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Analytical Analysis for Constant Wall Temperature for A Pipe with Power Law Fluid

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ABSTRACT: A completely developed laminar flow-based velocity distributions availed by a model governed by a power-law rheology of fluid has been utilized, while glutinous indulgence was considered. The conceptual analysis of the performance of heat transfer was executed under an unchanged wall temperature case. A significant feature of such approach is permitting a commonplace distribution of neighboring mid-temperature as well as the fluid's simpleton velocity distribution. All of these mechanisms had been tested by a relativity with the prevalent results. The Brinkman number's effects and rheological materials on the home Nusselt number's distribution were studied. It has been shown that the notion associated with the Nusselt number stubbornly relies upon the power law based index value. That Nusselt number strikingly gets reduced in the 0 < n < 0.1 range. Nonetheless, for n > 0.5, again, for n > 1, Nusselt number values are approaching an invariable value.

KEYWORDS: laminar; velocity; temperature; Nusselt number

INTRODUCTION

A good understanding of heat convection using non-Newtonian fluids within circular tube is critical towards the design in connection with numbers of thermal instruments. From this perspective, heat transfer shortcomings in relation with this kind have been inspected by many researchers [1-6].The fundamental Graetz problem that had firstly been resolved analytically by Graetz [7], has now been the classic Graetz –Nusselt problem within a singular segment flow, which ignores the axial heat conduction effects, sources of thermal power within fluid, and gluey dissipation. It is acknowledged to be one of the most significant resolutions for science of heat transfer wile it presides over the convective heat transfer fluid flow with the case of fluid using recognized velocity profile as well as gets related with the outcome of the rate of heat transfer in a completely developed fluid flowing flow . This kind of resolution lets temperature profile as measured from the motion and energy centric joined equations [1, 8, 9].

An all-inclusive analytically studied power-law fluid flow, which is fully developed within a circular pipeline for both wall temperature and standardized wall heat flux, has been done[10], however the authors overlooked the gluey dissipation effects. They have demonstrated that the value of Nusselt number for an power-law fluid in is provided as:

$$Nu = \frac{8(1+3n)(1+5n)}{31n^2+12n+1} \tag{1}$$

With n indicate the index of power-law. Where n =1, Eqn. (1) result as:

$$Nu = \frac{48}{11} = const. \tag{2}$$

Nonetheless, Graetz problem is escalated as issues that concentrate on disorderly flows, forced convection within a permeable means, non-Newtonian flowing, and the impacts of gluey indulgence for Newtonian liquid and that incorporate heat conduction effects [2, 5, 10-13]. Among the above cited works, no studies are available in relation with the gluey dissipation effects upon the heat transfer for not Newtonian liquid.

Hence, the purpose of the current study has been to resolve the problem related to heat transfer with forced convection mathematically, in a pipeline submitted to consistent wall temperature with completely developed zone, it has been kind of Graetz problem, deriving full mathematical resolutions appropriate to a liquid heat distribution as well as Nusselt number (local). As the current study highlights heat transfer with adequately huge Peclet number(Pe), the study considers the axial heat conduction to be ignorable. Nevertheless, gluey dissipation has been taken into consideration. Numerical measurements have been performed to expose the rheological properties effects as well as Brinkman number(Br) effects upon the Nusselt number (local) distribution.

In the literature, they has been studied the Newtonian fluid slug flow forced convection with reference for flow with turbulent or to flow with laminar for law value of the Prandtl number Moreover, we can describe with good thermal contact slug flow forced convection a solid rod moving through a heated sleeve. The researchers have been found some of the most important results reviewed by Shah and Bhatti [2]. The region with thermal entrance has been examined by Golos [3] and by Tyagi a.nd Nigam studied heat transfer and the momentum on a the power law fluids continuous moving surface for the power law fluids. Recently, significant consideration has been dedicated to the problem of how of how to foresee the conduct of the heat transfer and flow behavior of fluids with non-Newtonian. The principle purpose behind this is most likely that fluids (such as pulps, molten plastics, emulsions, slurries, etc.), which don't comply with the Newtonian postulate that the stress tensor is legitimately corresponding to the deformation tensor, are delivered mechanically in expanding amounts and are in this way now and again similarly prone to be pumped in a plant as the more common Newtonian fluids. Researchers have been attempting to set up a scientific model to express the connection between the stress and deformation and heat transfer for the non-Newtonian fluids.[14,12,15] .The main objective of this study is to investigate, in depth, the analytical solution of constant wall temperature for fluid flow inside a pipe.

FORMULATION OF MATHEMATICAL MODEL

Fig (1) Notations and axes of the fully developed constant wall temperature on pipe

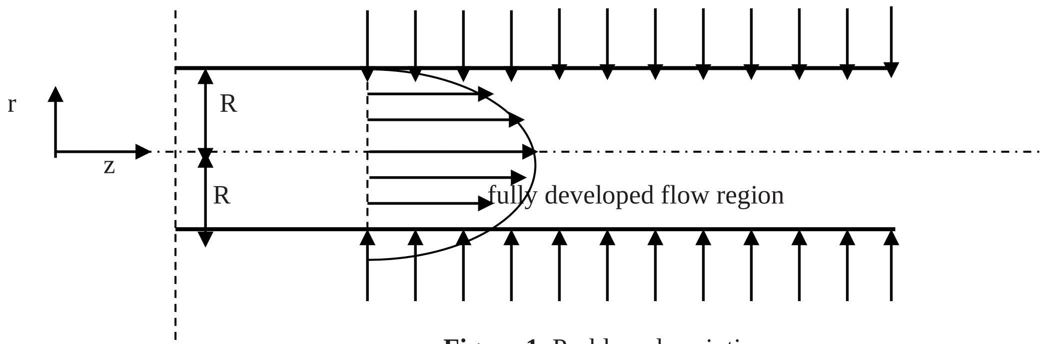

Figure 1. Problem description

The shear stress (τ_{rz}) over a viscous fluid is formulated [3, 14-16] as:

$$\tau_{rz} = -n\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr} \tag{3}$$

n is represented the parameter index of power-law model.
There are three cases as: pseudo-plastic fluid where $n<1$, Newtonian fluid for $n=1$, and dilatant fluid when $n>1$
There are simplifying assumptions as following:
1. The flow is laminar and steady.
2. Physical properties are constant.
3. The effects of Natural convection are neglected

$$\frac{u(r)}{u_m} = \frac{1+3n}{1+n}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] \tag{4}$$

With substituted Eqn. (3) with Eqn. (4), the expression for shear stress can be written as:

$$\tau_{rz}=m\left[\left(\frac{u_m}{R}\right)\left(\frac{3n+1}{n}\right)\left(\frac{r}{R}\right)^{\frac{1}{n}}\right]^n \qquad (5)$$

for steady-state heat , heat balancing can be taken as follows[1, 6, 8, 14]:

$$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\rho c_P u(r)\left(\frac{\partial T}{\partial r}\right)-\tau_{rz}\left(\frac{du}{dr}\right) \qquad (6)$$

Where k, c and ρ are the thermal conductivity specific heat, and density, respectively.

The applicable boundary conditions are :

$$BC1:\ \ r=0\ \ \ at\,centerline\ \ \ \ r=R\ \ at\ \ Wall$$
$$BC2:\ \ at\ \ \ \ r=0\ \ \ \ \ \ \frac{\partial T}{\partial r}=0$$
$$BC3:\ \ at\ \ \ \ r=R\ \ \ \ \ \ T=T_w$$

Now, coupling Eqn. (4) with Eqn. (6) represents;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{u_m}{\alpha}\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]\left(\frac{\partial T}{\partial z}\right)+\frac{m}{k}\left(\frac{3n+1}{n}\right)^{n+1}\left(\frac{u_m}{R}\right)^{n+1}\left(\frac{r}{R}\right)^{\frac{n+1}{n}} \qquad (7)$$

thermal diffusivity α is defined by:

$$\alpha=\frac{k}{\rho c_p} \qquad (8)$$

Introduce the dimensionless quantities as follows:

$$\xi=\frac{r}{R};\eta=\frac{z}{2RPe};\theta=\left(\frac{T_w-T(r)}{T_w-T_c}\right) \qquad (9)$$

The dimensionless partial differential energy equation, eqns (7,9) can be written as :

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right)=\frac{A_0}{4}\left(\frac{1+3n}{1+n}\right)\left[1-\xi^{\frac{n+1}{n}}\right]-Br\left(\frac{1+3n}{n}\right)^{1+n}\xi^{\frac{1+n}{n}} \qquad (10)$$

By introduce Brinkman number (*Br*) as :

$$Br=-\frac{mu^{n+1}}{kR^{n-1}\left(T_w-T_c\right)} \qquad (11)$$

Where the Peclet number (*Pe*) is written as :

$$Pe=\frac{2u_mR}{\alpha} \qquad (12)$$

Now the dimensionless boundary is :

$$\begin{aligned} &BC1:\ at \quad \xi = 0 \quad \theta = 1 \\ &BC2:\ at \quad \xi = 0 \quad \frac{d\theta}{d\xi} = 0 \\ &BC3:\ at \quad \xi = 1 \quad \theta = 0 \end{aligned} \tag{13}$$

For constant wall temp.so the θ (ξ ,η) is a function of dimensional cylindrical coordinate (ξ) only .

$$\text{So :}\ \frac{\partial\theta}{\partial\eta} = A_0 = \text{constant} \tag{14}$$

Solving Eqn. (14) with Eqn. (10) represents as:

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) = \frac{1}{4}\left(\frac{1+3n}{1+n}\right)\left[1-\xi^{\frac{1+n}{n}}\right]\left(\frac{\partial\theta}{\partial\eta}\right) - Br\left(\frac{1+3n}{n}\right)^{1+n}\xi^{\frac{1+n}{n}} \tag{15}$$

We can formulate new variables as:

$$N = \left(\frac{1+3n}{n}\right)^{1+n} Br \tag{16}$$

$$C_0 = \frac{A_0}{4}\left(\frac{1+3n}{n}\right) \tag{17}$$

With

$$\beta = \frac{n+1}{n} \tag{18}$$

By solving Eqns. (15-18) gets ;

$$\frac{1}{\xi} = \frac{d}{d\xi}\left(\xi\frac{d\theta}{d\xi}\right) = C_0\left(1-\xi^{\beta}\right) - N\xi^{\beta} \tag{19}$$

By solving differential equation , the temperature profile can written as follows:

$$\theta(\xi) = C_0\left(\frac{1}{4}\xi^2 - \frac{1}{(\beta+2)^2}\xi^{\beta+2}\right) - \frac{N}{(\beta+2)^2}\xi^{\beta+2} + C_1\ln\xi + C2 \tag{20}$$

By getting all constants , the dimensionless temperature profile can formulated as follows:

$$\theta(\xi) = C_0\left(\frac{1}{4}\xi^2 - \frac{1}{(\beta+2)^2}\xi^{\beta+2}\right) - \frac{N}{(\beta+2)^2}\xi^{\beta+2} + 1 \tag{21}$$

By applying the other boundary condition the constant (C_0) written as :

$$C_0 = 4\left[\frac{N-(\beta+2)^2}{4-(\beta+2)^2}\right] \qquad (22)$$

Coupling Eqn (up) with Eqn. (21), the local radial temperature profile written as:

$$\theta(\xi) = \left(\frac{N-C_0}{(\beta+2)^2}\right)\xi^{\beta+2} + \frac{C_0}{4}\xi^2 + 1 \qquad (23)$$

In another form:

$$\frac{T_w - T(r)}{T_w - T_c} = \left(\frac{N-C_0}{\left(\frac{3n+1}{n}\right)^2}\right)\left(\frac{r}{R}\right)^2 + 1 \qquad (24)$$

The bulk temperature is written as[14 ,2] :

$$T_b = \frac{2\tau\rho c_p \int_0^{2\tau}\int_0^R T(r)u(r)rdrd\theta}{2\tau\rho c_p \int_0^{2\tau}\int_0^R u(r)rdrd\theta} \qquad (25)$$

The bulk temperature in dimensionless form becomes:

$$\theta_b = \frac{\int_0^1 \theta(\xi)\phi(\xi)\xi d\xi}{\int_0^1 \phi(\xi)\xi d\xi} \qquad (26)$$

and ϕ is the velocity profile in dimensionless form as :

$$\phi = \frac{u}{u_m} \qquad (27)$$

By substitution Eqns. (4) and Eqn. (26) into Eqn. (23) , temperature profiles and velocity becomes:

$$\theta_b = \frac{\int_0^1 \left\{\left[\left(\frac{N-C_0}{(\beta+2)^2}\right)\xi^{\beta+2} + \left(\frac{C_0}{4}\right)\xi^2 + 1\right]\left(1-\xi^{\beta}\right)\right\}\xi d\xi}{\int_0^1 \left(1-\xi^{\beta}\right)\xi d\xi} \qquad (28)$$

By integrating , Eqn. (28), the bulk temperature can be defined as the dimensionless forms as follows:

$$\theta_b = \left(\frac{2\left(N - C_0\right)}{\left(\beta+2\right)\left(\beta+3\right)\left(2\beta+3\right)} \right) + \left(\frac{C_0\left(\beta+2\right)}{8\left(\beta+4\right)} \right) + 1 \quad (29)$$

And the heat transfer can be written as :

$$q_w = h\left(T_w - T_b\right) = k\frac{dT}{dr}\Big|_{r=R} \quad (30)$$

We can definw the local Nusselt number (*Nu*) as :

$$Nu = \frac{hD}{k} = \frac{2R\frac{dT}{dr}\Big|_{r=R}}{\left(T_w - T_b\right)} \quad (31)$$

And :

$$Nu = 2\frac{\frac{d\theta}{d\xi}\Big|_{\xi=1}}{\left(\theta_w - \theta_b\right)} \quad (32)$$

when applying BC in Eqn. (23) with $\theta_w = \theta\Big|_{\xi=1}$, the eqn(up) is written e as:

$$\theta_w = \theta\Bigg|_{\xi=1} = \left(\frac{N - C_0}{\left(\beta+2\right)^2} \right) + \frac{C_0}{4} + 1 \quad (33)$$

Now from Eqn. (23), the temperature gradient for $\left(\frac{d\theta}{d\xi}\Big|_{\xi=1} \right)$ is written as:

$$\left(\frac{d\theta}{d\xi}\Big|_{\xi=1} \right) = \left(\frac{2N + \beta C_0}{2\left(\beta+2\right)} \right) \quad (34)$$

With coupling the Eqns. (28), (33) and (34) and Eqn. (31) , The Nusselt number (*Nu*) can be written as:

$$Nu = \left[\frac{31n^2 + 12n + 1}{8\left(1+3n\right)\left(1+5n\right)} + 2^{n-1} Br \left(\frac{1+3n}{n} \right)^n \right]^{-1} \quad (35)$$

RESULTS AND DISCUSSIONS

Due to the unavailability of the effect of dissipation (Br=), the resolution has been self-governing regarding if there is wall cooling or temperature. Nonetheless, gluey dissipation causes internal temperature of the liquid all the time; therefore, the solution would vary in accordance with the process that happens. As a criterion, Brinkman number (Br) has been selected to show the comparative significance of gluey removal. The briefness and status within an adequate range,(-1 < Br < 1). Wherever the values of Brinkman number is identical for temperature of wall (T_w <$T_c$and Br<0) case, which means that heat has been provided across walls into liquid, although the conversed notion is also true against negative values related to Br, which means cases of wall cooling (T_w <$T_c$and Br<0).

As mentioned earlier, the thermal limit preconditions are considered, appropriate to pipeline wall is the persistent wall temperature. Considering this limit condition, both wall cooling and wall temperature cases have been treated and tested separately [17-20].

Figures demonstrate the heat profiles become dimensionless through this parameter for wall temperature, no gluey dissipation and wall cases of wall cooling, individually, while such profiles are rooted in Eqn. (29). All of these plots ultimately make the clear notion about the aforementioned impacts of enhanced dissipation. As anticipated, enhanced dissipation grows towards the bulk heat of the liquid because of internal temperature of the liquid. For the case of wall temperature, this grows up within the liquid heat actively decreases the difference of heat between the fluid and the wall, as would be demonstrated later that has been pursued with a reduction in the heat transfer. Whenever the wall cooling has been implemented, because of the internal temperature impact of the gluey dissipation on the profile of the liquid temperature, the heat difference becomes enhanced with a growing of Br. In practice, the cooling of wall has been implemented for lessen of the heat bulk heat for a liquid. Hence, a degree of gluey dissipation can alternate the overall balance of heat. In the event that Brinkman number crosses a distinct limiting value, internally generated heat through gluey dissipation process tend to come on over the impact of the wall cooling.

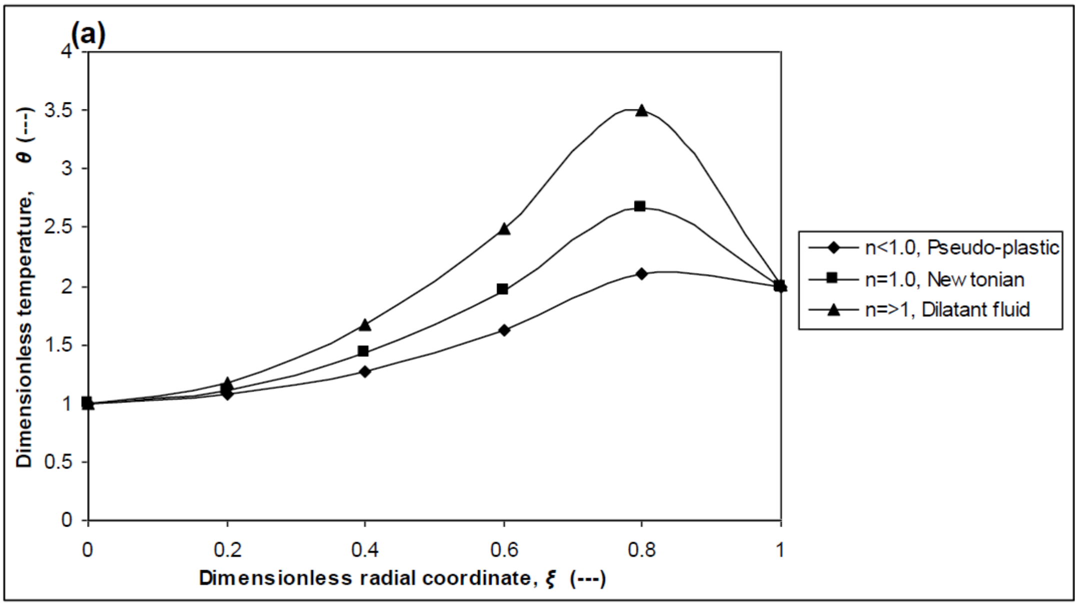

Figure 2. Fluid temperature profiles

Figure (3) symbolizes the variety of the Nusselt number for persistent wall temperature case with rheological power-law index (properties) with Brinkman number of different values. While the profiles of the downstream and asymptotic Nusselt number are demonstrated concisely for the temperature of wall (Brinkman number > 0). Whenever the cooling of wall (Brinkman number <0) has been implemented to lessen the temperature of bulk for a liquid, as earlier described, size of dissipation can alternate the balance of heat on a whole. With the value of the growing of Br towards the downbeat direction, Nusselt number attains a certain asymptotic value. According to what has been noticed so far, whenever Br turns into becoming the infinity appropriate to either the case of wall temperature or wall cooling, Nusselt number gains the indifferent asymptotic value. It is because the heat created internally through gluey dissipation processes would help balance the impact of the wall cooling. Nominally, the Nusselt number on the basis of gluey effects appropriate to both wall cooling and wall temperature will be less than the non-gluey dissipation centric Nusselt number.

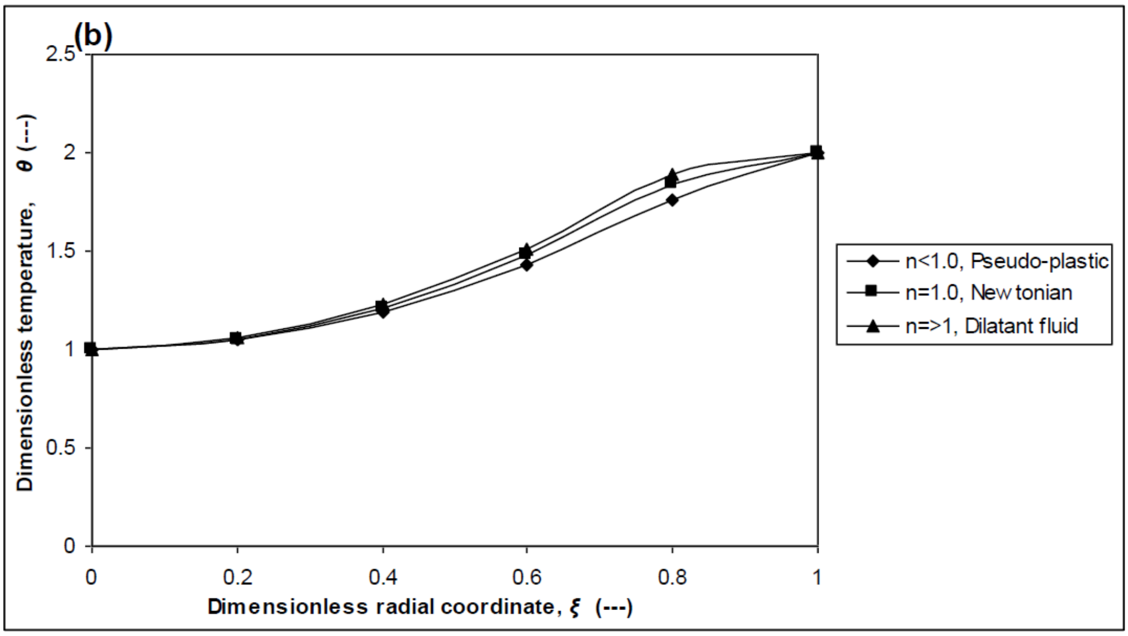

Figure 3. Power law index Effects on fluid temperature profiles

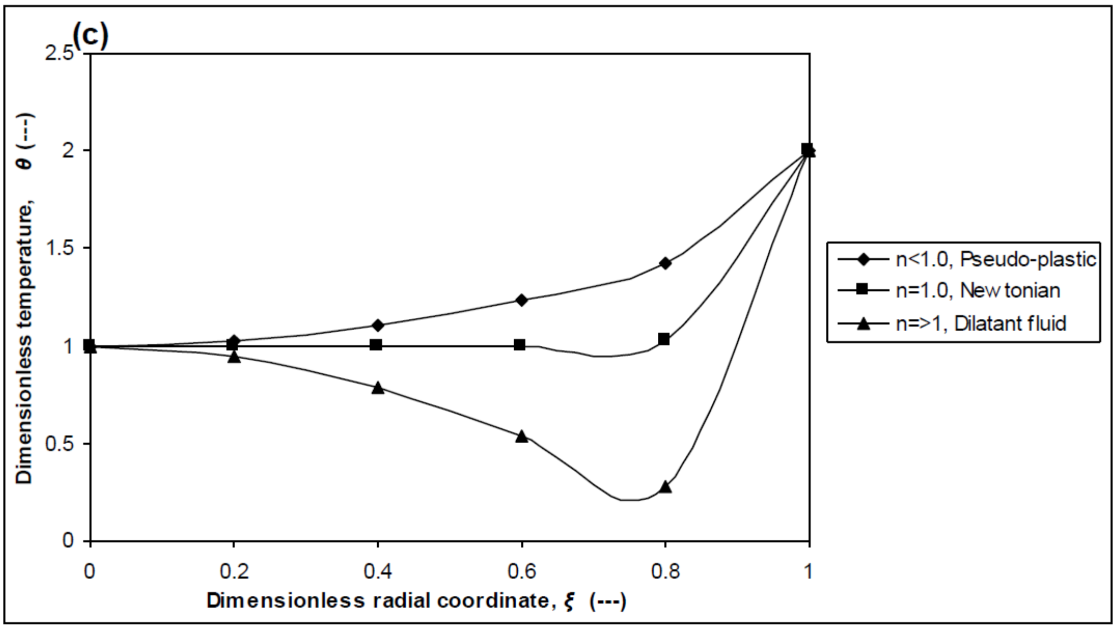

Figure 4. Fluid temperature profiles

As figure (4) corresponds to the variety of the Nusselt number on the basis of the Brinkman number against the persistent wall temperature case along with variant values regarding the power-law index (n). Practically, it's an anticipated outcome, when the Eqn. (41) has been intimately tested. For wall temperature case, along with value of the growing of the Brinkman number, Nu gets reduced to accomplish values of persistent. It is because the variation of heat that drives the heat transfer becomes lessened. At Br=0.5, heat provided by wall into liquid has been balanced through the internal heat creation because of the gluey temperature. For Br>0.5, internally created heat by the gluey dissipation supersedes the wall heat. Whenever Br=1.0, Nu gets equal to an asymptotic worth. Nominally, Nusselt number on the basis of Newtonian liquid has been higher than that for dilatants and pseudo-plastic fluids.

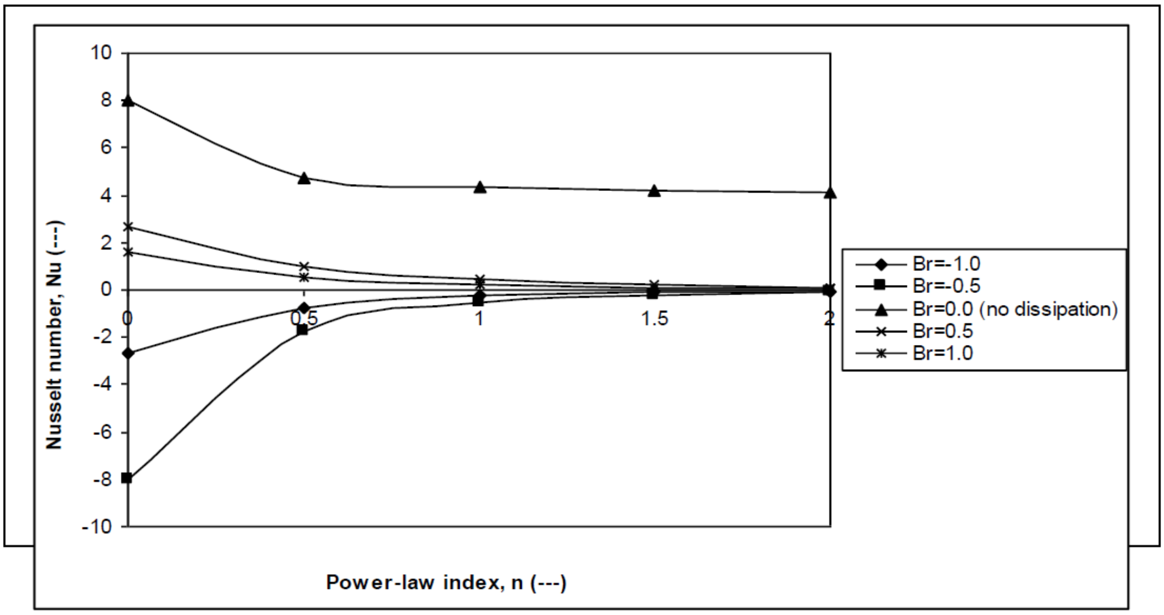

Figure 4. Nusselt number Figure

CONCLUSION

The problem of forced convection regarding the heat transfer, with gluey dissipation within a pipeline, which subjects to persistent wall temperature, is resolved mathematically that has been a kind of Graetz problem. Full analytical resolutions for the liquid temperature as well as local Nu (Nusselt number) have been availed. The impact of the Br and the rheological power-law index (properties) upon local Nu profile are demonstrated through numerical measurements. The local Nu in thermal zone is about to grow with a decrement in an index of power-law replica (n). It exposed the gluey dissipation within a liquid is likely to remarkably create impact on laminar flowing heat transfer. In terms of the Graetz problem, the current mathematical system can be implemented to heat transfer for a pipeline and a concentrated annulus. And a route between parallel shields; it is also about to get implemented to heat transfer within a channel using a portable wall because no restriction is available on a form of the profile of velocity of the fluid.

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