_{js'm'l'wt} \cdot x_{ismlwt} \right)), 1 \right\} \quad \forall i, t \quad (14)$$

$$F_{it} \geq S_{it} + \sum_{m=1}^{Mt} \sum_{l=1}^n \sum_{w=1}^{nw} (se_{wimt} \cdot x_{i1mlwt}) \quad \forall i, t \quad (15)$$

$$F_{it} \geq \sum_{k=1, k \neq i}^n \sum_{j=1, j \neq i}^n \sum_{m=1}^{Mt} \sum_{s=2}^n \sum_{l=1}^n \sum_{l'=1}^n \sum_{l''=1, l'' < l'}^n \sum_{w=1}^{nw} \sum_{w'=1}^{nw} (S_{it} + [(r_{it})^{a_{iw}} \cdot (l - l'' - 1)^{b_{iw}} \cdot \text{setup}_{jiwmt}]) \cdot x_{js-1ml'w''t} \cdot x_{kh_{it}ml''wt} \cdot x_{ismlwt} \quad \forall i, t, a_{iw} < 0, b_{iw} \geq 1 \quad (16)$$

$$F_{it} \geq \sum_{j=1, j \neq i}^n \sum_{m=1}^{Mt} \sum_{s=2}^n \sum_{l=1}^n \sum_{l'=1}^n \sum_{w=1}^{nw} \sum_{w'=1}^{nw} (S_{jt} + [(r_{it})^{a_{iw}} \cdot \text{setup}_{jiwmt}]) \cdot x_{js-1ml'w''t} \cdot x_{ismlwt} \quad \forall i, t, a_{iw} < 0 \quad (17)$$

$$C_{it} = \sum_{l=1}^n \sum_{m=1}^{Mt} \sum_{w=1}^{nw} \sum_{s=1}^n (F_{it} + p_{imt}) \cdot x_{ismlwt} \quad \forall i, t \quad (18)$$

$$C_{max} \geq C_{in} \quad \forall i \quad (19)$$

$$T_i \geq C_{in} - d_i \quad \forall i \quad (20)$$

$$T_i \geq 0 \quad \forall i \quad (21)$$

$$T_{max} \geq T_i \quad \forall i \quad (22)$$

$$S_{it}, F_{it}, C_{it}, C_{max}, T_i, T_{max}, h_{it}, r_{it} \geq 0$$

$$x_{ismlwt} = \{0,1\}$$

The objective is to minimize the weighted sum of make span and maximum tardiness that is shown in Equations (1) and Eq(2) ensures that which worker do each job at each stage (the sequence of jobs determine by Equations (2) and (3) explains that in each position on each machine at each stage can be only one job processed. Equation (4) ensures that in each position by each worker at each stage can be only one job processed. Equation (5) ensures that if in the one sequence on one machine at one stage is not any job for processing by worker that is assigned to this sequence therefore it is not possible to process one job to next sequence on the same machine at the same stage (it causes to produce a feasible sequence on each machine at each stage). Equation (6) ensures that if in the route of each worker at each stage is not any job for processing by worker that is assigned to this route therefore it is not possible to process one job to next sequence by the same worker at the same stage (it causes to produce a feasible route on each worker at each stage). Equations (7) and (8) generate a feasible sequence on each machine done by workers in the solution space. Equations (9)-(12) calculate starting time of each job at each stage by worker that prepare machine for processing them. Equation (13) determines number of setup on the same machine that are done before of job i at stage t by the same worker. Equation (14) determines number of setup on the different machine that are done before of job i at stage t by the same. Equations (15)-(17) calculate leaving time of each job at each stage by worker that is prepared machine for processing them. Equation (18) calculates completion time of each job at each stage. Equation (19) determines the maximum completion time (Make span). Equations (20) and (21) determines tardiness of each job. Maximum of tardiness calculated by Equation (22).

In order to confirm that the mathematical model works as intended, an example is provided and the results of the execution with details of schedule of jobs on machines are presented. Suppose that the weight of make span is 0.2 and there are 6 jobs, 3 workers, the processing times, setup times, due dates, are shown in Tables 1-5.

Table 1. The processing time of example.

| | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <i>Stage 1</i> | 2 | 4 | 8 | 2 | 2 | 6 |
| <i>Stage 2</i> | 1 | 3 | 6 | 9 | 2 | 3 |

Table 2. The sequence dependent set up time between jobs at stage 1 of example.

| | | | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|----------------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <i>Stage 1</i> | <i>Worker 1</i> | <i>Job 1</i> | 0 | 6 | 6 | 6 | 7 | 6 |
| | | <i>Job 2</i> | 7 | 0 | 4 | 7 | 6 | 6 |
| | | <i>Job 3</i> | 5 | 6 | 0 | 6 | 6 | 7 |
| | | <i>Job 4</i> | 6 | 6 | 7 | 0 | 4 | 8 |
| | | <i>Job 5</i> | 7 | 8 | 7 | 5 | 0 | 7 |
| | | <i>Job 6</i> | 4 | 5 | 3 | 6 | 6 | 0 |
| | <i>Worker 2</i> | <i>Job 1</i> | 0 | 5 | 7 | 6 | 6 | 4 |
| | | <i>Job 2</i> | 7 | 0 | 6 | 8 | 5 | 4 |
| | | <i>Job 3</i> | 2 | 6 | 0 | 7 | 7 | 3 |
| | | <i>Job 4</i> | 5 | 2 | 7 | 0 | 7 | 5 |
| | | <i>Job 5</i> | 6 | 8 | 8 | 1 | 0 | 6 |
| | | <i>Job 6</i> | 4 | 5 | 5 | 9 | 8 | 0 |
| | <i>Worker 3</i> | <i>Job 1</i> | 0 | 4 | 6 | 5 | 4 | 3 |
| | | <i>Job 2</i> | 6 | 0 | 7 | 3 | 5 | 5 |
| | | <i>Job 3</i> | 7 | 5 | 0 | 6 | 9 | 4 |
| | | <i>Job 4</i> | 8 | 6 | 5 | 0 | 7 | 3 |
| | | <i>Job 5</i> | 4 | 7 | 4 | 6 | 0 | 3 |
| | | <i>Job 6</i> | 3 | 8 | 7 | 8 | 5 | 0 |

Table 3. The sequence dependent set up time between jobs at stage 2 of example.

| | | | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|----------------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <i>Stage 2</i> | <i>Worker 1</i> | <i>Job 1</i> | 0 | 6 | 5 | 5 | 5 | 5 |
| | | <i>Job 2</i> | 7 | 0 | 6 | 4 | 6 | 6 |
| | | <i>Job 3</i> | 5 | 6 | 0 | 6 | 7 | 4 |
| | | <i>Job 4</i> | 6 | 7 | 7 | 0 | 5 | 6 |
| | | <i>Job 5</i> | 7 | 8 | 8 | 5 | 0 | 4 |
| | | <i>Job 6</i> | 4 | 4 | 9 | 6 | 4 | 0 |
| | <i>Worker 2</i> | <i>Job 1</i> | 0 | 5 | 5 | 4 | 7 | 4 |
| | | <i>Job 2</i> | 3 | 0 | 4 | 6 | 5 | 6 |
| | | <i>Job 3</i> | 4 | 6 | 0 | 7 | 3 | 3 |
| | | <i>Job 4</i> | 7 | 7 | 4 | 0 | 6 | 5 |
| | | <i>Job 5</i> | 8 | 4 | 6 | 4 | 0 | 4 |
| | | <i>Job 6</i> | 9 | 7 | 4 | 5 | 6 | 0 |
| | <i>Worker 3</i> | <i>Job 1</i> | 0 | 8 | 3 | 7 | 1 | 7 |
| | | <i>Job 2</i> | 1 | 0 | 5 | 5 | 4 | 4 |
| | | <i>Job 3</i> | 9 | 7 | 0 | 8 | 8 | 7 |
| | | <i>Job 4</i> | 4 | 8 | 4 | 0 | 4 | 3 |
| | | <i>Job 5</i> | 5 | 6 | 5 | 4 | 0 | 7 |
| | | <i>Job 6</i> | 6 | 8 | 6 | 5 | 4 | 0 |

Table 4. The set up time of jobs at stage 1 and 2 of example.

| | | | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|----------------|-----------------|------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <i>Stage 1</i> | <i>Worker 1</i> | <i>Machine 1</i> | 5 | 6 | 1 | 7 | 2 | 6 |
| | | <i>Machine 2</i> | 2 | 8 | 4 | 6 | 6 | 8 |
| | | <i>Machine 3</i> | 4 | 9 | 6 | 7 | 4 | 7 |
| | <i>Worker 2</i> | <i>Machine 1</i> | 6 | 8 | 5 | 7 | 8 | 9 |
| | | <i>Machine 2</i> | 2 | 9 | 4 | 6 | 9 | 8 |
| | | <i>Machine 3</i> | 8 | 7 | 1 | 4 | 8 | 7 |
| | <i>Worker 3</i> | <i>Machine 1</i> | 6 | 9 | 6 | 9 | 9 | 7 |
| | | <i>Machine 2</i> | 7 | 9 | 5 | 7 | 7 | 8 |
| | | <i>Machine 3</i> | 8 | 9 | 4 | 6 | 8 | 6 |
| <i>Stage 2</i> | <i>Worker 1</i> | <i>Machine 1</i> | 8 | 9 | 8 | 9 | 8 | 1 |
| | | <i>Machine 2</i> | 9 | 2 | 9 | 7 | 9 | 4 |
| | | <i>Machine 3</i> | 7 | 7 | 2 | 8 | 7 | 5 |
| | | <i>Machine 4</i> | 6 | 7 | 9 | 9 | 9 | 10 |
| | <i>Worker 2</i> | <i>Machine 1</i> | 8 | 8 | 7 | 9 | 8 | 2 |
| | | <i>Machine 2</i> | 9 | 7 | 2 | 7 | 6 | 6 |
| | | <i>Machine 3</i> | 7 | 5 | 7 | 6 | 7 | 4 |
| | | <i>Machine 4</i> | 8 | 9 | 8 | 9 | 8 | 9 |
| | <i>Worker 3</i> | <i>Machine 1</i> | 9 | 8 | 9 | 8 | 5 | 9 |
| | | <i>Machine 2</i> | 8 | 1 | 6 | 6 | 6 | 7 |
| | | <i>Machine 3</i> | 9 | 4 | 8 | 8 | 8 | 8 |
| | | <i>Machine 4</i> | 7 | 5 | 8 | 2 | 10 | 10 |

Table 5. The due date of example.

| | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <i>Due date</i> | 24 | 18 | 15 | 21 | 28 | 14 |

Table 6. The learning coefficient.

| | | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|-----------------------------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Learning coefficient | <i>Worker 1</i> | -1 | -2 | -3 | -2 | -2 | -1 |
| | <i>Worker 2</i> | -1 | -2 | -2 | -2 | -1 | -3 |
| | <i>Worker 3</i> | -1 | -1 | -1 | -1 | -1 | -3 |

Table 7. The forgetting coefficient.

| | | <i>Job 1</i> | <i>Job 2</i> | <i>Job 3</i> | <i>Job 4</i> | <i>Job 5</i> | <i>Job 6</i> |
|-------------------------------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Forgetting coefficient | <i>Worker 1</i> | 1 | 2 | 1 | 2 | 1 | 1 |
| | <i>Worker 2</i> | 2 | 1 | 1 | 3 | 3 | 2 |
| | <i>Worker 3</i> | 1 | 1 | 1 | 1 | 2 | 2 |

The objective function is to minimize the weighted sum of makespan and maximum tardiness. The following Figure and Table illustrate the solution obtained of solution space in details from the mathematical model (Figure 2 and Table 8).

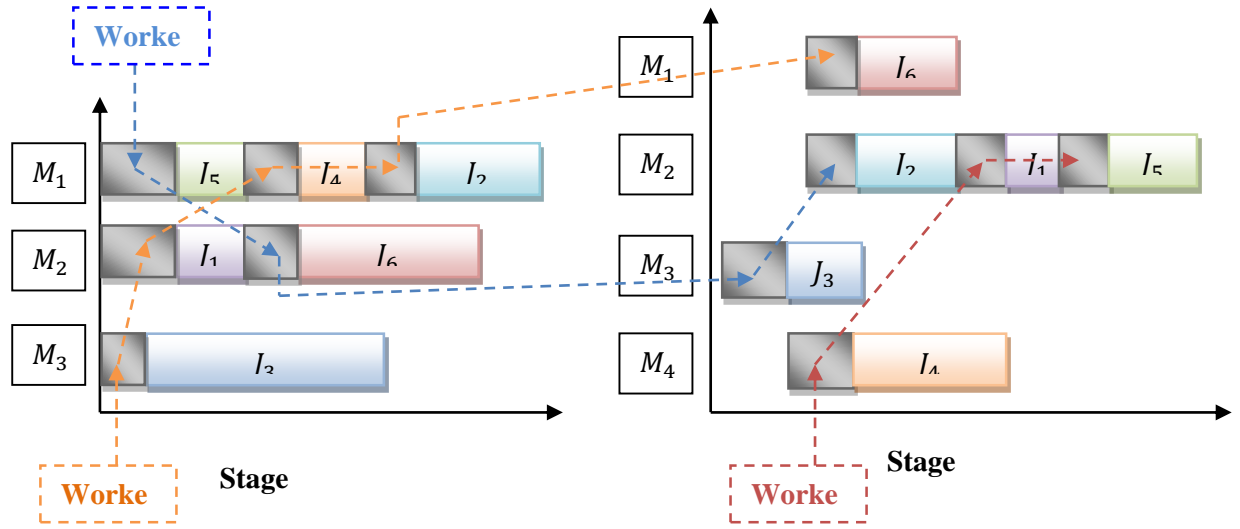


Figure 2. Presentation of solution graphically.

Completion times of jobs at stage 1 are as follows:

$$C_{11} = \{\text{Max}\{0, F_{31}\} + Se_{2121} + P_{11}\} = \{1 + 1 + 2\} = 4;$$

$$C_{21} = \{C_{41} + Setup_{4211} * (r_{21})^{a_{21}} + P_{21}\} = \{10 + 1 + 3\} = 15;$$

$$C_{31} = \{Se_{2331} + P_{31}\} = \{1 + 8\} = 9;$$

$$C_{41} = \{\text{Max}\{C_{51}, F_{11}\} + Setup_{5421} * (h_{41})^{b_{41}} + P_{41}\} = \{\text{Max}\{4, 2\} + 1 * (2)^{+2} + 2\} = 10;$$

$$C_{51} = \{Se_{5111} + P_{51}\} = \{2 + 2\} = 4;$$

$$C_{61} = \{\text{Max}\{C_{11}, F_{51}\} + Setup_{5421} * (h_{61})^{b_{61}} + P_{61}\} = \{\text{Max}\{2, 4\} + 1 + 6\} = 11;$$

Completion times of jobs at stage 2 are as follows:

$$C_{12} = \{\text{Max}\{C_{22}, F_{42}\} + Setup_{2132} + P_{12}\} = \{\text{Max}\{20, 12\} + 1 + 1\} = 22;$$

$$C_{22} = \{\text{Max}\{C_{21}, F_{32}\} + Se_{1222} * (h_{22})^{b_{22}} + P_{22}\} = \{\text{Max}\{15, 10\} + 2 + 3\} = 15;$$

$$C_{32} = \{\text{Max}\{C_{31}, F_{61}\} + Se_{1332} + P_{32}\} = \{\text{Max}\{9, 5\} + 2 + 6\} = 17;$$

$$C_{42} = \{C_{41} + Se_{3442} + P_{42}\} = \{10 + 2 + 9\} = 21;$$

$$C_{52} = \{\text{Max}\{C_{12}, F_{12}\} + Setup_{1532} * (r_{52})^{a_{52}} + P_{52}\} = \{\text{Max}\{22, 12\} + 4 + 2\} = 28;$$

$$C_{62} = \{\text{Max}\{C_{61}, F_{22}\} + Se_{2612} + P_{62}\} = \{\text{Max}\{11, 11\} + 2 + 3\} = 16;$$

Tardiness of jobs is calculated as follows:

$$T_1 = \text{Max}\{0, C_{12} - d_1\} = \text{Max}\{0, 22 - 24\} = 0;$$

$$T_2 = \text{Max}\{0, C_{22} - d_2\} = \text{Max}\{0, 15 - 18\} = 0;$$

$$T_3 = \text{Max}\{0, C_{32} - d_3\} = \text{Max}\{0, 17 - 15\} = 2;$$

$$T_4 = \text{Max}\{0, C_{42} - d_4\} = \text{Max}\{0, 21 - 21\} = 0;$$

$$T_5 = \text{Max}\{0, C_{52} - d_5\} = \text{Max}\{0, 28 - 28\} = 0;$$

$$T_6 = \text{Max}\{0, C_{62} - d_6\} = \text{Max}\{0, 16 - 14\} = 2;$$

$$T_{\text{max}} = \text{Max}\{0, 0, 2, 0, 0, 2\} = 2$$

Table 8. Decision variable of example.

| Number of job | Completion time at stage 1 | Completion time at stage 2 | Tardiness |
|---------------|----------------------------|----------------------------|-----------|
| Job 1 | 4 | 22 | 0 |
| Job 2 | 15 | 15 | 0 |
| Job3 | 9 | 17 | 2 |
| Job4 | 10 | 21 | 0 |
| Job5 | 4 | 28 | 0 |
| Job6 | 11 | 16 | 2 |

The objective value is equal to $(0.2*28+0.8*2) = 7.2$.

Conclusion and future works

In this paper a new mathematical model is presented for scheduling flexible flow shop problem with learning and forgetting effects. The processing of jobs is done automatically and rule of manpower is preparing of machine to process of jobs. This problem is Np-hard, therefore it is worthwhile to apply meta-heuristic algorithms to find the good solution in the solution space. The transportation time between stages is not considered, future researchers can be considered it.

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