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Optimization for Multiple Traveling Salesman Problem Based on Genetic Algorithm

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ABSTRACT: As a classical NP-hard problem, traveling salesman problem is of great significance, at the same time, solving multiple traveling salesman problem has much more practical significance. In the past, research on multiple traveling salesman problem is usually limited to set the optimization criterion to the minimal sum of all the traveling salesman path, comparatively, the literature about multiple traveling salesman problem of minimizing the maximum of all the traveling salesman path is rare. To solve the multiple traveling salesman problem of minimizing maximum value of all the traveling salesman path, this paper firstly introduces the mathematical model for the multiple traveling salesman problem, then by using genetic algorithm to optimize this problem, and proposes a matrix decoding method. Matrix decoding method is suitable for solving multiple traveling salesman problem of asymmetric and symmetric distance. In this paper, multiple traveling salesman problem of asymmetric distance is as sample to have tests and the performances of different crossovers are compared.

KEYWORDS: Genetic algorithm; Multiple traveling salesman problem; Optimization; Decoding method.

INTRODUCTION

Traveling Salesman Problem, short for TSP, is a well-known combinatorial optimization problem: Given n cities, and one traveling salesman departs from an initial city and visits each city only once and then returns to the original departure city, asking for the shortest circuit path [1]. If it is described by graph theory, it is known weighted graph G = (C, L), and find out Hamilton circle with the smallest total weights.

Multiple Traveling Salesman Problem (MTSP) is an extension of the Traveling Salesman Problem (TSP). MTSP refers to the set of N cities, M traveling salesmen depart from target city and take a travel route respectively so that each city has one and only one traveling salesman passed, and finally they back to the original departure city, and asking for the shortest total path [2].In practical problems, research about MTSP has great use value, such as: pipeline laying, transportation, topology design of computer network, route selection, postman messenger, etc., all of them can be abstracted into a TSP or MTSP.

Genetic Algorithm is a computational model that simulates genetics of Darwin's theory of biological evolution and biological evolution process of natural selection mechanism, and it is a method that through simulating natural evolution process to find the optimal solution. It is firstly proposed by Professor Holf and in the University of Michigan of American in 1975, and it is an effective method to solve complex combinatorial optimization problems. Genetic algorithm has been very widely used in solving TSP and MTSP.

However, in solving MTSP, only the shortest total distance can be guaranteed, a balance of each traveling salesman’s path can be rarely achieved, leading to an unreasonable allocation result that the traveling salesman’s path is unequal and resources cannot get completely rational use.

MATHEMATICAL MODEL OF MULTIPLE TRAVELING SALESMAN PROBLEM

With the point 0 represents the salesman’s initial departure city, called the source point, point 1, ..., l respectively represent a city that m salesmen need to visit.

Define variables:

$$x_{ijk} = \begin{cases} 1 & \text{traveling salesmankpass arc } \left(\text{i, j}\right) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$y_{ki} = \begin{cases} 1 & \text{traveling salesmankvisit cityi} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Among which, c_{ij} represents expended costs, such as cost, distance, time, etc. when the traveling salesman passes the corresponding arcs (i, j).

Then the following models can be got:

The objective function is:

$$Z = \min(\sum_{k=1}^{m} z_k) \tag{3}$$

In which:

$$z_k = \sum_{i=0}^{l}\sum_{j=0}^{l} c_{ij} x_{ijk}; k = 1, 2, \cdots, \mathrm{m} \tag{4}$$

Constraint is

$$\sum_{k=1}^{m} y_{ki} = \begin{cases} m & i = 0 \\ 1 & i = 1, 2, \cdots, \ \mathrm{l} \end{cases} \tag{5}$$

$$\sum_{i=0}^{l} x_{ijk} = y_{kj} \tag{6}$$

Among which: j=0,1,…, j; k=0,1,…,m.

$$\sum_{j=0}^{l} x_{ijk} = y_{ki} \tag{7}$$

Among which: j=0,1,…, j; k=0, 1,…,m.

$$X = (x_{ijk}) \in S \tag{8}$$

In the formula (8): S refers to slip eliminate the constraint namely the elimination of solution which can not constitute a complete route [3].

In the above mathematical model ofMTSP, the formula (3) represents to make the sum of the distance ofm traveling salesmen is minimum; the formula (4) represents the respectively completed route of the various salesmen; the formula (5) represents to depart from the initial target city 0, all the city has only one traveling salesman strictly access once; the formula (6) represents that a terminal city of any arc has and only has anorigin city connected with it; the formula (7) represents thatan origin city of any arc has and only has a terminal city connected with it; the formula (8) represents the elimination of solution which can not constitute a complete route.

GENETIC ALGORITHM DESIGN

The keyto apply genetic algorithm into MTSP is to usean appropriate decoding method and effective encoding way.

Genetic algorithm repeatedly has selection, crossover, mutation and other basic genetic manipulation to the population, and constantly generates new generation which can better adapt to the environment than the parent until meets the required condition [4].

Individual Encoding

Suppose point 0 represents the initial departure city of traveling salesman, called the source point, point 1, ..., l represents a city m traveling salesmen need to visit. Based on the characteristics of MTSP, when use genetic algorithm to solve MTSP, MTSP can convert to a TSP by the method of adding m-1 virtual symbols. The m-1 virtual symbols are respectively l + 1, ..., l + m-1.In the visiting path of traveling salesman, each appearing virtual symbol represents that the traveling salesman returns to the departure city 0,namely initial departure city, so as form a loop. Each loop represents a visiting path of traveling salesmanin MTSP. It is worth noting that, in order to avoid the emergence of an extraordinary child path, C_{00} =M，M must be assumed to be set to become an infinitely large positive number [5].

If the city is 0, 1, ..., 8, a chromosome encodingof MTSP for three traveling salesmen is:

2	5	7	3	9	4	10	1	8	6

The path of three traveling salesmen can be expressed as:

0-2-5-7-3-0;

0-4-0;

0-1-8-6-0。

In arithmetic operation process of the initial population or population, there may be situationthat two virtual symbols bear are connected closely and the virtual symbols are at both ends of a chromosome. As the first case chromosome appears:

9	5	7	3	2	4	10	1	8	6

The path of three traveling salesmen can be expressed as:

0-0-0;

0-5-7-3-2-4-0;

0-1-8-6-0。

Since C_{00} =M，M is set to be an infinitely large positive number, the maximum value of three traveling salesmen’s path is M. Since the purpose of this paper is to minimize the maximum pathof the traveling salesman, in population selection and copy process, this chromosome will be gradually eliminated.

As the second case of chromosome appears:

4	5	7	3	2	9	10	1	8	6

The path of three traveling salesmen can be expressed as:

0-4-5-7-3-2-0;

0-0-0;

0-1-8-6-0。

As described in the first case, this kind of chromosome willbe gradually eliminated in population selection and copy process.

Group Size Selection

In the convergence process of genetic algorithm, suitable group size has very important significance. If the group size is too small, it can notget satisfactory results; if thegroup size is too large, it will lead to complex calculation. According to experience, it is appropriate to make the group size 10 to 160.

Fitness Function

In selection operation, there may be a genetic algorithm deceptive problem. In the early stages of genetic evolution, it will often produce some extraordinary individuals.If the selection method is based on the proportion, these abnormal individuals tend to influence the selection process because of the too prominent competitiveness, and influence the global optimize performance of genetic algorithm; in the later stages of genetic evolution, that is when the algorithm is approaching convergence, because the differences offitness value of individuals among populations is relatively small, the possibility of continuing optimization lower, so it is most likely to get a local optimal solution. Poorly designed fitness function may cause such problems.

Due to minimizing the total distance is the optimization goal of the algorithm, so that making the objective function as the exponential transform to get corresponding fitness function:

$$f = \alpha \exp(-\beta \times Z) \tag{9}$$

Among which: α、β are arithmetic number.

Selection

Selection is used to cross an individual or determine reorganization, as well as alternate individual to produce child individuals. The first step ofselection is to calculate the fitness value, according to the fitness value of proportion to distribute, that is according to the probability of proportion in the fitness value of each individual to decide the leave possibility of their child individuals. If an individual i, whose fitness value is f_i, then its selected probability can be expressed as:

$$P_i = f_i / \sum_{k=1}^{M} f_k \tag{10}$$

Then,to calculate the cumulative probability for each chromosome:

$$q_i = \sum_{i=1}^{M} p_i \tag{11}$$

The second step is to select mating individualsbased on roulette selection. In order to select mating individuals, it need to have repeated rounds of selection, each round will generate a uniform random number between [0, 1], and then the random number will be the selection pointer to identify alternative individuals.

Crossover and Mutation

Crossover is the key to genetic manipulation. If the crossover probability is large, the ability of genetic algorithm to open up new search space will be enhanced, however, the possibility of better damage gene strings is relatively large, the convergence speed is reduced, and has unstable nature; if the crossover probability is small, the genetic algorithm search may fall into a passive state. For MTSP, the chromosome crossover can adopt cycle crossover (CX), ordered crossover (OX) and partially matched crossover (PMX) and so on.

CX is a crossover operation proposed by Oliver et al. based on TSP.In circular crossover operation, thecode sequence in child individuals is generated based on any parent individuals.OX is a crossover operation based on the path representation proposed by Davis et al. in 1985 according to TSP, this operation can be retain the arrangement and combine orderly structural units of different arrangement together.In crossover of two parent individuals, usually by selecting a portion of achromosome in parent individual 1, and conserving corresponding order of the chromosome in parent individual 2, thereby generating a child individual. PMX is a crossover operation based on the path representation proposed by Goldberg et al. in 1985 according to TSP. Partial crossover matching operation requires that randomly select two cross points, thereby determining amatching section, according to the mapping relationship given by the intermediate section between two cross points of two parent individuals, thereby generating the two child individuals.

In genetic manipulation, mutation belongs to supplementary search operation, and the main role of mutation is to maintain the diversity of population.A lower mutation probability may prevent certain important genes lost in groups, however, at the same time, it also reduces the ability of genetic algorithm to open up new search space; a higher mutation probability may make the operationof genetic algorithm tend to be purely random search, however, this approach reduces the stability and convergence speed of algorithm.For MTSP, chromosome mutation can adopt way of exchange mutation that is to randomly exchange genes in two positions.

Decoding

Suppose there areMTSP of eight cities (including source point cities) and three traveling salesmen. Wherein a chromosome is as follows:

6	7	8	4	1	9	5	2	3

Taking thischromosome as an example to have decoding operation on it, the specific process is as follows:

From the above description of the meaning of the chromosome encoding shows:

Cities and sequencevisitingby traveling salesman 1:

0-6-7-0;

Cities and sequence visiting by traveling salesman 2:

0-4-1-0;

Cities and sequence visiting by traveling salesman 3:

0-5-2-3-0。

Because there are eight travel cities in this problem, and the distance matrix between the cities is the matrix 8 × 8, so suppose that the matrix is represented as D. Because it is an addition of virtual symbol, without increasing the number of virtual city, therefore, the matrix D here is not the matrix of 10 × 10. In order to prevent the emergence of 0-0-0 walking path, in the matrix D, supposing the distance between the city 0 and the city 0 as a great number and the value is 10,000. Here, firstly set the matrix D as asymmetric distance matrix, which is also applicable for the decoding method following asymmetric distance matrix. Matrix D is as follows:

$$D = \begin{array}{cccccccccc} & \text{City 0} & & & & & & & \text{City 7} & \\ \left[\right. & 10\,000 & 13 & 10 & 11 & 9 & 8 & 18 & 21 & \left.\right] \text{City 0} \\ & 13 & 0 & 5 & 9 & 14 & 11 & 21 & 7 & \\ & 10 & 5 & 0 & 13 & 6 & 21 & 16 & 17 & \\ & 11 & 9 & 13 & 0 & 11 & 13 & 19 & 12 & \vdots \\ & 9 & 14 & 6 & 11 & 0 & 14 & 16 & 18 & \\ & 8 & 11 & 21 & 13 & 14 & 0 & 8 & 19 & \\ & 18 & 21 & 16 & 19 & 16 & 8 & 0 & 22 & \\ & 21 & 7 & 17 & 12 & 18 & 19 & 22 & 0 & \text{City 7} \end{array} \qquad (12)$$

In specific programming by computer, the following decoding steps can be used:

1)By the visiting path of traveling salesman 1 can obtain:

$$\begin{cases} x_{061} = 1 \\ x_{671} = 1 \\ x_{701} = 1 \end{cases} \tag{13}$$

Therefore, the reachability matrix of traveling salesman 1 is as follows:

$$X_1 = \begin{array}{c} \begin{array}{cccccccc} \text{City 0} & & & & & & & \text{City 7} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{City 0} \\ \\ \\ \vdots \\ \\ \\ \\ \text{City 7} \end{array} \tag{14}$$

Similarly, the reachability matrixes of traveling salesman 2, 3 are as follows:

$$X_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

$$X_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

2)The walking distance of each salesman:

Dot productthe about to reach matrix X_1 and the matrix D of traveling salesman 1 to obtain matrix D_1, the sum of value that is not 0 in matrix D_1 is the distance that the traveling salesman visited the city. This paper set the visiting cities distance of traveling salesmen 1, 2, 3 are respectively z_1、z_2、z_3.

$$D_1 = X_1 * D \tag{17}$$

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 \\ 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

z_1 =18+22+21=61 (19)

Similarly

z_2 =9+14+13=36 (20)

z_3 =8+21+13+11=53 (21)

3)Finally, according to the formula (19), (20), (21) to obtain the maximum walking distance of all traveling salesmen:

$Z = \max(z_1, z_2, z_3)$ =61 (22)

CASE SIMULATION

The above content introduces the steps that genetic algorithm optimizesMTSP; this section is simulated by the data of asymmetric distance in TSPLIB, and compares the performance of different crossovers.

MTSP of 17 Cities (br17. atsp)

In br17. atsp data there are total 17 cities, and the distance between the cities is asymmetrical. Suppose there are3traveling salesmen, and simulate on it.

Here, each chromosome adopts respectively CX, OX and PMX, set crossover probability as 0.55; mutationadopts exchangingmutation and mutation probability is set to be 0.25; the size of the population is set to be 100; the maximum number of iteration is set to be 100. Programming it by Matlab language and randomly running 10 times, the results are shown in Table 1.

Table 1. The simulation results of br17. Atsp.

Crossover operator	Optimal solution	Worst solution	Average solution
OX	28	31	28.9
CX	28	31	28.3
PMX	28	31	28.9

As used herein, the optimal solution of the routes are as follows:

1-8-9-17-11-13-12-1,

1-7-16-6-4-5-15-1,

1-3-14-10-2-1.

Among which: the passing route of three traveling salesmen are respectively 13, 28 and 11.

This case is a small-scale MTSP, and the data obtained from Table 1 shows that the results obtained by the algorithm in this paper is quite stable.

MTSP of 124 Cities (kro124p)

In kro124p data there are a total of 100 cities, and the distance between the cities is asymmetrical. Set the numbers of traveling salesmenare 4,8,12, and simulate on it.

Here, each chromosome adopts respectively CX, OX and PMX, set crossover probability as 0.55;mutation adopts exchanging mutation and mutation probability is set to be 0.25; the size of the population is set to be 100; the maximum number of iteration is set to be 500. Programming it by Matlab language, andwhen the numbers of traveling salesmen are 4,8,12, randomly running 10 times respectively, the results are shown in Table 2.

Table 2. The simulation results of kro124p.

Number of traveling salesman	**Crossover operator**	**Optimal solution**	**Worst solution**	**Average solution**
4	CX	24174	26084	25251.0
	OX	24610	29936	26297.0
	PMX	21454	25966	23663.0
8	CX	15091	17154	16054.5
	OX	15022	17134	16333.9
	PMX	14157	16211	14894.6
12	CX	11436	13192	12196.5
	OX	11891	13604	12707.8
	PMX	10965	13272	11974.9

This case is a large-scale MTSP, and the data obtained from Table 2 shows thatunder the above parameter setting situation, both in terms of the average solution or optimal solution, based on a path representation PMX is always better than CX and OX.

CONCLUSION

This paper analyzesMTSP ofminimizing maximum distance of multiple traveling salesmen by genetic algorithm, and proposes a feasible method of matrix decoding. Matrix decoding method is suitable for solving MTSP of asymmetric and symmetric distance. In this paper, MTSP of asymmetric distance is as sample to have tests and the performances of different crossovers are compared.

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