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Dynamic Modeling of Parallel Mechanism Based on Particle System

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ABSTRACT: As the traditional Lagrange method considers the connecting rod of the parallel mechanism as a particle, the accuracy of dynamic model tends to be low. In this study, a new dynamic modeling method based on particle system is presented for a six-pyramid parallel mechanism. The kinetic energy and the potential energy of the platform are calculated in terms of energy, and the inertia matrix is obtained. On the premise of fast calculation speed, the connecting rod is considered as a particle system. The kinetic energy and the potential energy of the connecting rod are calculated, and then the dynamic model of the six-pyramid parallel mechanism is established. To verify the correctness of the mathematical model, the prototype and its control system are developed based on dSPACE real-time simulation system. The comparative experiments are done according to a six-dimension trajectory task, and the force variation of the actuators calculated with the traditional Lagrange method and that calculated with the method based on particle system are compared with the actual variation. The results show that the dynamic model using the particle method can reduce errors by up to 60% than that using the traditional method. This research has laid the foundation for further application of parallel mechanisms in the mechanical industry.

KEYWORDS: Parallel mechanism; Dynamics; Particle system; Prototype.

INTRODUCTION

As the parallel mechanism has advantages such as high stiffness, fast response and fewer accumulated errors, it has been one of the hot points in the research field so far [1]. The research on parallel mechanism mainly includes two aspects: kinematics [2] and dynamics [3]. The dynamics is mainly about exploring the mapping relationship between the input of the driving force and the output of the pose of load platform. A reasonable dynamics modeling method is the precondition of the research and will lay a solid foundation for high precision real-time control and structural parameter optimization [4]. Dynamic modeling methods which are commonly used include the Lagrange method [5], the Newton Euler method [6], and the Kaine equation method [7], which are essentially equivalent in establishing the dynamic model of parallel mechanism [8]. When describing the parallel mechanism in terms of energy, the Lagrange method has the simplest explicit structure by not taking internal forces into account and and is therefore widely used nowadays.

Zheng [9] established the dynamic model of a 3-PRS-XY parallel mechanism using the Lagrange method for the specific requirement of forward feedback control. The dynamic model of 3-DOF stabilized platform was established by Lagrange method, and the controllers were capable of overcoming the disturbances under the mathematical model [10]. Dynamic model of a 2-DOF quasi-sphere parallel platform was established by Chen [11]; the effective inertia variation of the mechanism is discussed to improve the design of its components. The characteristic of a 3-DOF rotational platform was analyzed based on Lagrange method by Gao [12], and the third degree equation of the mechanism was obtained in respect to its torque and output angle.

The traditional Lagrange method usually considers the connecting rod as a particle and ignores the rotational inertia, the coriolis force and the centrifugal force in order to obtain a simplified dynamic model. But the connecting rod is a rigid body with rotational inertia in fact. Based on the research in literature [13] and using a six-pyramid parallel mechanism as the research object, this study presents a novel dynamic modeling method based on particle system, which regards the connecting rods as a particle system. Ensuring the speed of calculation, the kinetic energy and potential energy expressions are obtained to improve the accuracy of the dynamic model. The research will lay a good foundation for further application of parallel mechanisms in mechanical industry.

ANALYSIS OF THE SIX-PYRAMID PARALLEL MECHANISM

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Structure and working principle

The structure of the six-pyramid parallel mechanism is presented in Figure 1.

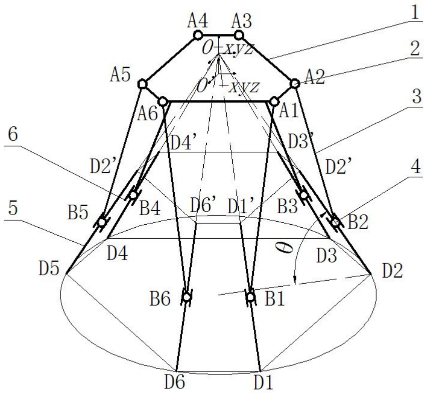


Figure 1. Sketch map of six-pyramid parallel mechanism.

The six-pyramid parallel mechanism consists of the load platform (1), the Hooke joint (2), the connecting rod (3), the spherical joint (4), the lead rail (5) and the sliding pair (6). The system has defined two coordinate systems: one is a moving coordinate system *O'-x'y'z'*, which is fixed to the center of the load platform; the other is the inertial coordinate system *O-xyz*, whose origin is the intersection of the direction of extension line of the six lead rails. The initial state of the mechanism is defined as the load platform in parallel with the *xy* plane of the inertial coordinate system *O-xyz.*

The six lead rails *DiD'i* (*i*=1-6) are arranged inclined to the ground with the angle θ from 0 to 90 degrees, and the performance of the spoke inclined parallel mechanism varies when the angle θ changes. The lower end of the connecting rod is connected with the motor rotor through a spherical joint at *Bi* (*i*=1-6), and the upper end of connecting rod is connected with the load platform through a Hooke joint at *Ai* (*i*=1-6). The six motor rotors of the six-pyramid parallel mechanism do reciprocating motion along the lead rails in the range of *DiD'i* (*i*=1-6) in order to make the motion of the load platform translate along the three axis and rotate around the three axis of the inertial coordinate system *O-xyz* .

Calculation of the degree of freedom

The degree of freedom of the six-pyramid parallel mechanism can be calculated according to Grubler formula:

$$f_0 = 6m - \sum_{i=1}^{n} f_i \tag{1}$$

In the formula, f_0 is the degree of freedom of the six-pyramid parallel mechanism; m is the number of the moving parts; n is the number of the kinematic pairs; f_i is the product of the number of restricted freedom of kinematic pairs and the number of the kinematic pairs.

The moving parts of the six-pyramid parallel mechanism include six motor rotors, six connecting rods and a load platform, which are 13 in total. The kinematic pairs of the six-pyramid parallel mechanism include six sliding pairs, six Hooke joints and six spherical joints, which are 18 in total. The number of restricted freedom of spherical joint is 3; the number of restricted freedom of Hooke joint is 4 and the number of restricted freedom of sliding pair is 5. Then the degree of freedom of the six-pyramid parallel mechanism can be calculated as:

$$f_0 = 6 \times 13 - 6 \times (5 + 4 + 3) = 6 \tag{2}$$

It is thus clear that the degree of freedom is 6; the mechanism can realize linear motion along the three coordinate axes and realize rotary motion around the three coordinate axes. When the mechanism is working, only the motor rotors and the connecting rods are moving. Compared to the Stewart platform whose hydraulic cylinders are attached to the connecting rods, the six-pyramid parallel mechanism has the advantage of small rotational inertia which is the precondition to realize high speed and high precision control.

DYNAMIC MODELING BASED ON PARTICLE SYSTEM

Kinetic energy and potential energy of the platform

Suppose the mass of the load platform of the six-pyramid parallel mechanism is m, the pose of the load platform is $\boldsymbol{P}=[x_p,y_p,z_p,\alpha,\beta,\gamma]^T$ and the velocity of the center of mass is $\boldsymbol{V_c}=[\mathbf{V}\boldsymbol{p},\boldsymbol{\omega_p}]^T$, given that the kinetic energy of the platform KE_1 can be divided into translational energy and rotational energy according to the rigid body dynamics, KE_1 can be expressed as

$$KE_1=\frac{1}{2}[m\boldsymbol{V}_p^2+\boldsymbol{\omega}_p^T\boldsymbol{RIR}^T\omega_p] \tag{3}$$

where $\boldsymbol{I}$ is the inertia of the load platform in relative coordinates O'-$x'y'z'$, and $\boldsymbol{R}$ is the space transformation matrix. It should be noted that the load platform of the six-pyramid parallel mechanism is a regular rigid body and the inertia $\boldsymbol{I}$ has a simple form in a moving coordinate system:

$$\boldsymbol{I}=\begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \tag{4}$$

where I_x is the rotational inertia of the load platform around the x axis; I_y is the rotational inertia of the load platform around the y axis; and I_z is the rotational inertia of the load platform around the z axis. We can transform Formula (3) to get the inertial matrix of the load platform -- $\boldsymbol{M_1}$. As the expression is complicated, the non-zero elements of the matrix are listed as follows:

$$\begin{aligned}
&M_{11}=M_{22}=M_{33}=m\\
&M_{14}=m\sin\gamma\cos\beta\\
&M_{15}=m\cos\gamma\\
&M_{24}=-m\cos\gamma\cos\beta\\
&M_{25}=m\sin\gamma\\
&M_{41}=m\sin\gamma\cos\beta\\
&M_{42}=-m\cos\gamma\cos\beta\\
&M_{44}=m\cos^2\beta+I_x\\
&M_{46}=-I_x\sin\beta\\
&M_{51}=m\cos\gamma\\
&M_{52}=m\sin\gamma\\
&M_{55}=m+I_y\cos^2\alpha+I_z\sin^2\alpha\\
&M_{56}=(I_y-I_z)\cos\beta\sin\alpha\cos\alpha\\
&M_{64}=-I_x\sin\beta\\
&M_{65}=(I_y-I_z)\cos\beta\sin\alpha\cos\alpha\\
&M_{66}=I_x\sin^2\beta+I_y\cos^2\beta\sin^2\alpha+\\
&I_z\cos^2\beta\cos^2\alpha
\end{aligned} \tag{5}$$

The potential energy of the load platform is determined by the vertical distance from the center of the mass relative to the inertial coordinate system O-xyz, which is independent from the pose, velocity and acceleration in other directions. When the output pose of the load platform is $\boldsymbol{P}=[x_p,y_p,z_p,\alpha,\beta,\gamma]^T$, the potential energy of the load platform P_1 has a simple form:

$$P_1=mgz_p \tag{6}$$

Kinetic energy and potential energy of connecting rods

In fact, the mass of the connecting rods is unevenly distributed. A connecting rod consists of motor rotors, the spherical joint, the linkage and the Hooke joint. Suppose that the mass of each connecting rod is m_L, each rod is divided into n segments of the same length and the kinetic energy and potential energy of the segments are calculated. Since each segment of the connecting rod has a different quality, we can regard each segment as a particle to calculate its kinetic energy respectively. If the quality of a particle j ($j=1-n$) from the lower end to the upper end is m_{Li}, then the kinetic energy of the particles of the connecting rod KE_{2ij} is:

$$KE_{2ij} = \frac{1}{2} m_{Lj} v_{ij}^2 \tag{7}$$

If the velocity of the upper end of the connecting rod i is $\boldsymbol{V_{Ai}}$, and the velocity of the lower end of the corresponding connecting rod is $\boldsymbol{V_{Bi}}$, then the velocity of the arbitrary particle of the connecting rod can be expressed as:

$$v_{ij} = \frac{j}{n}(\boldsymbol{V_{Ai}} + \boldsymbol{V_{Bi}}) \tag{8}$$

$\boldsymbol{V_{Ai}}$ and $\boldsymbol{V_{Bi}}$ can be also expressed using the generalized velocity of the load platform as:

$$\begin{cases} \boldsymbol{V_{Ai}} = \boldsymbol{G^{Ai}G^{c}\dot{P}} \\ \boldsymbol{V_{Bi}} = \boldsymbol{J\dot{P}} \end{cases} \tag{9}$$

where $\boldsymbol{G^{Ai}G^{c}}$ is the transformation matrix of velocity, which reveals the relationship of generalized velocity between the input of the motor rotors and the output of the center of mass of the load platform; $\boldsymbol{J}$ is the Jacobian matrix of the six-pyramid parallel mechanism. The kinetic energy of the connecting rod i can be expressed via Formula (7-9) as:

$$KE_{Li} = \frac{1}{2}\sum_{j=1}^{n} m_{Lj} \frac{j^2}{n^2} \boldsymbol{\dot{P}}^T (\boldsymbol{G^{Ai}G^{c}} + \boldsymbol{J})^T (\boldsymbol{G^{Ai}G^{c}} + \boldsymbol{J})\boldsymbol{\dot{P}} \tag{10}$$

Thus, the kinetic energy of the six connecting rods KE_2 is

$$KE_2 = \sum_{i=1}^{6} KE_{Li} \tag{11}$$

and the inertia matrix of the six connecting rods $\boldsymbol{M_2}$ is

$$\boldsymbol{M}_2 = \sum_{i=1}^{6}\sum_{j=1}^{n} m_{Lj} \frac{j^2}{n^2} (\boldsymbol{G^{Ai}G^{c}} + \boldsymbol{J})^T (\boldsymbol{G^{Ai}G^{c}} + \boldsymbol{J}) \tag{12}$$

For the connecting rods which are considered as the system of particles, each particle of the connecting rod has a different position in the inertial coordinate system *O-xyz*, and is only related to the difference in the direction of axis z between corresponding Hook joint of the load platform and the corresponding motor rotor. Take the particle j of the connecting rod i as research object, the coordinate of the particle can be expressed as:

$$P_{ij} = m_{Lj} g \frac{z_{Ai} + (j+n) z_{Bi}}{n} \tag{13}$$

where,z_{Ai} is the value of the direction of axis z in *O-xyz* of the corresponding Hooke joint; z_{Bi} is the value of the direction of axis z in *O-xyz* of the corresponding motor rotor. Then, the potential energy of the connecting rod i can be obtained, and the potential energy of the six connecting rods P_2 can be expressed as:

$$P_2 = \sum_{i=1}^{6}\sum_{j=1}^{n} m_{Lj} g \frac{z_{Ai} + (j+n) z_{Bi}}{n} \tag{14}$$

Dynamic model and analysis

The Lagrange function $\boldsymbol{\Gamma}$ is defined as the difference between kinetic energy *KE* and potential energy *KP*. For six degrees of freedom parallel mechanism, the numerical value of Coriolis force and centrifugal force are too small to measure. Definite driving force τ of the system, Lagrange equation can be written in the form of vector:

$$\frac{d}{dt}\left(\frac{\partial \boldsymbol{\Gamma}}{\partial \boldsymbol{\dot{P}}}\right) - \frac{\partial \boldsymbol{\Gamma}}{\partial \boldsymbol{P}} = \boldsymbol{\tau} \tag{15}$$

If the kinetic energy and potential energy of the mechanism are expressed as below:

$$\begin{cases} KE = \frac{1}{2}\dot{\boldsymbol{P}}^T \boldsymbol{M}(\boldsymbol{P})\dot{\boldsymbol{P}} \\ KP = F(\boldsymbol{P}) \end{cases} \tag{16}$$

Then, the Lagrange equation of the six-pyramid parallel mechanism takes the following standard form:

$$\boldsymbol{M}(\boldsymbol{P})\ddot{\boldsymbol{P}} + \boldsymbol{G}(\boldsymbol{P}) = \boldsymbol{\tau} \tag{17}$$

where ***M***(***P***) is the inertia matrix of the six-pyramid parallel mechanism, which includes the inertia matrix of the load platform and the inertia matrix of connecting rods; ***G***(***P***) is the partial derivatives of the potential energy of the six pyramid parallel mechanism. If the inertia matrix of the system ***M***(***P***) and the potential energy of the system *KP* are known, the expression of the generalized force can be derived by ignoring the coriolis force and centrifugal force:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{P})\ddot{\boldsymbol{P}} + \frac{\partial KP}{\partial \boldsymbol{P}} \tag{18}$$

According to formula (3), (6), (12), (14) and (18), the relationship between driving forces of the motors and the pose of the load platform can be obtained, and the kinetic equation of the six-pyramid parallel mechanism can be expressed as:

$$\boldsymbol{F} = (\boldsymbol{J}^T)^{-1}[(\boldsymbol{M}_1 + \boldsymbol{M}_2)\ddot{\boldsymbol{P}} + \frac{\partial KP_1}{\partial \boldsymbol{P}} + \frac{\partial KP_2}{\partial \boldsymbol{P}}] \tag{19}$$

PROTOTYPE AND EXPERIMENTS

Prototype and its control system

For further research on the six pyramid parallel mechanism and its performance, the prototype of this mechanism has been developed which is shown in Figure 2.

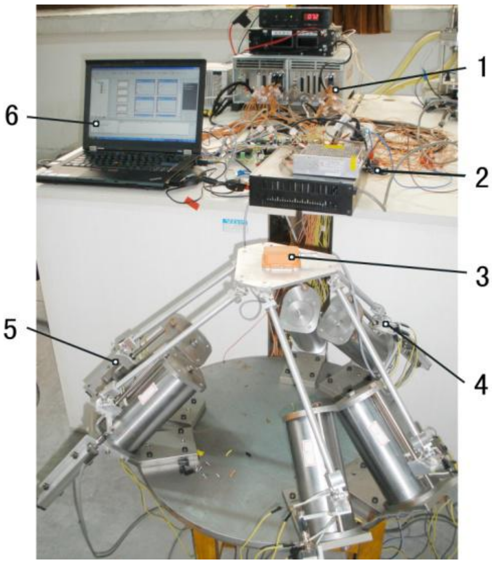


Figure 2. Prototype of six pyramid parallel mechanism.

The control system consists of the dSPACE real time simulation system (1), the current sensor (2), the pose sensor (3), the linear motor (4), the displacement sensor (5) and the upper monitor (6). As the force of the linear motor is not easy to be measured directly, the current is measured to calculate the driving force according to the formula of Ampere force:

$$F = BIL \tag{20}$$

When we input a six-dimension trajectory task to the upper monitor, the forces of the linear motors can be calculated based on the dynamic model. Orders are provided from dSPACE to linear motors, and corresponding signals are measured by current sensors and displacement sensors. Based on the feedback of force and displacement signals to dSPACE, closed-loop control is realized to improve the accuracy of the motion of load platform according to the target trajectory. At the same time, the displacement along the three axes and the rotation around the three axes are measured by pose sensor, and the signals are delivered to upper monitor for data recording and real-time monitoring.

Measurement of motor resistance

High performance linear motors developed by our research group are used as the driving source for the six-pyramid parallel mechanism. From the formula of Ampere force, we know that the current and the driving force of the linear motor are in direct proportion, and the linear motor constant *BL*=18, so the relationship between the current and motion trajectory of the motors is

$$\boldsymbol{I} = \frac{\boldsymbol{\tau} + \boldsymbol{F_0}}{BL} \tag{21}$$

In this formula, $\boldsymbol{F_0}$ is the resistance of the motor. The mass parameters of the components must be measured and calculated before the experiments. The related parameters of the mechanism are listed and shown in Table 1.

Table (1). Mass parameters.

Name	mass/g
Motor rotor	78.6
Connecting rod	89.4
Load platform	306.3

When the linear motor is idling and doing a uniform upward movement, the friction caused by velocity changes can be ignored. From the force analysis of the motor rotor, we know that the resultant consists of the downward vertical gravity, the ampere force and the motor resistance. Then, the expression of motor resistance can be obtained by Newton's second law:

$$F_0 = BIL - m_d g \cos 45^{\circ} \tag{22}$$

where m_d is the mass of the motor rotor. The value of the current is measured by the current sensor. With the corresponding structure parameters, we can get the value of each motor resistance, as shown in Table 2.

Table 2. Current and motor resistance.

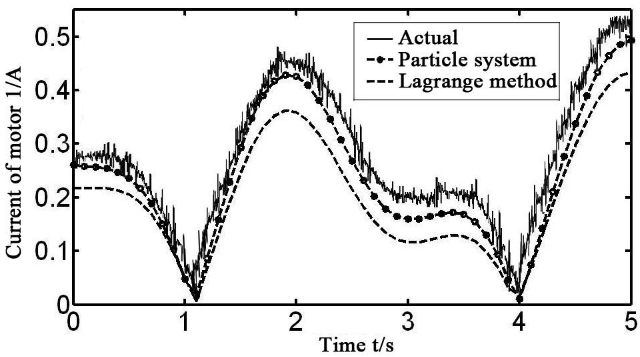
Motor number	Current/A	Motor resistance/N
1	0.12	1.6
2	0.20	3.04
3	0.18	2.68
4	0.15	2.14
5	0.14	1.96
6	0.17	2.5

Comparative experiments

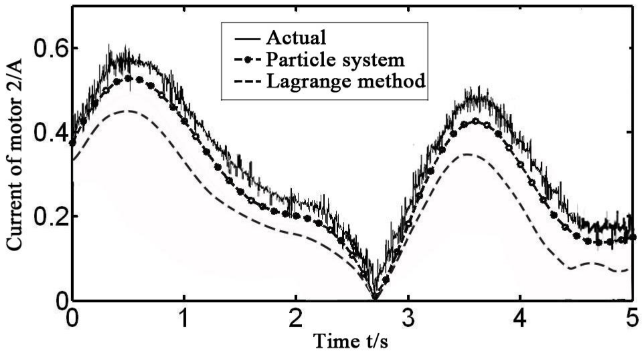
The correctness of the dynamic model based on particle system of the six-pyramid parallel mechanism is the foundation for the research of the next step. So a comparative experiment was conducted according to the six-dimension trajectory tasks as below:

$$\boldsymbol{P} = \begin{pmatrix} x_p \\ y_p \\ z_p \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 5\sin(t) \\ 5\sin(2t) \\ 3\sin(t) \\ 0.0436\sin(t) \\ 0.0436\sin(2t) \\ 0.0872\sin(t) \end{pmatrix} \tag{23}$$

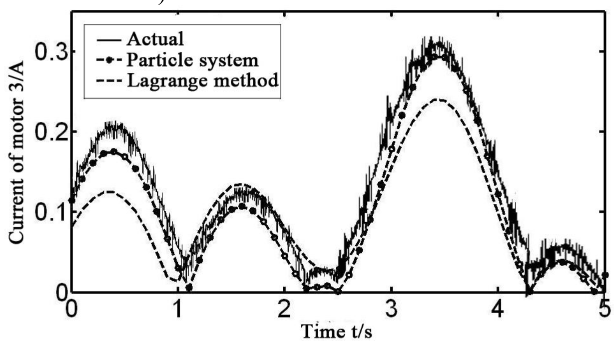
In order to compare the accuracy of the modeling method based on the particlesystem-with that based on the traditional Lagrange dynamics method, the currents of the six motors are measured using both methods as mentioned above, and the variations of the currents are drawn on the same graph along with the changes of the actual current variation, as shown in Figure 3.



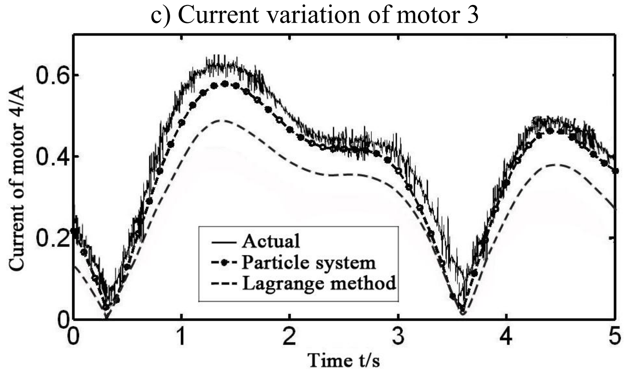
a) Current variation of motor 1



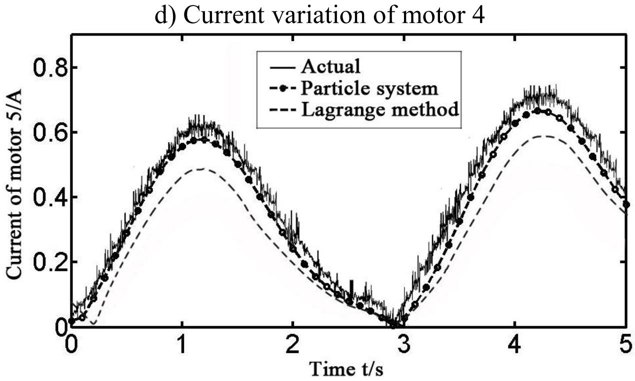
b) Current variation of motor 2



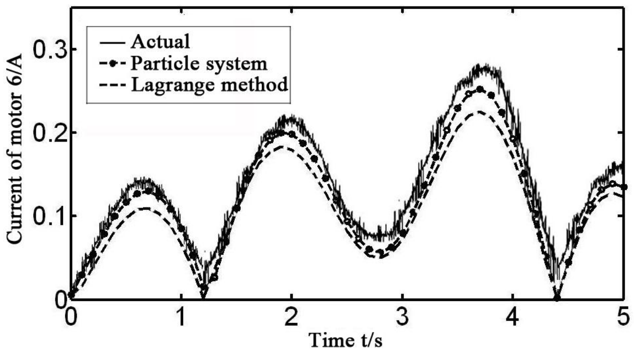
c) Current variation of motor 3



d) Current variation of motor 4



e) Current variation of motor 5



f) Current variation of motor 6

Figure 3. Comparison of theoretical and measured current variation.

The result has shown that the current of the six linear motors fluctuates between 0 to 0.62A, and the largest driving force to realize the target trajectory is 11.16N. Both theoretical driving currents using dynamic modeling methods based on particle system and traditional Lagrange method have the same rules with the actual driving current. For different motors, the errors of the two modeling method are different. It is because the acceleration of each motor is different, resulting in different motion of inertia of connecting rods. The major reasons why the theoretical value is less than the measured value include the following aspects:

(1) The accuracy of the dynamic modeling method. As the motion of the inertia of connecting rods, and the centrifugal and coriolis forces of the system are ignored when establishing the model, the actual driving current is bigger than the theoretical current. From Figure 3, we can see that the dynamic modeling method based on the particle system can increase the accuracy to a certain extent, the error can be reduced from 10% to 4% compared with traditional Lagrange dynamics method, and the accuracy can be increased to 60%.

(2) The friction of the components of the mechanism. The friction force of the motors and the gravity of the motor rotors have been considered in the modeling method. However, several forces which are not easy to measure, such as the friction of sensors and the friction of moving pairs, can cause the actual current value to be higher than the theoretical current value.

(3) Control accuracy. The six-pyramid parallel mechanism has six input signals and six output signals, and any error caused by the lack of control accuracy will cause interference to each other within the mechanism. Such problem can only be solved by improving the motion trajectory and control precision.

To sum up, for a given six-dimension trajectory task, the theoretical current variation of the motors obtained with the dynamic method based on particle system are consistent with the actual current variation, and the dynamic method presented in this paper is effective and correct.

CONCLUSIONS

(1) This paper presents a six-pyramid parallel mechanism, the structure characteristics of the mechanism are analyzed, and the dynamic model is established by calculating the kinetic and potential energy based on the particle system.

(2) The prototype of the mechanism has been produced based on dSPACE real time simulation system. The relationship between the driving forces and the current has been revealed, and comparative experiments have been done to prove the dynamic method based on the particle system is effective and correct.

(3) The accuracy of the dynamic modeling method based on the particle system and that based on the traditional Lagrange dynamics method have been compared with the actual data. The result shows that the dynamic modeling method based on particle system can reduce the error from 10% to 4%, and the accuracy can be increased to 60%.

CONFLICT OF INTEREST

This article content has no conflict of interest.

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