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Failure Analysis of Composites Plate with Central Opening Hole Subject to Arbitrary Tension Load

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ABSTRACT: The most practical composite structure contains holes as parts of the basic design, such hole cause high-stress gradients. The understanding of the effect of the cutout on the failure criteria of such composite plates is very important in the design of structures. This paper is focused on the investigation of the failure loads around a hole in the composite plate (E-glass/ Epoxy) under the effect of various factors such as fiber volume fraction, loading angle, aspect ratio and fiber orientation. To achieve the objective, two techniques are used I- Analytical, the generalized formulation thus obtained is coded and analytical results are obtained by using MATLAB ver.17 was developed to implement Lekhnitskii's solution. II-Numerical, the simulation model was developed using finite element software, ANSYS 17.2 was used for analysis purpose. In this work, an experimental test procedure was design and implemented to estimate the strength parameter for composite plate in which used these parameters in the analysis. Based on the results presented hither, the failure index of a composite plate can be safely changed by utilize suitable material properties ($\nu_f \geq 0.3$), fiber orientation ($\theta \leq 45°$) and aspect ratio ($a/b \geq 1.5$). In general, it can be stated that the failure index becomes more severe as the aspect ratio of the ellipse becomes more pronounced. To validate the approach the numerical results by using the finite element technique are compared with the predictions of the analytical models, the results show good agreement.

KEYWORDS: Strength Parameter; Failure Index; Fiber Orientation; Aspect Ratio; Fiber Volume Fraction; Loading Angle

INTRODUCTION

Fiber reinforced composite materials are unique in application because they may possess, high strength to weight ratios, resistance to fatigue and corrosion, low coefficients of thermal expansion and there exists the opportunity to uniquely tailor the fiber orientations to a given geometry, applied load and environment [1].

Different holes shapes in structural elements are needed to reduce the weight of the structure or provide access to other parts of the structure. In some cases, structural elements are being failure during their service life. It is well known that the presence of a cutout or hole in a stressed member creates highly localized load at the vicinity of the cutout [2]. The value of stress concentration factor (SCF) may be estimate either experimentally using the photoelastic method, analytical as well as numerical methods. For composite materials, the value of the utmost stress concentration factor can be greater or less than 3, and the locations of the utmost stress point could shift depending on the loading and the fiber orientations [3]. In particular, laminated composites are commonly used in aeronautical and aerospace industries as the main part of the structure rather than aluminum or other metallic materials. One of the first research in this field was reported by Forchet, M.M. [4], where the stress concentration factor for holes, grooves, and fillets were determined by using different loading (pure tension, compression and bending) and using photoelasticity method for determining the maximum value of stress from the change in photometric properties of certain solids due to the external load supplied on the body under elastic conditions. Jin, K.K., Huang, Y. Lee Y.H. And Ha**.** [5] determine the micro stress within a unidirectional composite under various mechanical and thermal loading conditions. Based on linear stress-strain relations, three unit cell models, square, hexagonal, and diamond fiber arrays, are analyzed and compared using three-dimensional finite element methods. In Venice [6]**,** studied the numerical analysis performed to predict the engineering properties of the multilayered plate and the stress-strain distributions for various lamination angles of continuous fiber composite laminate is described. Bailer, R. and Hicks.[7] developed theoretical method for determining elastic behavior of end loaded plates completely perforated with closely spaced circular holes.

The primary objective of this work is to compute the failure index around the hole in composite plate under uniform tension loading by the application of complex functions. Also, the effects of geometrical and parameters on failure loads around the hole will be studied. The model is also used to compare the results from analytical expression with numerical analysis using Ansys program. This knowledge will assist engineers, researchers and composite community to use composite material for aerospace and automotive.

THEORETICAL BACKGROUND

An analytical method uses various mathematical expressions to predict the stress analysis of a composite plate with cutout shapes in structure element [8].

Lekhnitskii's Solution for Composite Materials

Consider an infinite anisotropic plane subjected to a uniform traction load, P with an angle α from the x-axis as shown in Fig.(1). Equilibrium equations into two dimensional for non – attendance of the body force can be expressed as;

$$\left.\begin{aligned}\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=0\\ \frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}=0\end{aligned}\right\} \quad (1)$$

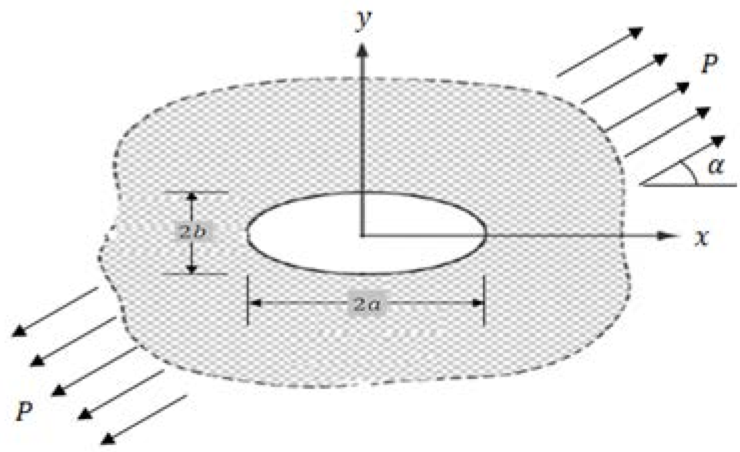

Figure 1. An infinite composite containing a normal elliptical opening

The strain compatibility equations can be expressed as;

$$\frac{\partial^2 \varepsilon_x}{\partial y^2}+\frac{\partial^2 \varepsilon_y}{\partial x^2}=2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} \quad (2)$$

For the plane stress condition, the following stresses and strains are zero

$$\sigma_{zz}=\sigma_{xz}=\sigma_{yz}=0, \varepsilon_{xz}=\varepsilon_{yz}=0$$

Consider the generalized Hooke's law in an anisotropic material, the in-plane strains can be expressed as

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{16}\\ a_{21} & a_{12} & a_{26}\\ a_{61} & a_{62} & a_{66}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \quad (3)$$

Where, a_{ij} is the elastic compliance. The Airy stress function can be expressed as [9]

$$\left.\begin{aligned}\sigma_x&=\frac{\partial^2 \phi}{\partial y^2}\\ \sigma_y&=\frac{\partial^2 \phi}{\partial x^2}\\ \tau_{xy}&=-\frac{\partial^2 \phi}{\partial x \partial y}\end{aligned}\right\} \quad (4)$$

Where,ϕ is a potential function that can be solved via boundary condition. Substituting Eq.(4) into Eq.(3) yield ;

$$a_{11}\frac{\partial^4\phi}{\partial y^4}+(2a_{12}+a_{66})\frac{\partial^4\phi}{\partial x^2\partial y^2}+a_{22}\frac{\partial^4\phi}{\partial x^4}-2a_{16}\frac{\partial^4\phi}{\partial x\partial y^3}-2a_{26}\frac{\partial^4\phi}{\partial y\partial x^3}=0 \quad (5)$$

The general solution of Eq.(5) in the form $\phi(x,y)=F(x+sy)$, where s is a complex variable. Therefore Eq.(5) becomes a characteristic equation expressed as;

$$a_{11}s^4-2a_{16}s^3+(2a_{12}+a_{66})s^2-2a_{26}s+a_{22}=0 \quad (6)$$

The roots of this characteristic equation are complex so $s_{1,3}=\alpha_1\mp i\beta$, $s_{2,4}=\alpha_2\mp i\beta$,
The general solution of ϕ is given by;

$$\phi(x,y)=F_1(x+s_1y)+F_2(,x+s_2y)+F_3(x+\bar{s}_1y)+F_4(x+\bar{s}_2y) \quad (7)$$

Because of ϕ is real Eq. (7) can be written as

$$\phi(x,y)=2Re[F_1(z_1)+F_2(z_2)] \quad (8)$$

Where, $z_j=x+s_jy, \quad j=1,2$

By solving the characteristic equation and applying the boundary conditions, the in-plane stress can be expressed as follows [10];

$$\left.\begin{aligned}\sigma_x&=2Re[s_1^2\bar{F}_1+s_2^2\bar{F}_2]\\ \sigma_y&=2Re[\bar{F}_1+\bar{F}_2]\\ \tau_{xy}&=-2Re[s_1\bar{F}_1+s_2\bar{F}_2]\end{aligned}\right\} \quad (9)$$

Savin's Solution for Stress in Composite

Since it is traction free on the surface around the hole as shown in Fig. (1), we can express the traction are;

$$\left.\begin{aligned}f_1&=\int Y_n ds=2Re[\Phi_1+\Phi_2]=0\\ f_2&=\int X_n=-2Re[s_1\Phi_1+s_2\Phi_2]=0\end{aligned}\right\} \quad (10)$$

The potential function Φ_1 and Φ_2 obtained from Savin's solution can be expressed as [10];

$$\left.\begin{aligned}\Phi_1(z_1)&=B_1z_1+\psi_0(z_1)\\ \Phi_2(z_2)&=(B_2+iC_1)z_2+\psi_0(z_2)\end{aligned}\right\} \quad (11)$$

The coefficient can be solved as;

$$\beta_1=\left(\frac{cos^2\alpha+(cos^2\alpha)sin^2\alpha+\alpha_2 sin2\alpha}{2((\alpha_2-\alpha_1)^2+(\beta_2^2-\beta_1^2))}\right),$$

$$\beta_2=\left(\frac{(cos^2\alpha-2\alpha_1\alpha_2)sin^2\alpha-cos^2\alpha+\alpha_2 sin2\alpha}{2((\alpha_2-\alpha_1)^2+(\beta_2^2-\beta_1^2))}\right),$$

$$C_1=P\left(\frac{(\alpha_1-\alpha_2)cos^2\alpha+(\alpha_2(\alpha_1^2-\beta_1^2)-\alpha_1(\alpha_1^2-\beta_2^2)sin^2\alpha+(((\alpha_1^2-\beta_1^2)-(\alpha_2^2-\beta_2^2)sin\alpha cos\alpha}{2\beta_2((\alpha_2-\alpha_1)^2+(\beta_2^2-\beta_1^2))}\right)$$

Where;

$$\psi_0(z_1)=-\frac{iP(a-is_1b)}{4(s_1-s_2)}\left(\frac{b(s_2sin2\alpha+2cos^2\alpha}{z_1+\sqrt{z_1^2-(a^2-s_1^2b^2)}}+\frac{ia(2s_2sin^2\alpha+sin2\alpha}{z_1+\sqrt{z_1^2-(a^2-s_1^2b^2)}}\right)$$

$$\psi_0(z_2)=-\frac{iP(a-is_2b)}{4(s_1-s_2)}\left(\frac{b(s_1sin2\alpha+2cos^2\alpha)}{z_2+\sqrt{z_2^2-(a^2-s_2^2b^2)}}+\frac{ia(2s_1sin^2\alpha+sin2\alpha)}{z_2+\sqrt{z_2^2-(a^2-s_2^2b^2)}}\right)$$

Substituting in Eq.(10) yield, the stress state at each point in the composite plate due to the remote stresses can be expressed as:

$$\left.\begin{aligned}\sigma_x&=\sigma_x^\infty+2Re[s_1^2\bar{\psi}_0(z_1)+s_2^2\bar{\psi}_0(z_2)]\\ \sigma_y&=\sigma_y^\infty+2Re[\bar{\psi}_0(z_1)+\bar{\psi}_0(z_2)]\\ \tau_{xy}&=\tau_{xy}^\infty-2Re[s_1\bar{\psi}_0(z_1)+s_2\bar{\psi}_0(z_1)]\end{aligned}\right\} \quad (12)$$

Where, $\sigma_x^\infty, \sigma_y^\infty$ *and* τ_{xy}^∞ are in- plane stresses under uniform tension (P) acting at any angle (α) measured form the horizontal axis, $\sigma_x^\infty=Pcos^2\alpha$,$\sigma_y^\infty=Psin^2\alpha$, $\tau_{xy}=Psin\alpha\ cos\alpha$ .Therefore, the stress field around an elliptical hole in an anisotropic plate subjected to stresses and determined by Eq.(12).The transformation of orientated stress to principal stresses is performed by the following matrix expression[11];

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=\begin{bmatrix} m^2 & n^2 & 2mn\\ n^2 & m^2 & 2mn\\ -mn & mn & m^2-n^2\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \quad (13)$$

Where, m and n are cosine and sine of fiber orientation respectively.

Failure Criteria

The strength of a composite lamina may be characterized by five different material parameters. The Tsai-hill criterion is a phenomenological material failure theory, which is widely used for composite materials which have different strength in tension and compression. They are able to predict overall failure, but cannot predict the exact failure mode. The Tsai-hill criterion predicts failure when the failure index in ply reaches 1.The prediction of the failure of structural components is usually accomplished by comparing the stresses or strains to the material ultimate limits and this criterion may be expressed in the form [12];

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\tau_{12}^2 \geq 1 \quad (14)$$

Where,

$$F_1 = \frac{1}{X^t}+\frac{1}{X^c}, F_2 = \frac{1}{Y^t}+\frac{1}{Y^c}, F_{11} = \frac{-1}{X^t X^c}, F_{22} = \frac{-1}{Y^t Y^c}, F_{66} = \frac{1}{S^2}, F_{12} = 0.5\sqrt{F_{11}F_{22}}$$

And, X^t, X^c, Y^t *and* Y^c and Y^t are the tensile strengths of the material in the fiber and transverse directions, respectively and S is the shear modulus. Generally, these expressions are based on the process of adjusting an expression to a curve obtained by experimental tests in next section.

NUMERICAL ANALYSIS

The finite element technique is a powerful tool which can be used in many engineering applications by taking advantage of the continuously developing digital computers with fast capability. The commercial software Ansys 17.2 is user-friendly and easy to design the required model [1, 13]. The meshing process has been done by choosing the suitable element SHELL181 as shown in Fig.(2) with six degree of freedom at each node. A convergence study was carried out to determine how used mesh is suitable for accurate results and computing time during analysis. The location of the elements and therefore the nodes must also reflect any changes in material properties, geometry, constraint conditions and applied loads.

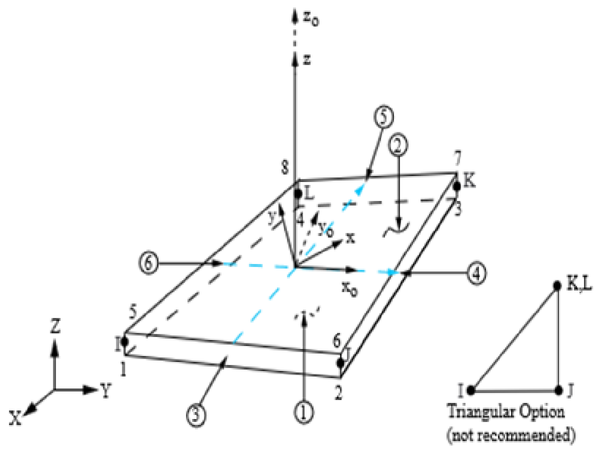

Figure 2. Shell181 element geometry

In Ansys program, due to the symmetric nature of the plate, only quarter models of composite plat with hole and simply supported (S.S.) boundary condition is presented in Fig. (3), where u and v is the displacement in x and y direction respectively. The geometry of a plate with various holes can be drawn relatively easily in Ansys program through Design Modeler.

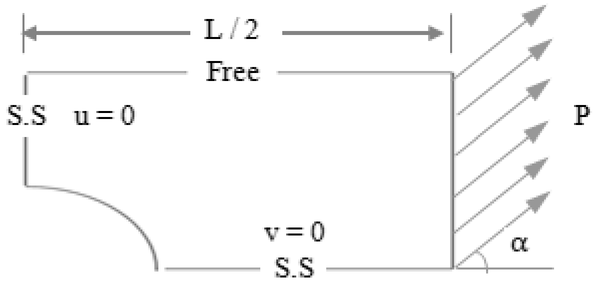

Figure 3. Quarter composite plat with hole

PROBLEM DESCRIPTON

The physical structure that was used in this work is a fiber reinforced composite plate and the material properties are shown in Table.1. A square plate of uniform thickness with an opening central hole subjected to the uniform a constant pressure P=10 MPa at an angle measured from the horizontal axis is considered. The model is analyzed for the different aspect ratio of the elliptical hole by keeping the dimension of the plate fixed. The width of 20 mm and thickness of the plate is 1 mm [14].

Table 1. Material properties of composite plate

Elastic properties	Fiber material (E-glass)	Matrix material (Epoxy)
Young's modulus, GPa	85.0	3.40
Poisson's ratio	0.20	0.30
Shear modulus, GPa	35.42	1.308

EXPERIMENTAL WORK

In the first stage, the majority of the experimental research to find the properties of composite plate by testing the specimen accordance to ASTM standards with five specimens prepared for each test conditions. This test was done by using the material testing machine as shown in Fig. (4). The test was carried out, the tensile strength was found by testing the specimen for tension test using ASTM D3039 standard. To ensure the accuracy of the experimental results, some tests were prepared at the same conditions. The sample is fix with strain gages in the linear and accidental direction. The loads are applied to the sample at a rate of 4 mm/*min*. A total of 30 to 40 data points for stress and strain is taken until the fiber starts to fracture. The step for obtainment accidental tensile strength is such as for finding the linear tensile strength just the sample dimension differs. The compression modulus and strength was obtainment with the combined loading compression test according to ASTM D3410. The shear modulus and shear strength were found in accordance to ASTM 7979. Finally, the experimental data is summarized in Table.2 for various fiber volume fraction and input into the Ansys program through engineering data [15-18].

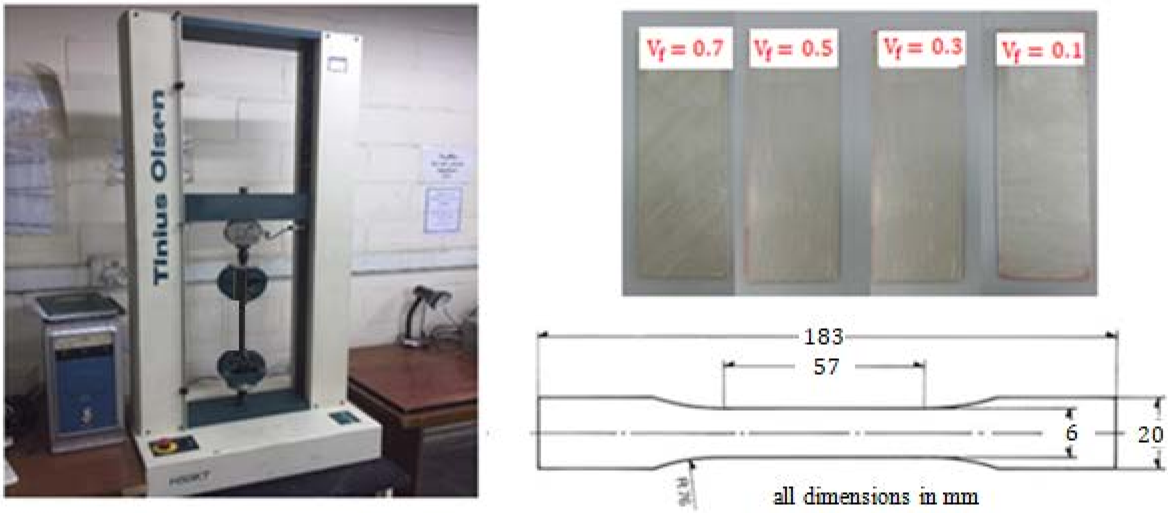

Figure 4. Machine and samples of tensile test for composite material

Table 2. Strength properties for composite ply at different fiber volume fraction

Fiber volume fraction, v_f	Longitudinal tensile strength, X^t (MPa)	Longitudinal compressive strength, X^c (MPa)	Transverse tensile strength, Y^t (MPa)	Transverse compressive strength, Y^c (MPa)	In-plane shear strength, S (MPa)
0.1	210.8	554.9	523.5	741.1	246.9
0.3	508.4	888.9	411.1	582.3	193.4
0.5	806.1	875.4	323.7	458.6	151.7
0.7	1103.6	520.8	205.1	290.6	94.4

RESULTS AND DISCUSSION

Results of the previously described analysis are presented in this section. The analytic method was implemented to research the influence of a variable factor on the failure index for the composite plate. The simulation was carried out using programs that have been written using Matlab code, in order to efficiently analyze the composite plate.

Figure 5 shows counter of stress fields under tension load (10 MPa) in x-direction at zero loading angle. Here we can see that the maximum stress is 27.591 MPa occur at the edge of the hole at periphery of hole.

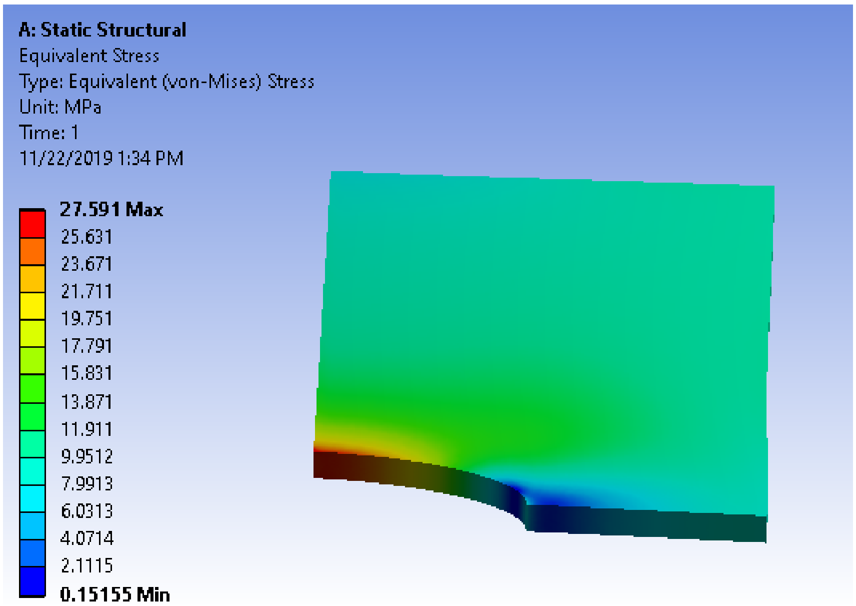

Figure 5. Distribution of max. stress in plate with hole ($b = 1, a = 3$) at $v_f = 0.2$ and $\theta = 0^{\circ}$

The value of the failure index of the composite plate is predicted by using Eq. (14). Based on result presented herein, the effect of loading angle on the failure index at different fiber volume fraction and fiber orientation as shown in figures below. According to Fig. (6) as the loading angle increases in this region, the failure index actually slightly increases in all other fiber volumes fraction, but the rate of this increase is not constant, due to strength are most sensitive to material properties. The maximum value of failure index occurred at loading angle equal to (α = 90°), whereas the minimum value of a failure index appears to occur at a loading angle equal to (α = 0°).

From Fig.(7) it can be seen that ,when the fiber angle equal to the maximum value ($\theta = 90°$), the failure index smoothly decrease with the increase of the loading angle, due to stress ratio is most sensitive to material properties and directly depend on the ratio of moduli's. Conversely when the fiber angle equal to the minimum value($\theta = 0°$), the fibers are parallel to the x-axis, the failure index increase with the increase of the loading angle. Another important point in these figures is observed that, when the fiber angle equals to ($\theta = 45°$), the failure index smoothly decrease with the increase of the loading angle, until the value of loading angle equals to (α = 45°) and after that, the failure index slowly increase with the increase of the loading angle.

The effect of hole size on the failure index predicted at different fiber volume fraction and fiber angle as shown in Figs. (8 and 9) respectively. It is observed that as the aspect ratio a / b increases the failure index reduced gradually and the curves become nearly flat. It is shown that from the figures, the rise in failure index in the vicinity of the hole is quite sharp as compared to the circular hole. In general, it can be concluded that the failure index is an approximately constant value when the aspect ratio greater than unity.

Figures (10) and (11) show that analytical results are in good agreement with finite element method using Ansys program, the discrepancy results obtained is less than 5 %. In general, the Ansys program gives a better approximation of failure behavior when the major diameter of the hole is small. But as the diameter is increased, the deviation of the Ansys results from the analytical one is higher and is equal to 12 %. Moreover, the deviation increases with loading angle. One reason for the deviation is that the approximate solutions and it's to avoid the discontinuities of stress that occur across inters element boundaries in compatible finite element method.

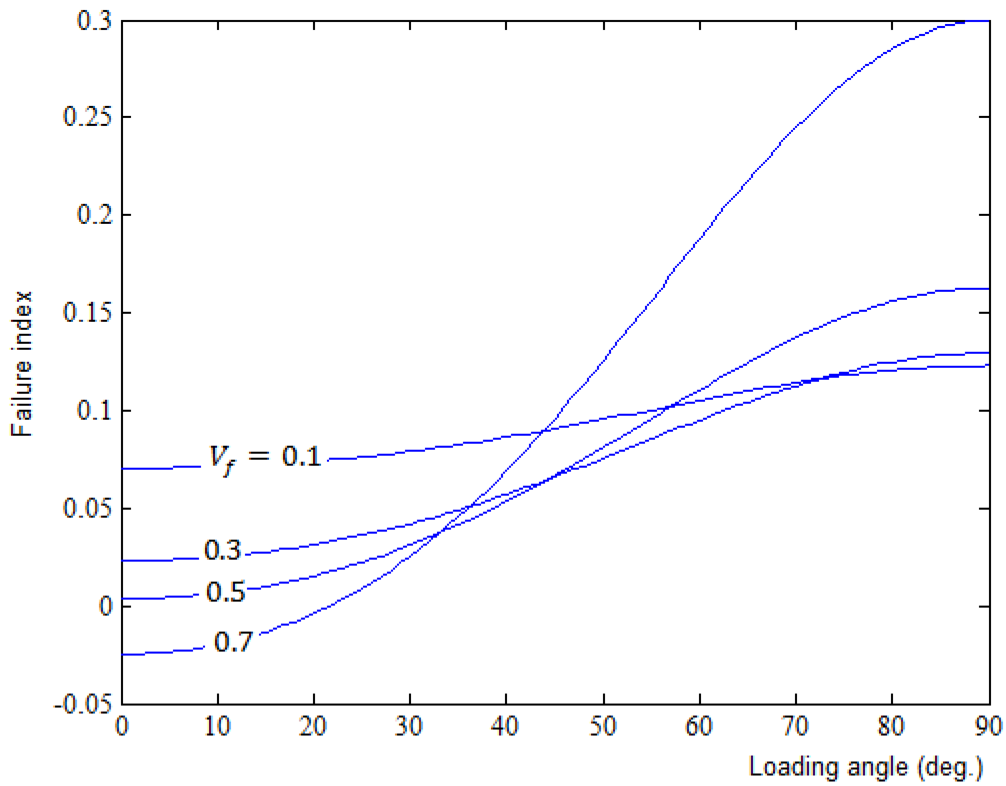

Figure 6. Failure index with loading angles at different fiber volume fraction of composite

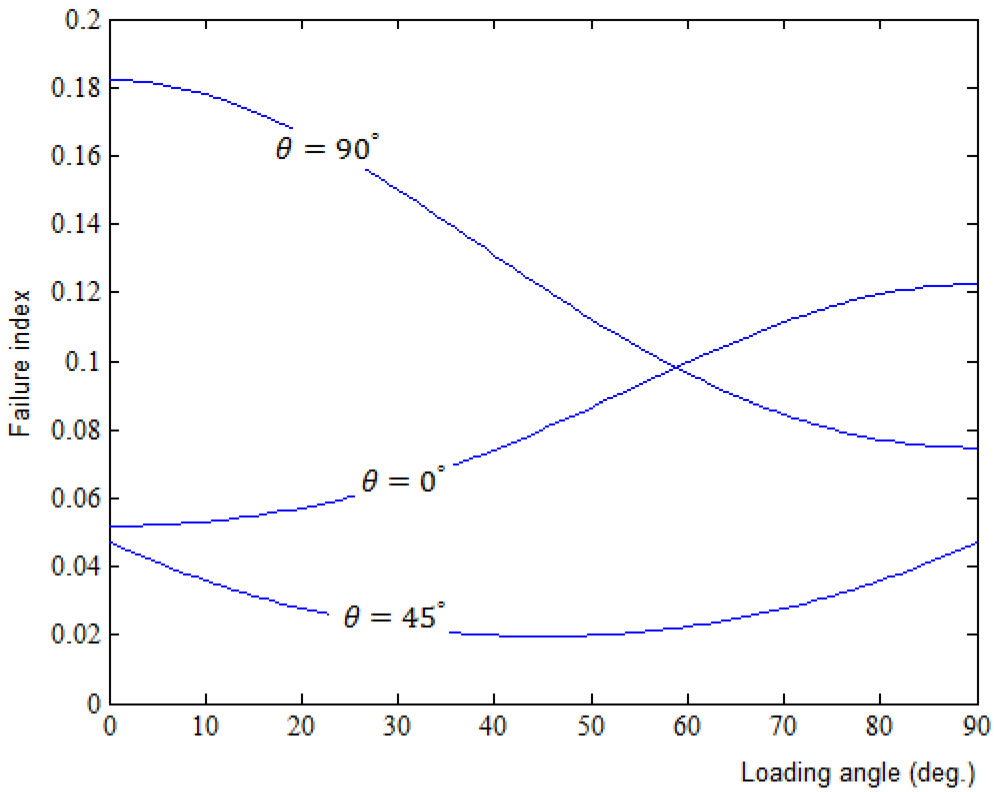

Figure 7. Failure index with loading angles at different fiber orientation of composite

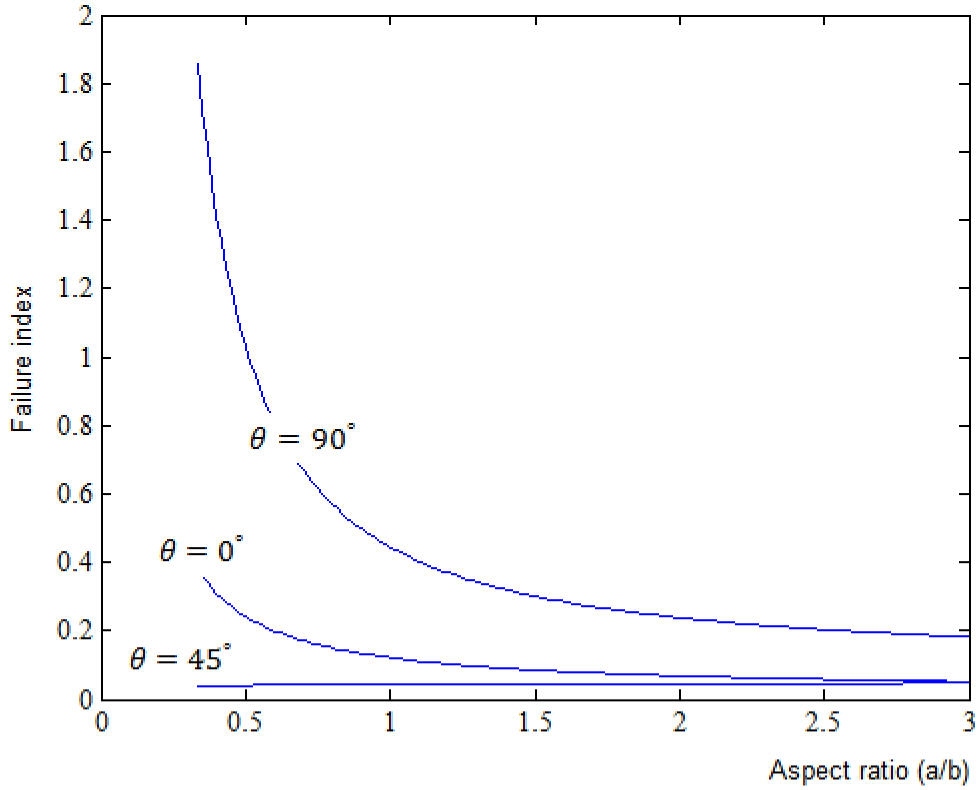

Figure 8. Failure index with aspect ratio at different fiber orientation of composite

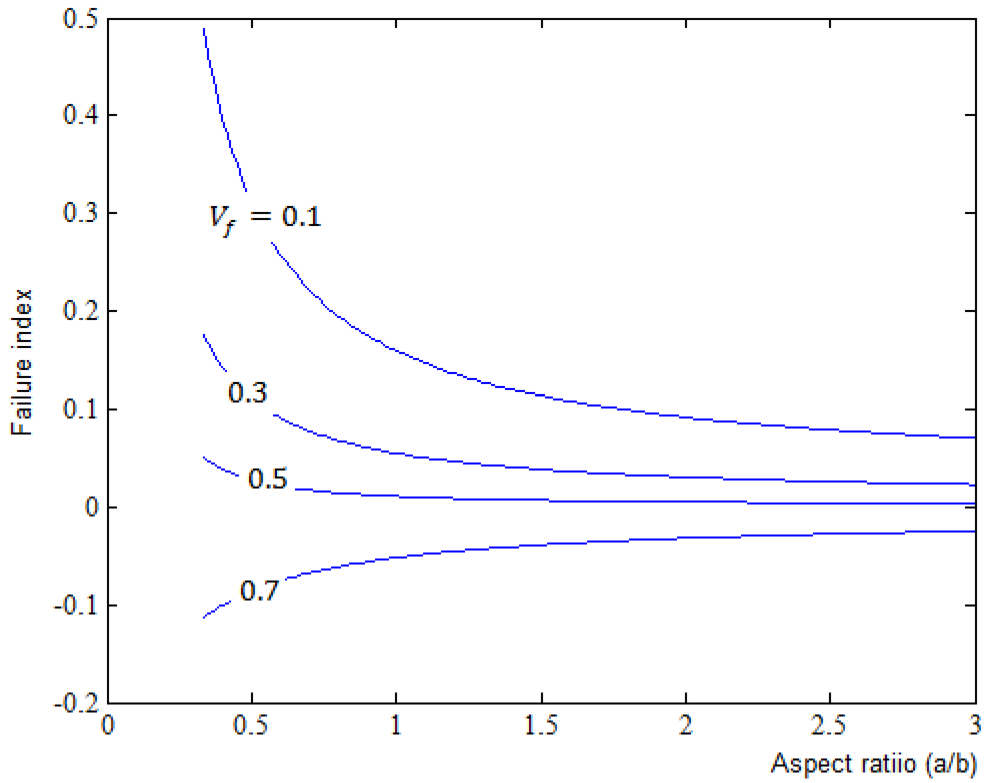

Figure 9. Failure index with aspect ratio at different fiber volume fraction of composite

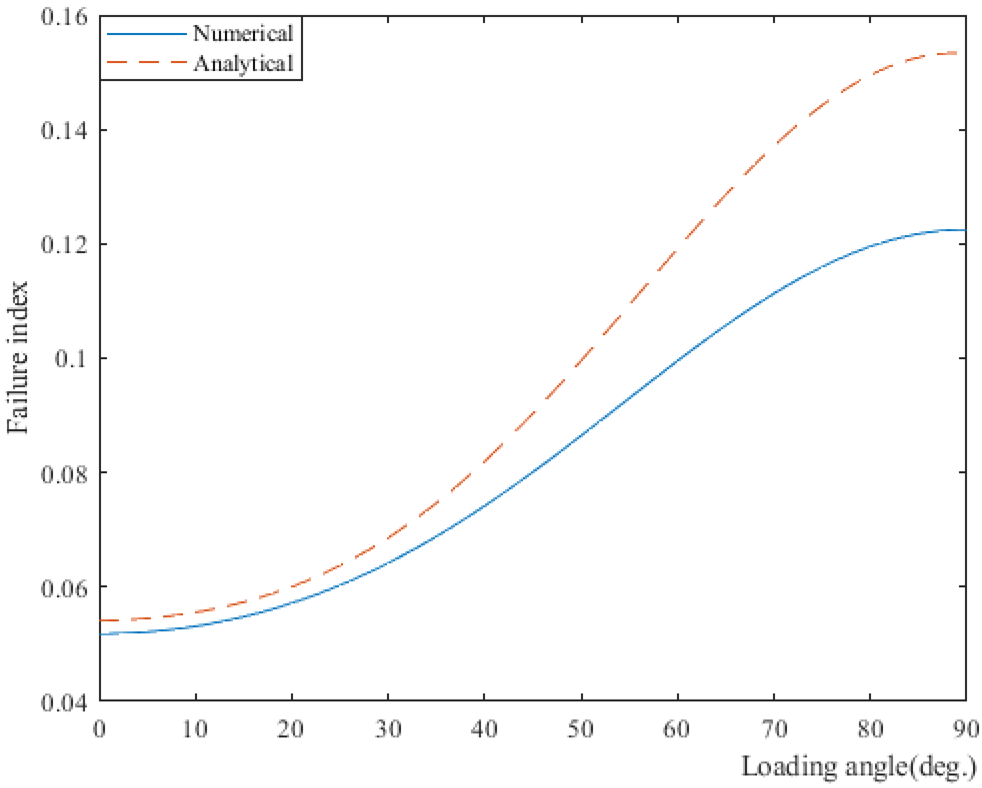

Figure 10. Failure index with loading angles for different technique at ($\nu_f = 0.2, \theta = 0^{\circ}, b = 1, a = 3$

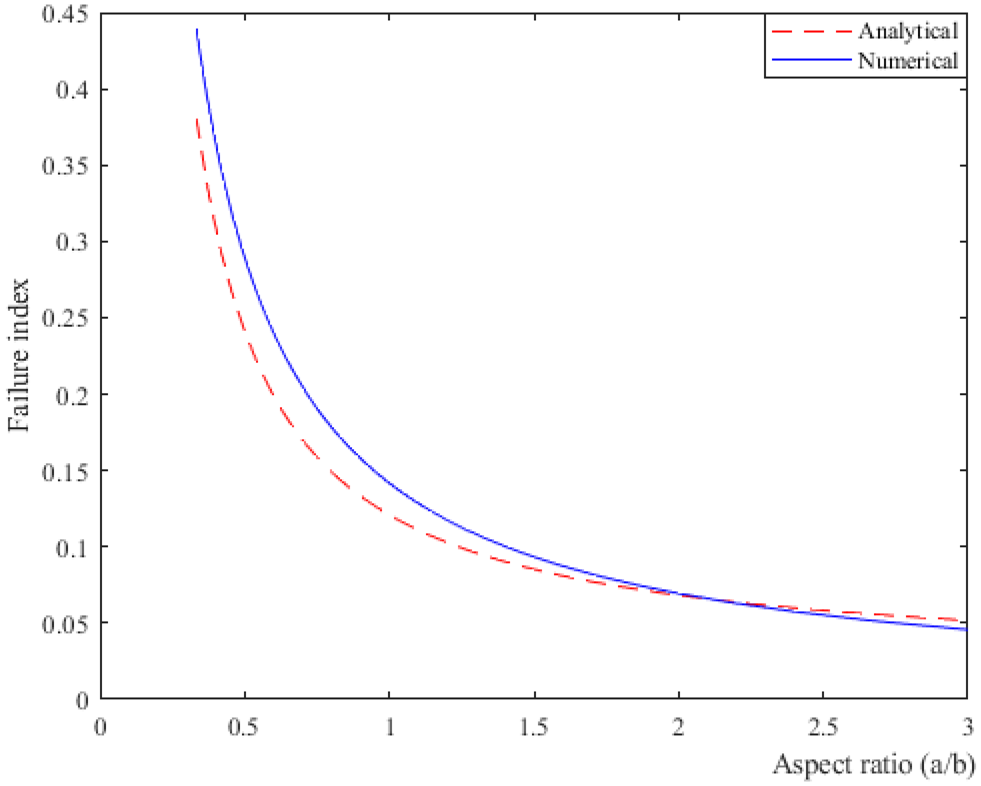

Figure 11. Failure index with aspect ratio for different technique ($\nu_f = 0.2, alf = 0, \theta = 0^{\circ}$)

CONCLUSIONS

In this work, failure behavior of composite plate (E-glass/Epoxy) with a central hole is investigated analytical and numerical under the effects of various factors. From the above mentioned discussions it is important to note that these;

The maximum failure index of the composite plate can be significantly varied by using appropriate fiber volume fraction and dimension of the hole with respect to the width of the plate.

The utmost value of failure index utilize at a ply angle of θ= 90° for E-glass/ Epoxy plate, as the fiber orientation decrease, the stress ratio also decreases until to reach a constant value.

As the fiber volume fraction increase, the failure index decreases. However, the rate of these decreases is not constant.

In all cases, the utmost failure index occurs at the corners of the elliptical hole. As the aspect ratio of the elliptical hole decrease, the failure index increasers.

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