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Analytical Solution of Free Vibration Characteristics of Partially Circumferential Cracked Cylindrical Shell

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ABSTRACT: Free vibration characteristics of a cylindrical shell of finite length containing a partial peripheral crack are studied. An analytical solution method is proposed to evaluate the natural frequency of the cracked cylindrical shell structures. The governing equations of motion are based on classical shell theory and are simplified by Love's theory. The bending rigidity equivalent to the shell (D) expressed by exponential function is calculated taking into account the effect of the crack. The influence of the crack length, crack depth, crack location, are investigated on the vibratory characteristics under simply supported (SS-SS) at each end boundary condition. Natural frequencies were obtained by solving general equations using "MATLAB" program. The results obtained from the proposed method were confirmed by comparing them with the results obtained from the literature. The results showed that the natural frequency decreases as the crack length, and depth decreased. also, the natural frequency decrease when the crack locates at the mid of the longitudinal line of the shell more than if it were in other locations. The results obtained by this work have a very good agreements with those been found by literatures.

KEYWORDS: Cylindrical shell, crack, natural frequency, free vibration, vibration characteristics.

INTRODUCTION

Cylindrical shells are a special type of shells and are one of the most significant constructions for modeling different kinds of engineering constructions e.g.: chimneys, pipe systems, storage tanks, reducing the water's temperature in "Cooling Towers" for energy stations, curved dams, shafts, at sea oil containers, automobile manufacturing, planes, and subs. They are very effective structures where stresses and thickness in the transverse direction are reduced, also due to their inherent lightweight characteristics, thin-walled cylindrical shells adoption are in a constant increase for applications in the engineering field. Numerous structures during life in service are unavoidably exposed to deterioration and damage resulting from different factors i.e. environmental erosion, various types of dynamic and fatigue loads, operating loads, etc. which leads to cracking and other flaws resulting in their damage. The most common serious defects are surface cracks in any type of plate or shell. Cracks that form in constructions of cylindrical shells may reduce the shells' stiffness strength and by that decrease its natural frequency. Therefore, has become essential to assess the flaw's influence on a cylindrical shell when in free vibration. Therefore, It is of no wonder that there's an immense attentiveness of the cylindrical shell's free vibration analysis.

Moreover, cylindrical shells have an expansive scope of uses in various industries to biomechanical fields. In this way, becoming acquainted with the crack issue in these structures better has been the target of numerous investigations up until now. Delale and Erdogan,1981 [1] utilized (line-spring method) made by Rice and Levy [2] in order to acquire a convergent outcome for a cylindrical shell containing a longitudinal or a circumferential part-through surface slit. Nikpour,1990 [3] introduced a strategy for understanding balanced vibrations for the cylindrical shell that have meager overlaid anisotropic including a circumferential sort split to uncover the profundity and position of the break which pre-existed on the shell. C. Wang and J.C.S. Lai ,2000 [4] introduced the approach to foresee the natural frequencies based on Love's equations of cylinder shells having a finite length without facilitating the motion's equations for variant boundary conditions. Roytman and Titova, 2002 [5] enhanced numerous mathematical Techniques for flexible vibrations of a tubular shell with a superficial - closed type fissures. Ip and Tse, 2002 [6] created the fissure detecting method in tubular composite material shells built on the natural frequenciess, and the styles of the modes at mode shape according to information at certain places. Javidruzi et al., 2004 [7] investigated the dynamic behavior of tabular l shells containing different types of fissures fissure subjected to fissure with set supports and lay open to an in-plane (compressive/tensile cyclic edge load).

Vaziri and Estekanchi, 2006 [8] found that the existence of fissures in tabular shells could harshly affect the buckling performance of shell constructions by decreasing their capability for carrying-loads as well as the insertion of buckling locally at the fissure zone. Xin et al., 2011 [9] investigated how the performance of free vibration and buckling of a tabular shell are affected by the length of a fissure, orientation, constant rotating speed, and the length diameter ratio. In addition to this the stability features of the fissured shell as well as how the fissure's length. Its orientation, basic speed of rotation, steady load factor, the factor for dynamic load and the damping ratio were also investigated. Yin and Lam, 2013 [10] proposed a numerical model for computing natural frequencies for a tubular cylinder shell of finite-length where the fissure is part through the circumferential surface. Moradi and Tavaf, 2013 [11] investigated with the aid of the quadrature method and Bees algorithm. On how to detect fissures in shell constructions that are tubular cylindrical.

Sarker et al., 2015 [12] presented a new approach founded on the Ritz procedure for finding impairments in tubular cylindrical structures. Sander's hypothesis for thin shells coupled with the Ritz's procedure is used for analyzing the dynamic performance of tubular cylindrical shells. K. Moazzeza et al., 2018 [13] presented a methodical model so as to study the performance of free vibration of a long cylindrical tubular shell, involving a changeable oriented surface fissure, in free vibration. M. A. Husain and M.A. Al-shammari ,2020 [14] introduced an analytical and numerical study of cylindrical shell containing a circumferential part-through fissure depend on bending rigidity and based on Donnell-Mushtari-Vlasov theory. Research works have done using analytical solutions or experimental techniques to investigate the effect of the studied parameters concerned with the cracks [24-25]. A new analytical method founded on the bending rigidity of a shell and based on Love's theory is presented to analyze the vibration behavior for a cylindrical shell involving a circumferential crack. The effect of several parameters crack on the natural frequencies is analyzed. The analytical results were verified with the results obtained from the literature.

MATHEMATICAL MODEL

Consider a finite length of circular cylindrical shell, the coordinate system (x ,Ө, and z) are taken in the middle surface as shown in Fig. (1). The shell is assumed to be made up of a homogeneous and isotropic material and has a uniform thickness h, which is small, as compared to its other dimensions. R is the radius of the cylindrical shell; l is the length of the shell. Assume there exists a circumferential surface crack with length l, mean radius R and thickness h. A circumferential crack with finite length of a and uniform depth of h_c is located at a distance x_c from one end on the external surface of the shell and paralleled to the length of shell.

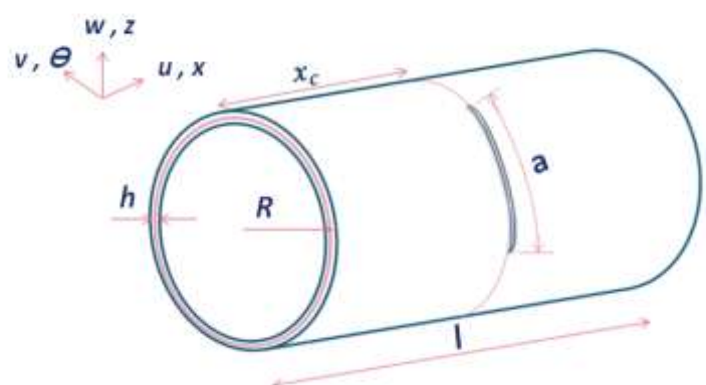


Figure 1. Coordinate System of a Thin Circular Cylindrical Shell

The governing equations of motion for cylindrical shell without crack in terms of displacement components u, v and w is to be derived based on the classical shell theory. [15]

$$\frac{\partial N_{xx}}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta} + f_x = \rho h \ddot{u} \tag{1}$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{Q_{\theta z}}{R} + f_\theta = \rho h \ddot{v} \tag{2}$$

$$\frac{\partial Q_{xz}}{\partial x} + \frac{1}{R}\frac{\partial Q_{\theta z}}{\partial \theta} - \frac{N_{\theta\theta}}{R} + f_z = \rho h\ddot{w} \quad (3)$$

where $\boldsymbol{\rho}$ is the mass density of the shell material, $(N_{xx}, N_{\theta x}, N_{x\theta}, N_{\theta\theta})$ and $(Q_{\theta z}, Q_{z\theta})$ denote the resultant forces and transverse shear forces acting on the mid-surface, respectively. (f_x, f_θ, f_z) represent the external forces along the x, Θ and z directions, respectively.

In terms of the shell displacements, the force and moment components can be expressed as [15]:

$$N_{xx} = C\left(\frac{\partial u}{\partial x} + \frac{\nu}{R}\frac{\partial v}{\partial \theta} + \frac{\nu}{R}\,\mathrm{w}\right) \quad (4)$$

$$N_{x\theta} = N_{x\theta} = C\left(\frac{1-\nu}{2}\right)\left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right) \quad (5)$$

$$N_{\theta\theta} = C\left(\frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{w}{R} + \nu\frac{\partial u}{\partial x}\right) \quad (6)$$

Where: $C = \frac{Eh}{1-\nu^2} = \frac{12D}{h^2}$ (7)

$$M_{xx} = D\left(-\frac{\partial^2 w}{\partial x^2} + \frac{\nu}{R^2}\frac{\partial v}{\partial \theta} - \frac{\nu}{R^2}\frac{\partial^2 w}{\partial \theta^2}\right) \quad (8)$$

$$M_{\theta\theta} = D\left(\frac{1}{R^2}\frac{\partial v}{\partial \theta} - \frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} - \nu\frac{\partial^2 w}{\partial x^2}\right) \quad (9)$$

$$M_{x\theta} = M_{x\theta} = D\left(\frac{1-\nu}{2}\right)\left(\frac{1}{R}\frac{\partial v}{\partial x} - \frac{2}{R}\frac{\partial^2 w}{\partial x\partial \theta}\right) \quad (10)$$

$$Q_{xz} = \frac{\partial M_{xx}}{\partial x} + \frac{1}{R}\frac{\partial M_{\theta x}}{\partial \theta} = D\left(-\frac{\partial^3 w}{\partial x^3} + \frac{1+\nu}{2R^2}\frac{\partial^2 v}{\partial x\partial \theta} - \frac{1}{R^2}\frac{\partial^3 w}{\partial x\partial \theta^2}\right) \quad (11)$$

$$Q_{\theta z} = \frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial M_{\theta\theta}}{\partial \theta} = D\left(\frac{1-\nu}{2R}\frac{\partial^2 v}{\partial x^2} + \frac{1}{R^3}\frac{\partial^2 v}{\partial \theta^2} - \frac{1}{R^3}\frac{\partial^3 w}{\partial \theta^3} - \frac{1}{R}\frac{\partial^{3w}}{\partial x^2\partial \theta}\right) \quad (12)$$

Finally, using Eqs. (4 to 6) and Eqs. (11,12), the equations of motion, Eqs. (1 to 3) can be expressed in terms of the displacement components *u, v* and *w* as: [15]

$$C\ \frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2R^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\nu}{R}\frac{\partial w}{\partial x} + \frac{1+\nu}{2R}\frac{\partial^2 v}{\partial x\partial \theta} + f_x = \rho h\ddot{u} \quad (13)$$

$$C\left(\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{R^2}\frac{\partial w}{\partial \theta} + \frac{1+\nu}{2R}\frac{\partial^2 u}{\partial x\partial \theta}\right) + D\left(\frac{1-\nu}{2R^2}\frac{\partial^2 v}{\partial x^2} + \frac{1}{R^4}\frac{\partial^2 v}{\partial \theta^2} - \frac{1}{R^4}\frac{\partial^3 w}{\partial \theta^3} - \frac{1}{R^2}\frac{\partial^3 w}{\partial x^2\partial \theta}\right) + f_\theta = \rho h\ddot{v} \quad (14)$$

$$D\left(-\frac{\partial^4 w}{\partial x^4} + \frac{1}{R^2}\frac{\partial^3 v}{\partial x^2\partial \theta}\ \frac{2}{R^2}\frac{\partial^4 w}{\partial x^2\partial \theta^2} - \frac{1}{R^4}\frac{\partial^4 w}{\partial \theta^4} + \frac{1}{R^4}\frac{\partial^3 v}{\partial \theta^3}\right) - C\left(\frac{1}{R^2}\frac{\partial v}{\partial \theta} + \frac{w}{R} + \frac{\nu}{R}\frac{\partial u}{\partial \theta}\right) + f_\theta = \rho h\ddot{w} \quad (15)$$

The equations of motion for cylindrical shell, eqs. (13 to 15) can be simplified using the Love's theory. When external forces are absent ($f_x = f_\theta = f_z = 0$), the equations for motion are reduced to: [15]

$$C\left(\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2R}\frac{\partial^2 u}{\partial \theta^2} + \frac{\nu}{R}\frac{\partial w}{\partial x} + \frac{1+\nu}{2R}\frac{\partial^2 v}{\partial x\partial \theta}\right) = \rho h\ddot{u} \quad (16)$$

$$C\left(\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 v}{\partial \theta^2}+\frac{1}{R^2}\frac{\partial w}{\partial \theta}+\frac{1+\nu}{2R}\frac{\partial^2 u}{\partial x\partial \theta}\right)+\frac{D}{R^2}\left(\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{1}{R^2}\frac{\partial^3 w}{\partial \theta^3}-\frac{\partial^3 w}{\partial x^2\partial \theta}\right)= \rho h\ddot{v} \quad (17)$$

$$C\left(-\frac{1}{R^2}\frac{\partial v}{\partial \theta}-\frac{w}{R^2}-\frac{\nu}{R}\frac{\partial u}{\partial \theta}\right)+\frac{D}{R^2}\left(-R^2\frac{\partial^4 w}{\partial x^4}+\frac{\partial^3 v}{\partial x^2\partial \theta}-2\frac{\partial^4 w}{\partial x^2\partial \theta^2}-\frac{1}{R^2}\frac{\partial^4 w}{\partial \theta^4}+\frac{1}{R^2}\frac{\partial^3 v}{\partial \theta^3}\right)= \rho h\ddot{w} \quad (18)$$

The solution is presumed harmonic during free vibration as: [15]

$$u(x,\theta,t)=U(x,\theta)e^{i\omega t} \quad (19)$$

$$v(x,\theta,t)=V(x,\theta)e^{i\omega t} \quad (20)$$

$$w(x,\theta,t)=W(x,\theta)e^{i\omega t} \quad (21)$$

where ω is the frequency of vibration.

Substituting Equations. (19 to 21) into Equations. (16 to 18) gives: [15]

$$C\left(\frac{\partial^2 U}{\partial x^2}+\frac{1+\nu}{2R^2}\frac{\partial^2 U}{\partial \theta^2}+\frac{\nu}{R}\frac{\partial W}{\partial x}+\frac{1-\nu}{2R}\frac{\partial^2 V}{\partial x\partial \theta}\right)+\rho h\omega^2 U=0 \quad (22)$$

$$\left(\frac{1-\nu}{2}\frac{\partial^2 V}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 V}{\partial \theta^2}+\frac{1}{R^2}\frac{\partial W}{\partial \theta}+\frac{1+\nu}{2R}\frac{\partial^2 U}{\partial x\partial \theta}\right)+\frac{D}{R^2}\left(\frac{1-\nu}{2}\frac{\partial^2 V}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 V}{\partial \theta^2}-\frac{1}{R^2}\frac{\partial^3 W}{\partial \theta^3}\frac{\partial^3 W}{\partial x^2\partial \theta}\right)+\rho h\omega^2 V= 0 \quad (23)$$

$$C\left(\frac{1}{R^2}\frac{\partial V}{\partial \theta}-\frac{W}{R^2}-\frac{\nu}{R}\frac{\partial U}{\partial x}\right)+\frac{D}{R^2}\left(-R^2\frac{\partial^4 W}{\partial x^4}+\frac{\partial^3 V}{\partial x^2\partial \theta}-2\frac{\partial^4 W}{\partial x^2\partial \theta^2}-\frac{1}{R^2}\frac{\partial^4 W}{\partial \theta^4}+\frac{1}{R^2}\frac{\partial^3 V}{\partial \theta^3}\right)+\rho h\omega^2 W= 0 \quad (24)$$

The effect of a circumferential crack on a cylindrical shell is obtained by calculating the bending stiffness (D) by an exponential function as given by: [16]

$$D=\frac{D_o}{1+S.e^{(-2\alpha|x-x_c|/h)}}=D_o\,.f(x) \quad (25)$$

Where: $D_o=\frac{Eh^3}{12(1-\nu^2)}$ (25-A)

$$f(x)=\frac{1}{1+S.e^{(-2\alpha|x-x_c|/h)}} \quad (25\text{-B})$$

$$S=\frac{I_o-I_c}{I_c} \quad (25\text{-C})$$

$$I_o=\frac{\pi}{4}(R_o^4-R_i^4) \quad (25\text{-D})$$

$$I_c=\frac{\pi}{4}(R_o^4-R_i^4)-[\frac{\pi}{4}(R_o^4-R_c^4)*\frac{a}{2\pi} \quad (25\text{-E})$$

$$\alpha=-0.02391+0.027616\,\frac{x_c}{l}+0.002666\,\frac{h_c}{h}+0.00415\,\frac{a}{2\pi R} \quad (25\text{-F})$$

I_o , I_c : are the second moment of areas of the non-cracked and cracked cylindrical shell, respectively. R_o, R_i are the outer and inner radius of the shell, respectively. R_c is the radius of cracked shell, *h* is the thickness of shell, *x* is the position along the shell, x_c the position of the crack, and α is a non-dimensional value which is determined by multiple linear regression method and using numerical results.

Fig [2] shows the boundary conditions, which are simply supported, for the cracked cylindrical shell at x=0 & x=*l* and are represented by the following equations [15]:

$$V(o,\theta)=0 \quad (26)$$

$$W(o,\theta) = 0 \tag{27}$$

$$\left(-\frac{\partial^2 W}{\partial x^2} + \frac{\nu}{R^2}\frac{\partial V}{\partial \theta} - \frac{\nu}{R^2}\frac{\partial^2 W}{\partial \theta^2}\right)(o,\theta) = 0 \tag{28}$$

$$\left(\frac{\partial U}{\partial x} + \frac{\nu}{R}\frac{\partial V}{\partial \theta} + \frac{\nu}{R}\mathrm{W}\right)(o,\theta) = 0 \tag{29}$$

$$V(l,\theta) = 0 \tag{30}$$

$$W(l,\theta) = 0 \tag{31}$$

$$\left(-\frac{\partial^2 W}{\partial x^2} + \frac{\nu}{R^2}\frac{\partial V}{\partial \theta} - \frac{\nu}{R^2}\frac{\partial^2 W}{\partial \theta^2}\right)(l,\theta) = 0 \tag{32}$$

$$\left(\frac{\partial U}{\partial x} + \frac{\nu}{R}\frac{\partial V}{\partial \theta} + \frac{\nu}{R}\mathrm{W}\right)(l,\theta) = 0 \tag{33}$$

$$f(0) = A_1 \tag{34}$$

$$f(l) = A_2 \tag{35}$$

$$f'(0) = B_1 \tag{36}$$

$$f'(l) = B_2 \tag{37}$$

$$f''(0) = E_1 \tag{38}$$

$$f''(l) = E_2 \tag{39}$$

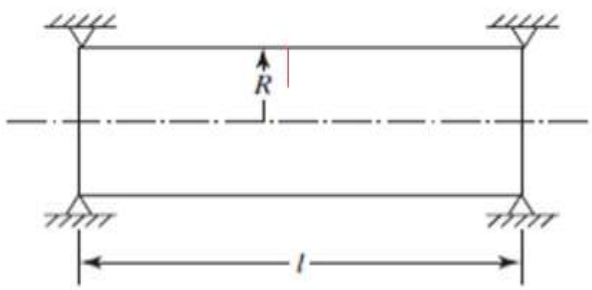


Figure 2. simply supported shell [15]

The following solutions are presumed to gratify the boundary conditions of Equations (26 to 39): [15]

$$U(x,\theta) = C_1 \cos\frac{m\pi x}{l}\, cos\, n\ (\theta - \phi_o) \tag{40}$$

$$V(x,\theta) = C_2\, sin\frac{m\pi x}{l}\, \sin n\ (\theta - \phi_o) \tag{41}$$

$$W(x,\theta) = C_3 \sin\frac{m\pi x}{l}\, \cos n\ (\theta - \phi_o) \tag{42}$$

where C_1 , C_2 and C_3 are constants and ϕ_o is the phase angle.

m is the number of displaced half-waves along the length of the shell known as "Longitudinal Half-Waves"; whereas n is the number of displaced circumferential half- waves "Circumferential Waves" [see Fig. (3)].

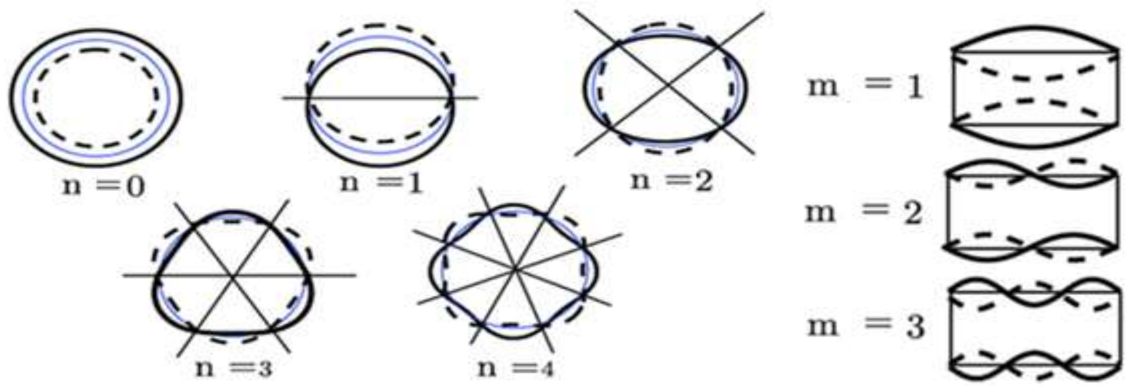


Figure 3. circumferential and longitudinal modes of cylindrical shell

The assumed solution [Eqs. (40 to 42)] fulfills the boundary conditions [Eqs. (26 to 39)]. After compensating an equation of the bending stiffness D for a cracked cylindrical shell in eqs. (22 to 24), Substituting eqs. (40 to 42) and eqs. (26 to 39) in the cracked shell's equations for motion and obtain:

$$C_1[-A_1\,\lambda^2 - A_1\,a_1\,n^2 + \Omega] + C_2[a_2\,\lambda\,n\,A_1] + C_3[\nu\,\lambda\,A_1] = 0 \tag{43}$$

$$C_1[-B_1\,a_1\,n] + C_2[B_1\,a_1\,\lambda(1+\mu)\,] + C_3[B_1\,a_1\,\lambda\mu n)\,] = 0 \tag{44}$$

$$C_1\,[\,0\,] + C_2\,[\,2D_o\,B_1\,a_1\,\lambda\,n\,] + C_3[2D_o\,B_1\lambda^3 + 2D_o\,B_1\,a_1\,\lambda\,n^2\,] = 0 \tag{45}$$

Where: $\lambda = \frac{m\pi R}{L}$, $\mu = \frac{h^2}{12R^2}$, $\Omega = \frac{(1-\nu^2)R^2\rho}{E}\omega^2$, $a_1 = \frac{1-\nu}{2}$, $a_2 = \frac{1+\nu}{2}$

Equations (43 to 45) can be written in matrix form as:

$$\begin{bmatrix} -A_1\,(\lambda^2 + a_1 n^2) + \Omega & A_1 a_2 \lambda n & \nu\lambda\,A_1 \\ -B_1\,a_1 n & B_1(a_1\lambda + a_1\lambda\mu\,) & B_1\,a_1\,\lambda\mu n \\ 0 & 2D\,B_1\,a_2\,\lambda\,n & 2DB_1(\lambda^3 + \lambda n^2) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{46}$$

The expansion of Eq. (46) steers to the frequency equation:

$$g_1\Omega + g_2 = 0 \tag{47}$$

Where: $g_1 = \lambda^2\,(1+\mu) + n^2(1+\mu) - a_2 n\mu$ (48)

$$g_2 = -A_1[\lambda^4\,(1+\mu) + \lambda^2(n^2 + n^2\mu - a_2 n\mu + a_1 n^2 + a_1 n^2\mu - a_2 n^2) + n^4(a_1 + a_1\mu - a_2) + n^2\,a_2(\nu - a_1 n\mu] \tag{49}$$

$$\Omega = -\frac{g_2}{g_1} \tag{50}$$

Or: $\omega^2 = -\frac{g_{2\,E}}{(1-\nu^2)R^2\rho g_1}$ (51)

RESULTS AND DISCUSSION

The free vibration analysis is achieved for the intact cylindrical shells and that containing a crack under the simply supported shell (SS-SS) to both ends of the shell boundary conditions. The Validation of the present work for un-cracked cylinder shells is verified, by the free vibration analysis of a perfect cylindrical shell is performed with (the shell's radius R = 242 mm, shell's length l = 609.5 mm, shell's thickness h = 0.65 mm), The properties of Aluminium: Young's modulus (E = 68.9 MPa), Aluminium's density (ρ = 2700 Kg/m^3) and Poison's ratio (ν = 0.35).

Table 1 shows that the results (analytical, experimental and numerical) obtained from the literature are united agreement with the present results of the fundamental frequencies.

Table 1. Comparison the results obtained from the literature with the present work.

Boundary	References	Frequency(Hz)
SS-SS	Experimental (Cook, 1981)[17]	163 and 169
	Analytical (Bolotin 1964)[18]	168.13
	Numerical (Sewall and Naumann, 1968)[19]	166.22
	Numerical (Javidruzi et al., 2004)[7]	166.40
	Numerical (Xin et al. 2011)[9]	168.73
	Analytical (Husain, and Al-shammari)[14]	168.47
	Numerical (Husain, and Al-shammari)[14]	165.39
	Present analytical analysis	167.28

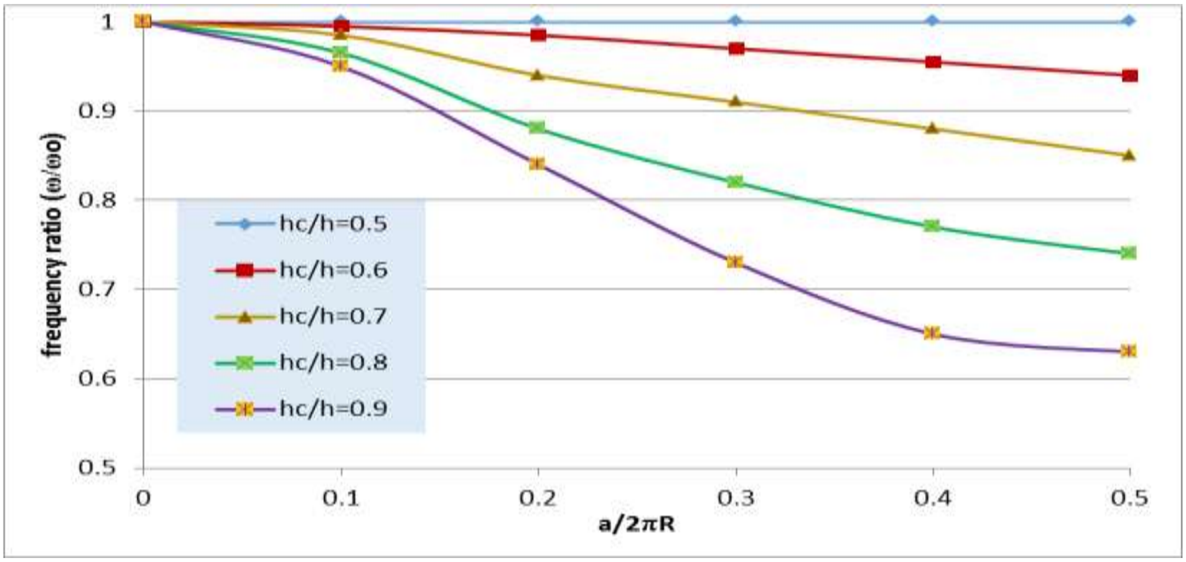
The performance of the circular cylindrical shell structures having a circumferential part-through crack is investigated on the vibration characteristics. Two thin shells (shell 1, and shell 2) are modeled of aluminum material (6061) as shown in table (2). The influences of different parameters like (length crack, crack depth, and location of the crack) on the natural frequencies are examined and discussed as follow:

Table 2. Dimensions of the cylindrical shell structures

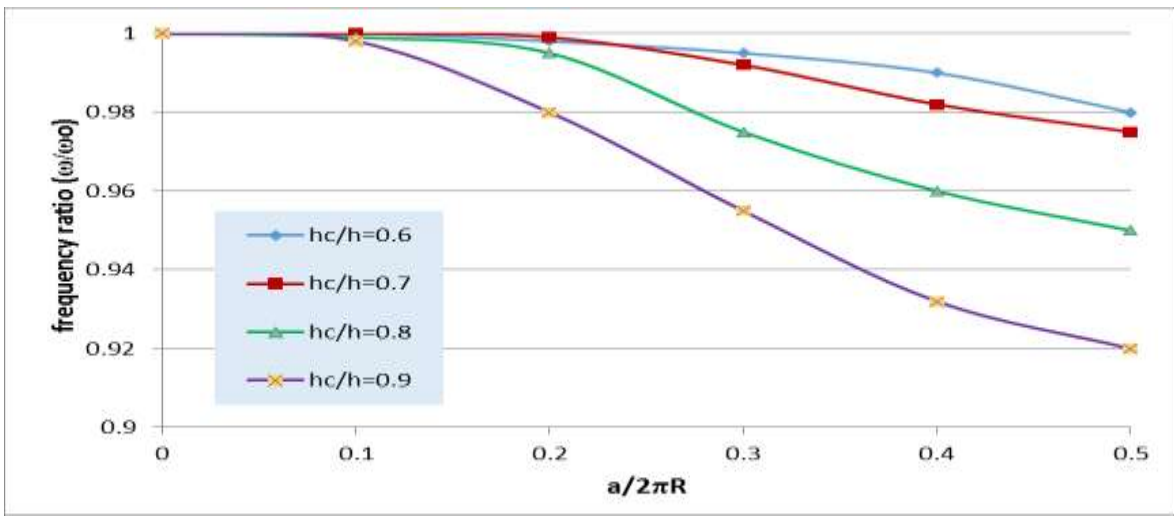
Property	Shell radius R (mm)	Ratio of thickness to radius (R/h)	Ratio of length to radius (L/D)
Shell 1	80	50	1.5
Shell 2	500	500	1

Crack size (length and depth of the fissure) is an important factor that impacts the natural frequency of the fissured shell. So as to estimate the effect of fissure size on the natural frequency, two cylindrical shells of different dimensions (shell 1, shell 2) containing circumferential crack and a set crack located in the middle of the shell were studied with different crack depths and crack lengths to consider the effect of the crack size. A crack has a significant effect on the shell's natural frequency as well as its mode shapes and stiffness. It is found that the shell's stiffness will decrease as a reaction to an increase in the crack depth and this will lead to a reduction of the shell's fundamental natural frequency. However, the natural frequency is little affected by a crack that has a depth of less than 0.5 from the thickness of the shell even if the crack length reaches half of the circumferential of the shell as shown in figure (4).

The impact of the fissure depth on the natural frequency becomes important when it exceeds about 0.5 of the thickness of the cylindrical shell. The existence of the closed circumferential fissure significantly reduces the first natural frequency of the cylindrical shells further than other vibration modes. The natural frequencies have been normalized by the fundamental frequency of the corresponding un-cracked cylindrical shell. This reduction in the natural frequency is gradually at first as the fissure length increases, this influence becomes significant when the fissure length overtake around 10% of the perimeter of the cylindrical shell. The effect of cracking in cylindrical shell 2 is more pronounced than in cylindrical shell 1 because shell 2 shorter and thinner compared with shell 1. Figure (4) shows the results of the fundamental natural frequency (non-dimensional frequency ratio value for all fundamental frequency of un-cracked shell) for different depths of the closed crack differ with the increase of the length crack.



a. For cylindrical shell 2



b. For cylindrical shell 1

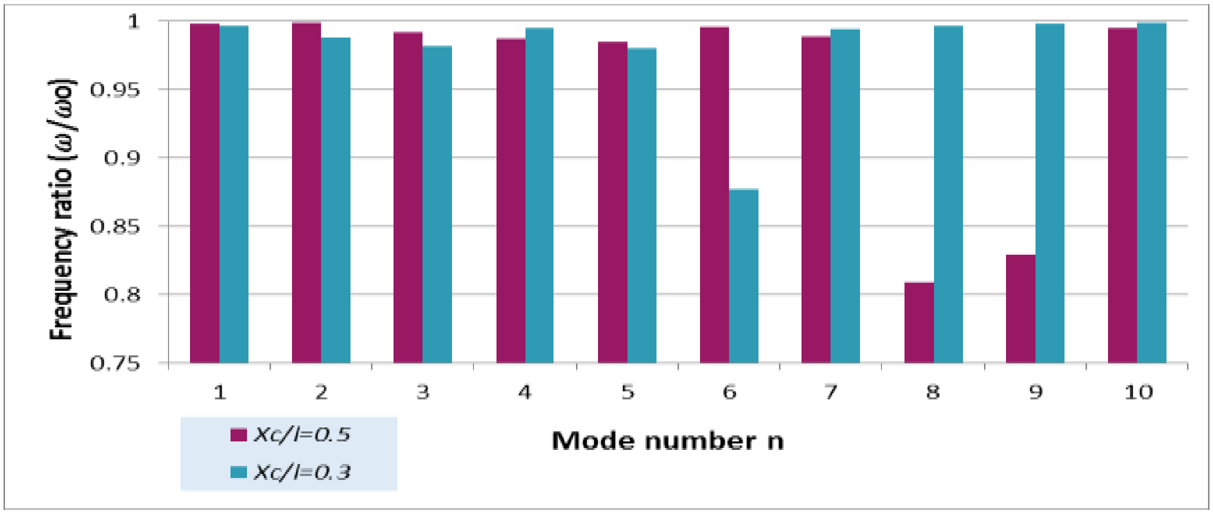
Figure 4. The effects of Normalized crack length (a/2πR) and crack depth (h_c/h) on the change of frequency rate for a cracked shell with x_c / l = 0.5.

Crack location is another significant parameter in knowing the effects of the fissure on the natural frequency of the fissured shell. The variations in the natural frequencies caused by the fissure of the first axial mode (m=1) for ten modes (n≤10) for the SS-SS boundary conditions are calculated. Cylindrical shells 1, 2 containing circumferential surface crack with the relative crack length (a / 2 πR = 0.2), and the relative crack depth (h_c/h=0.8) are studied. Two representative normalized fissure locations (x_c/l = 0.3 & 0.5) are examined to the influence of fissure position on variations in natural frequencies for the circumferential first ten modes.

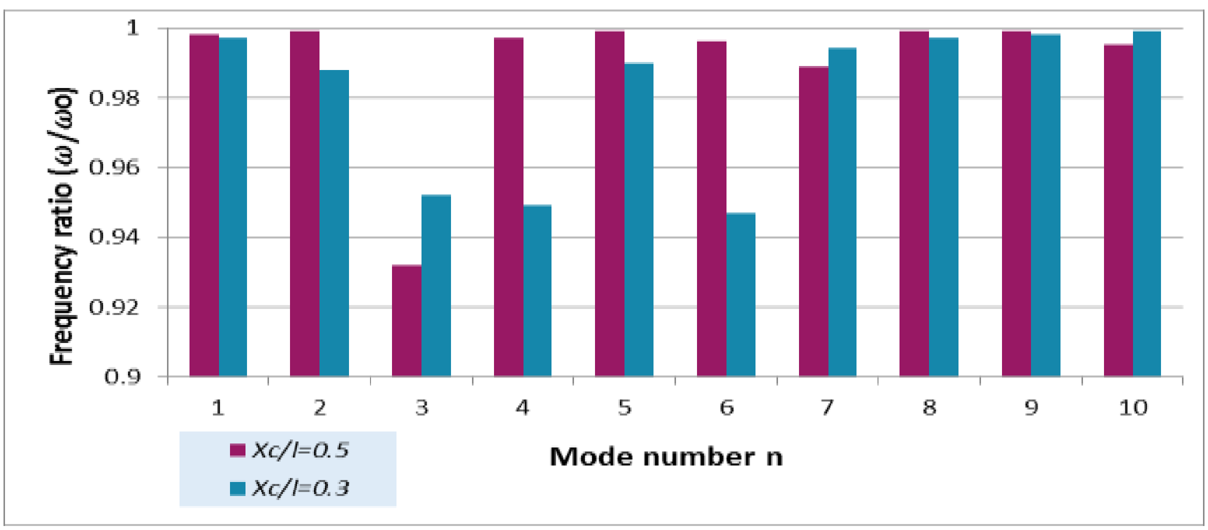
Figure (5) shows the variations in the natural frequency caused by the fissure in the cylindrical shell 1& 2. It is clear that some modes are greatly affected by the existence of the fissure more than others and this effect leads to a change in the natural frequency in it, but these modes differ according to the fissure location for a specific boundary condition.

From an observation of the figure [5(a)] for shell 2 it is found that the modes (6 and 8) are more affected for frequency reduction when the normalized fissure location is 0.3, whereas when it is 0.5, the modes (7 and 8) are more evident for frequency reduction. As the modes that affected by fissure existence rely on the fissure location (for a specific boundary condition), the variations in the natural frequencies caused by fissure for specific modes can be considered as a model for fissure detection through the model-compatibility technique.

It can be seen in figure [5(b)] for shell 1 that modes (3, 4and 6) show more affected when the normalized fissure location is 0.3, while when it is 0.5, only modes (4and 5) are most effective for frequency reduction. Although there are variations in the natural frequencies caused by the crack in some models with respect to shell 1, it is noted that these variations are relatively small compared to the variations in shell 2. And it can thus be concluded that the variations in natural frequencies caused by fissure may be undetectable for the long and thick shell.



a. For cylindrical shell 2



b. For cylindrical shell 1

Figure 5. Crack-induced changes in natural frequencies for different modes of cylindrical shell

Figure (6) shows the changing in ratio natural frequencies for Shell 1, 2 under the two boundary conditions with different crack locations. when the slit is located in the center of the shell's axial direction the maximum reduction of natural frequency is happening. The illustration for this consequence is that the maximum deflection zone is located at the center of the longitudinal direction of the cylindrical shell, in addition to the mode shape of the first circumferential mode is symmetry around the level at x / l = 0.5 is for this kind of conditional term. (as in approach M.A.Husain, and Al-shammari in 2020[14]).

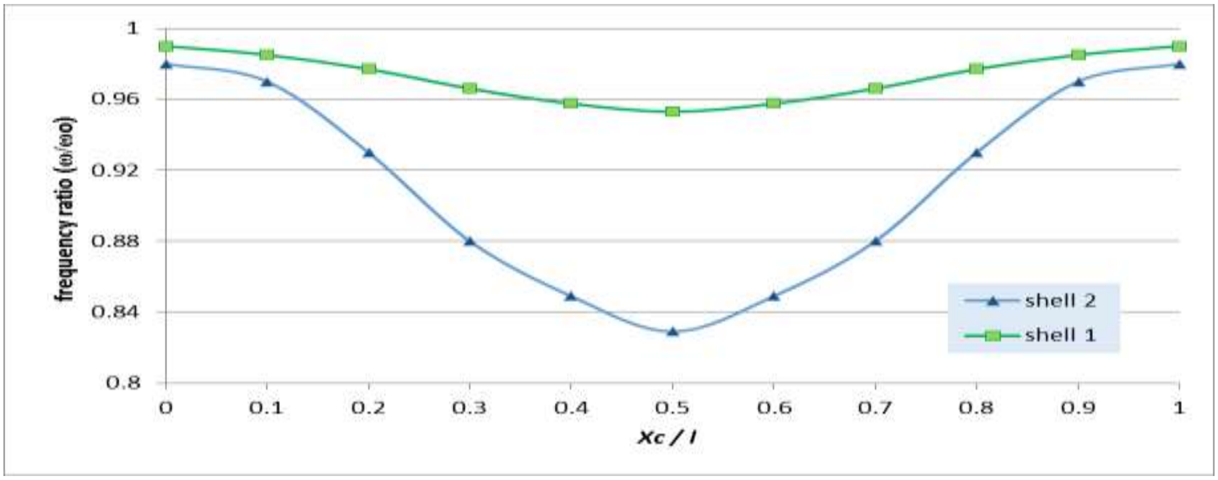


Figure 6. The influence of Normalized crack location on the change of frequency rate with h_c/h = 0.8, a/2πR = 0.2 for a cracked shell 1 and 2

CONCLUSION

The free vibration analysis of circular cylinder shell structures having a finite length and which involving a crack was studied. the effect of different parameters of crack such as (crack length, crack depth, and location of the crack), were investigated on the dynamic vibration characteristics of shell under SS-SS boundary conditions. A comparison was made between analytical results and literature, which showed a good agreement.

The essential findings able to be summed up as below:

1. This study contributed to providing a mathematical solution method for cracked cylindrical shells of finite length to analyze characteristics of free vibration because the effect of the crack is much greater in the short shells.

2. The presence of a crack affects the dynamic vibrational characteristics of the cylindrical shell as expected and reduces the natural frequencies.

3. It was found that the shell's stiffness will decrease as a reaction to an increase in the crack depth and this will lead to a reduction of the natural frequency for the specified modes.

4. The natural frequency is decreased continuously as the fissure length increases and this reduction depends on the crack depth. Also, the change of frequency in a more obvious method in shorter and thinner shells.

5. when the crack exists in the center of the longitudinal direction of the shell, the decrease in the natural frequency is higher than in other places.

6. the crack-created natural frequency changing in a certain number of modes depending on the location of the crack. These modes can be considered as a pattern that is constantly monitored to detect cracks on cylindrical shells.

7. The results of this work could be mostly utilized to detect the existence of a crack in the cylinder shell and illustrating its length, depth, location, and orientation of the crack according to the natural frequency of this cylindrical shell.

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