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A computational method for optimal midcourse guidance law with impact gust wind

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ABSTRACT: A computational method is described as a method used for constructing an optimal midcourse guidance law, which is based on the optimal control theory and initial boundary conditions. This proposed guidance law is derived from an optimal control theory with the boundary conditions which allow relative distance between missile and target at the final time, and at a small line-of-sight rate. A numerical simulation verifies the performance of this guidance law with the impact of harmonic wind. The simulation results demonstrate that the quality of effectiveness as well as the applicability of this proposed guidance law

INTRODUCTION

In general, a missile can be guided by a variety of methods, including proportional navigation guidance, and optimal guidance law. In reference 0, by considering the desired impact angle without violating the field-of-view limit, the authors studied a two-stage pure proportional navigation guidance law. Majumder et al. investigated a near-optimal solution in real-time for air-to-air engagement 0 which is applied to solve the midcourse guidance problem in real-time for air-to-air engagement. The authors also presented recent developments in this field.

In reference 0, the authors proposed an optimal midcourse guidance law with flight path angle and lead angle constrainted to reach a circular target area. This guidance law was derived by appling an optimal control theory which minimizes control energy weighted by a power of a range-to-go. However, in this research, the target was considered as a stationary target because it moves so slowly. For that target, Zhang et al. proposed a novel closed-form guidance law with impact time and impact angle constraints 0. This guidance law takes missile's normal acceleration as the control command directly. By simplifying missile dynamics under small heading error approximation, Chen and Wang derived an optimal guidance law with impact angle constraint against a stationary target 0. An impact time requirement is achieved by adding a feedback controller to the obtained optimal guidance law.

By solving an optimal control problem of minimizing the energy cost function weighted by a power of range-to-go, Park proposed an optimal guidance law with terminal angle constraint at the end of the midcourse phase 0. In reference 0, considering the final velocity vector constraint, an optimal terminal guidance law was developed for exoatmospheric interception using the optimal control theory. By taking the gravity difference model in this approach, the proposed guidance law requires much less fuel than the traditional ones in the exoatmospheric interception. In recent years, the combined guidance laws have been applied widely for air-to-air missiles. These guidance laws have several advantages, such as the significant distance of attack, high accuracy, and sizeable initial look-angle.

In the literature review, however, the targets move so slowly or stationarily. Therefore, the parameters of the target can not be introduced in the geometric dynamic equations that represent the relationship between the missile and target. Then, by applying the optimal control theory, the analytics guidance law can be easy to synthesize. For maneuver target, the guidance law is based on the proportional navigation guidance law with impact angle and impact time. This guidance law, however, is not compared to the optimal one. So, in this study, an optimal guidance law is proposed for a missile to attack a maneuvering target. This guidance law is derived from the optimal control theory with the boundary conditions such as allowing relative distance between missile and target at the final time, and at a small line-of-sight rate.

The major structure of this paper is as follows: In section 2, based on the dynamic missile model, a state equations system is deduced. An optimal midcourse guidance law is also presented in this section, which is based on the boundary conditions at the final time and optimal control theory that minimizes a range of weighted control energy. The simulation results and investigation of the proposed guidance law with the influence of wind are included in section 3, and the conclusion will be presented in section 4.

COMPUTATION OF AN OPTIMAL MIDCOURSE GUIDANCE LAW

Equation of motion

The desired trajectory of a missile is shown in Fig. 1.

Where, V_m and ψ_m are velocity and flight path angle of the missile, respectively;

(x, z) is the position of the missile.

(x_m^*, z_m^*) is the desired position of the missile.

ψ_m^* is the desired flight path angle of the missile.

h is the instant deviation that is defined as a relative distance from the target, which is perpendicular to the velocity vector.

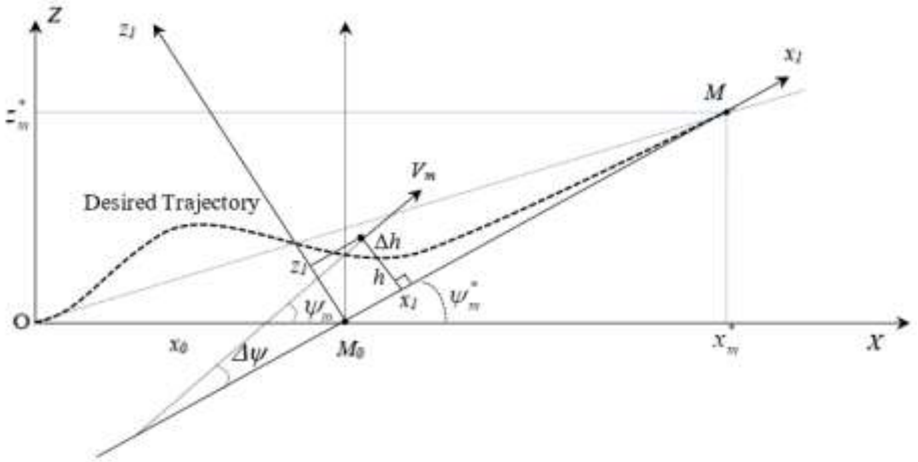


Figure 1. The desired trajectory of a missile.

The equations of motion in the coordinate's $x_0x_1z_1$ as follows:

$$\begin{cases} \Delta\dot{\psi} = \frac{g}{V_m} n_z \\ \dot{x}_1 = V_m \cos\Delta\psi \\ \dot{z}_1 = V_m \sin\Delta\psi \end{cases} \tag{1}$$

We assume that x_m is a variable, then, $z_m = f(x_1)$ is the desired trajectory in the coordinate's $x_0x_1z_1$. The function $f(x_1)$ must ensure rectilinear asymptote with the straight line x_0x_1.

The function $f(x_1)$ is described in detail as follows.

A function which represents the transition from Oxz coordinate to $x_0x_1z_1$ coordinate is given in Eq. 2.

$$z = K\frac{x - x_0}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \tag{2}$$

Where, Ox is the horizontal asymptote of the function z;

$z=0$ when $x\text{-}x_0 = 0$; and $z = z_{\max} = \frac{K}{\sigma} e^{-\frac{1}{2}}$ when $x - x_0 = \sigma$.

We aim to find the values K and σ so that the graph of the function z passes through the point (x_1, a) and reaches the maximum value at (x_2, b). Where, x_1, x_2, a, and b are known values. Using the division method, we get a solution x^* from which we find K and σ satisfy the requirement. The first and second derivatives of Eq. (2) are given as Eq. (3) and Eq. (4):

$$\sigma = \frac{x_2 - x_1}{1 - x^*}; K = be^{\frac{1}{2}}\sigma. \tag{3}$$

$$f_x = \frac{\partial z}{\partial x} = \frac{K}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \left(1 - \left(\frac{x - x_0}{\sigma}\right)^2\right)$$

$$f_{xx} = -\frac{K}{\sigma^2} \frac{x - x_0}{\sigma^2} \left(3 - \frac{(x - x_0)^2}{\sigma^2}\right) e^{-\frac{(x-x_0)^2}{2\sigma^2}} \tag{4}$$

It can be seen from Fig.1 that the deviation between a missile trajectory and the desired one is determined as follows: $\Delta h = h - z_m = z_1 - z_m$, two-sides derivation and combination with Eq. (1) we have:

$$\Delta \dot{h} = V_m \sqrt{1 + f_x^2} \sin(\Delta\psi - \psi_m) \tag{5}$$

Where, $f_x = \frac{\partial f(\mathrm{x}_1)}{\partial x_1}$; $\sin\psi_m = \frac{f_x}{\sqrt{1+f_x^2}}; \cos\psi_m = \frac{1}{\sqrt{1+f_x^2}}; \tan\psi_m = f_x$.

ψ_m is the desired flight path angle at the time t.

Let $\Delta\bar{\psi} = \Delta\psi - \psi_m$ is the bias between the flight path angle and the desired one. Two-sides derivative is:

$$\Delta\dot{\bar{\psi}} = \Delta\dot{\psi} - \dot{\psi}_m \tag{6}$$

Combining with the first equation of the Eq. (1), we have:

$$\begin{cases} \Delta\dot{\bar{\psi}} = \dfrac{g}{V_m} n_z - \dot{\psi}_m \\ \Delta\dot{h} = V_m \sqrt{1 + f_x^2} \sin(\Delta\bar{\psi}) \end{cases}$$

Let $n_{zm} = \sqrt{1+f_x^2}(n_z - \frac{V_{tbb}}{g}\dot{\psi}_m); \bar{V}_m = V_m\sqrt{1+f_x^2}$ (7)

We get:

$$\begin{cases} \Delta\dot{\bar{\psi}} = \dfrac{g}{\bar{V}_m} n_{zm} \\ \Delta\dot{h} = \bar{V}_m \sin(\Delta\bar{\psi}) \end{cases} \tag{8}$$

A. Computation of an optimal midcourse guidance law

Let us consider the cost function is as follows:

$$J = \tfrac{1}{2}\rho_1 \Delta\bar{\psi}_{t_f}^2 + \tfrac{1}{2}\rho_2 \Delta h_{t_f}^2 + \tfrac{1}{2}\int_{t_o}^{t_f} k n_{z0}^2 dt \to \min \tag{9}$$

$G = \frac{1}{2}\begin{bmatrix}\Delta\bar{\psi} \\ \Delta h\end{bmatrix}_{t_f}^T \begin{bmatrix}P^f\end{bmatrix}_{2\times 2} \begin{bmatrix}\Delta\bar{\psi} \\ \Delta h\end{bmatrix}_{t_f}$, $P^f = \begin{bmatrix}\rho_1 & 0 \\ 0 & \rho_2\end{bmatrix}$, P^f is the symmetric matrix;

From Eq. (7) and Eq. (8) the Hamiltonian function is as follows:

$$H = \lambda_{\Delta\bar{\psi}} \frac{g}{\bar{V}_m} n_{zm} + \lambda_{\Delta h} \bar{V}_m \sin \Delta\bar{\psi} + \frac{1}{2} k n_{zm}^2 \tag{10}$$

Co-state variables are defined as the following:

$$\begin{cases} \dfrac{d\lambda_{\Delta\bar{\psi}}}{dt} = -\dfrac{\partial H}{\partial \Delta\bar{\psi}} = -\lambda_{\Delta h} \bar{V}_m \cos \Delta\psi \\ \dfrac{d\lambda_{\Delta h}}{dt} = -\dfrac{\partial H}{\partial h} = 0 \end{cases} \tag{11}$$

The expression determines the optimal solution $\partial H / \partial n_{z_m} = 0$, we obtain:

$$n_{zm} = -\frac{g}{k} \frac{1}{\bar{V}_m} \lambda_{\Delta\bar{\psi}} \tag{12}$$

From Eq. (9) the Teminant function: $G = \frac{1}{2} \rho_1 \Delta\overline{\psi}_{t_f}^2 + \frac{1}{2} \rho_2 \Delta h_{t_f}^2$, we have converted boundary conditions:

$$\begin{cases} \lambda_{\Delta\bar{\psi}}\Big|_{t_f} = \dfrac{\partial G}{\partial \Delta\overline{\psi}}\Big|_{t_f} = \rho_1 \Delta\overline{\psi}\Big|_{t_f} \\ \lambda_{\Delta h}\Big|_{t_f} = \dfrac{\partial G}{\partial \Delta h}\Big|_{t_f} = \rho_2 \Delta h\Big|_{t_f} \end{cases}$$

In a matrix form:

$$\begin{bmatrix} \lambda_{\Delta\bar{\psi}} \\ \lambda_{\Delta h} \end{bmatrix}_{t_f} = \left[P^f \right] \begin{bmatrix} \Delta\overline{\psi} \\ \Delta h \end{bmatrix}_{t_f} ; P^f = P^f = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix} \tag{13}$$

The Eq. (8) rearranged in a matrix form is given:

$$\begin{bmatrix} \Delta\dot{\bar{\psi}} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \bar{V}_m & 0 \end{bmatrix} \begin{bmatrix} \Delta\bar{\psi} \\ \Delta h \end{bmatrix} + \begin{bmatrix} \frac{g}{\bar{V}_m} \\ 0 \end{bmatrix} n_{zm} \text{ where, } \sin \Delta\bar{\psi} \approx \Delta\bar{\psi}$$

$$\dot{x} = A^{(11)} x + Bu \text{ ; where, } A^{(11)} = \begin{bmatrix} 0 & 0 \\ \bar{V}_m & 0 \end{bmatrix}, \ B = \begin{bmatrix} \frac{g}{\bar{V}_m} \\ 0 \end{bmatrix}, \ x = \begin{bmatrix} \Delta\bar{\psi} \\ \Delta h \end{bmatrix}, \ u = n_{zm} \tag{14}$$

From Eq. (11) we can rewrite in a form as follows:

$$\begin{bmatrix} \dot{\lambda}_{\Delta\bar{\psi}} \\ \dot{\lambda}_{\Delta h} \end{bmatrix} = \begin{bmatrix} 0 & \bar{V}_m \cos \Delta\bar{\psi} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{\Delta\bar{\psi}} \\ \lambda_{\Delta h} \end{bmatrix} \text{, where } \dot{\lambda} = A^{(22)} \lambda \tag{15}$$

and $A^{(22)} = \begin{bmatrix} 0 & \bar{V}_m \cos \Delta\bar{\psi} \\ 0 & 0 \end{bmatrix}; \lambda = \begin{bmatrix} \lambda_{\Delta\bar{\psi}} \\ \lambda_{\Delta h} \end{bmatrix}$

From Eq. (12) and Eq. (14) we have: $\dot{x} = A^{(11)} x - B \frac{g}{k} \frac{1}{\bar{V}_m} [1 \quad 0] \lambda$

$$\dot{x} = A^{(11)}x + A^{(12)}\lambda \text{, where } A^{(12)} = -\frac{1}{k}\left(\frac{g}{\bar{V}_m}\right)^2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (16)$$

And from Eq. (15) and Eq. (16) we obtain:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A^{(11)} & A^{(12)} \\ A^{(21)} & A^{(22)} \end{bmatrix}\begin{bmatrix} x \\ \lambda \end{bmatrix} \text{, where } A^{(21)} = [0]_{2\times2} \quad (17)$$

From Eq. (13), the states at the final time are determined as follows:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix}_{t_f} = \begin{bmatrix} I_{2\times2} \\ P^f \end{bmatrix} x_{t_f} \quad (18)$$

The solution of Eq. (17) is $\begin{bmatrix} x \\ \lambda \end{bmatrix} = \Phi(t_f \text{-} t)\begin{bmatrix} x \\ \lambda \end{bmatrix}_{t_f}$. Substituting into Eq. (18), we have:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} \Phi_{11}(t_f - t) & \Phi_{12}(t_f - t) \\ \Phi_{21}(t_f - t) & \Phi_{22}(t_f - t) \end{bmatrix}\begin{bmatrix} I_{5\times5} \\ P^f \end{bmatrix} x_{t_f} \quad (19)$$

From Eq. (19), we get: $x = \left(\Phi_{11} + \Phi_{12}P^f\right)x_{tf} \Rightarrow x_{t_f} = \left(\Phi_{11} + \Phi_{12}P^f\right)^{-1}x$

$$\lambda = \left(\Phi_{21} + \Phi_{22}P^f\right)x_{t_f} = P^T(t)x \quad (20)$$

Where, $P^T(t) = \left(\Phi_{21} + \Phi_{22}P^f\right)\left(\Phi_{11} + \Phi_{12}P^f\right)^{-1}$

From Eq. (13) and Eq. (20) we obtain boundary conditions:

$$P^T(t_f) = P^f \quad (21)$$

Two-sides derivative of Eq. (20) we get:

$$\dot{\lambda} = P^T(t)\dot{x} + \dot{P}^T(t)x \quad (22)$$

From Eq. (15), (16), (20), and (22) we get:

$$A^{(22)}P^T(t)x = P^T(t)(A^{(11)}x + A^{(12)}P^T(t)x) + \dot{P}^T(t)x$$

$$\dot{P}(t) = P(t)\left(A^{(22)}\right)^T - \left(A^{(11)}\right)^T P(t) - P(t)A^{(12)}P(t) \quad (23)$$

$$\dot{P}(t) = \begin{bmatrix} \bar{V}_m(\cos\Delta\bar{\psi}P_{12} - P_{21})\frac{1}{k}\left(\frac{g}{\bar{V}_m}\right)^2 P_{11}^2 & -\bar{V}_{tl}P_{22} + \frac{1}{k}\left(\frac{g}{\bar{V}_m}\right)^2 P_{11}P_{12} \\ \bar{V}_m\cos\Delta\bar{\psi}P_{22} + \frac{1}{k}\left(\frac{g}{\bar{V}_m}\right)^2 P_{21}P_{11} & \frac{1}{k}\left(\frac{g}{\bar{V}_m}\right)^2 P_{21}P_{12} \end{bmatrix} \quad (24)$$

From Eq. (12) and Eq. (20) an optimal midcourse guidance law is determined as follows:

$$n_{zm} = -\frac{g}{k}\frac{1}{\bar{V}_m}\begin{bmatrix} 1 & 0 \end{bmatrix}P^T(t)x \quad (25)$$

To find P(t), it is necessary to solve the Riccati equation (24) with boundary conditions Eq. (21). However, this is very difficult because the Eq. (24) has no analytic solution. Therefore, here we find the approximate solution by adding constraints on the quality criteria of the system.

From Eq. (25) we have:

$$n_{zm} = -\frac{g}{k}\frac{1}{\bar{V}_m}(P_{11}\Delta\bar{\psi} + P_{21}\Delta h),$$

Let $k_1 = \frac{g}{k}\frac{1}{\bar{V}_m}P_{11}; k_2 = \frac{g}{k}\frac{1}{\bar{V}_m}P_{21}$ we get:

$$n_{zm} = -k_1\Delta\bar{\psi} - k_2\Delta h \tag{26}$$

So, instead of finding P_{11}, P_{21}, we find k_1 và k_2;

Substituting Eq. (26) into Eq. (7), we have:

$$\Delta\dot{\bar{\psi}} = -\frac{g}{V_m}(k_1\Delta\bar{\psi} + k_2\Delta h) \tag{27}$$

A two-sided derivation of Eq. (27) is given:

$$\Delta\ddot{\bar{\psi}} = -\frac{g}{\bar{V}_m}(k_1\Delta\dot{\bar{\psi}} + k_2\Delta\dot{h}) \tag{28}$$

Let $y_1 = \Delta\bar{\psi}; y_2 = \Delta\dot{\bar{\psi}}$, the Eq. (28) is rearranged:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -\frac{g}{\bar{V}_m}k_1 y_2 - gk_2 y_1 \end{cases} \text{ or } \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -gk_2 & -\frac{g}{\bar{V}_m}k_1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{29}$$

The characteristic equation of the equation system (29) is followed:

$$\alpha^2 + \frac{g}{\bar{V}_m}k_1\alpha + gk_2 = 0$$

$$\alpha_{1,2} = \frac{-\frac{g}{\bar{V}_m}k_1 \pm \sqrt{\left(\frac{g}{\bar{V}_m}k_1\right)^2 - 4gk_2}}{2} \tag{30}$$

The general solution of Eq. (28) is given:

$$y_1 = \Delta\bar{\psi} = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} \tag{31}$$

The Eq. (31) is the general solution form of the homogeneous Eq. (28). In order to errors in the stable mode become zero, just satisfy conditions $\alpha_1 < 0$ *and* $\alpha_2 < 0$. To avoid over-tuning, α_1 and α_2 are real numbers, which means that:

$$\left(\frac{g}{\bar{V}_{tl}}k_1\right)^2 > 4gk_2 \Rightarrow k_1 > 2\bar{V}_{tl}\sqrt{\frac{k_2}{g}}; k_2 > 0 \tag{32}$$

Because $k_1 > 0$ and $k_2 > 0$, then $\alpha_1 > \alpha_2$. Therefore, if $\alpha_1 < 0$, it will be sure that $\alpha_2 < 0$.

Besides, we have: $-1/\alpha_1 = \tau$, where τ is a time constant.

We add 2 constraints to the system. The first constraint is that the control law must ensure that the time constant is not higher than the allowed time constant. Then, we obtain:

$$-\frac{-\frac{g}{\bar{V}_m}k_1+\sqrt{\left(\frac{g}{\bar{V}_m}k_1\right)^2-4gk_2}}{2}=\frac{1}{T_{CP}}$$. Where, T_{CP} is the allowed time constant

$$k_2=\frac{1}{\bar{V}_m T_{CP}}\left(k_1-\frac{\bar{V}_m}{gT_{CP}}\right) \quad (33)$$

The second constraint is that a normal overload (the value of the control signal) is not higher than the allowed overload, which means that $|n_{zm}| \le n_{zm-CP}$..

Integrating with Eq. (26), we get:

$$n_{zm-CP}=k_1\Delta\bar{\psi}_{\max}+k_2\Delta h_{\max},$$

Where, $\Delta\bar{\psi}_{\max},\Delta h_{\max}$ are the maximum value of bias of flight path angle. Then,

$$k_1=\frac{n_{zm-CP}+\frac{1}{gT_{CP}^2}\Delta h_{\max}}{\frac{1}{\bar{V}_m T_{CP}}\Delta h_{\max}+\Delta\bar{\psi}_{\max}} \quad (34)$$

Substituting k_1 into Eq. (33), we get:

$$k_2=\frac{n_{zm-CP}+\frac{\bar{V}_m}{gT_{CP}}\Delta\bar{\psi}_{\max}}{\Delta h_{\max}+\bar{V}_m T_{CP}\Delta\bar{\psi}_{\max}} \quad (35)$$

From Eq. (7) and Eq. (26) we obtain:

$$n_z=-\frac{k_1\Delta\bar{\psi}+k_2\Delta h}{\sqrt{1+f_x^2}}+\frac{\bar{V}_m}{g}\dot{\psi}_m \quad (36)$$

So, the Eq. (36) is the optimal midcourse guidance law with the coefficients are determined as Eq. (34) and Eq. (35); and f_x, $\dot{\psi}_m$ are calculated as Eq. (3) and Eq. (4).

Determining the desired line-of-sight angle at final time t_f

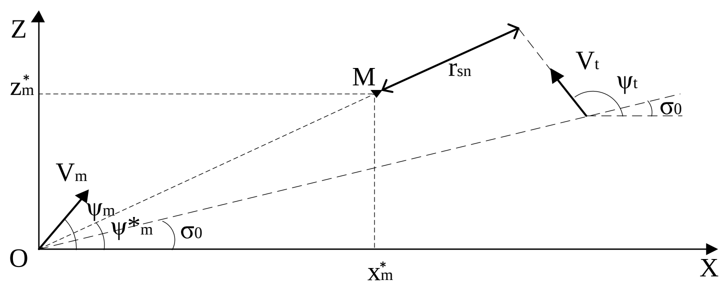


Figure 2. Determining a desired line-of-sight angle.

Based on Figure 2, we obtain:

$$\begin{cases}(t_{td}V_m+r_{td})\sin\Delta\psi=t_{td}V_t\sin\Delta\psi_t\\(t_{td}V_m+r_{td})\cos\Delta\psi=r_0+t_{td}V_t\cos\Delta\psi_t\end{cases} \quad (37)$$

Where, r is a relative distance between a missile and a target, then $r(t_f)=r_{td}$, and $r(t_0)=r_0$; t_f is a final time.

Solving this equations system, we get:

$$t_{td} = \frac{r_0}{V_m}\frac{-\left(k_r - k_V\cos\Delta\psi_t\right)+\sqrt{\left(k_r - k_V\cos\Delta\psi_t\right)^2+(1-k_V^2)(1-k_r^2)}}{(1-k_V^2)}$$

$$\psi_m^* = \arcsin\left(\frac{t_{td}V_t}{t_{td}V_m + r_{td}}\sin\Delta\psi_t\right)+\sigma_0 \tag{38}$$

$$z_m^* = t_{td}V_m\left(\frac{t_{td}V_t}{t_{td}V_m + r_{td}}\sin\Delta\psi_t\right);\ x_m^* = t_{td}V_{tbb}\frac{r_0 + t_{td}V_t}{r_{td} + t_{td}V_m}\cos\Delta\psi_t$$

However, since the missile trajectory is a high curve, we have to calculate the amount of compensation. Therefore, the desired line-of-sight (LOS) angle compensated by the missile's curved trajectory is given as follows:

$$\psi_m^* = \arcsin\left(\frac{t_{td}V_t}{t_{td}V_m + r_{td}}\sin\Delta\psi_t\right)+\sigma_0+\psi_d \tag{39}$$

The compensation of the flight path angle is calculated as follows:

$$\sin\psi_d \approx V_t t_{sl} / r_{td} \tag{40}$$

Where, t_{sl} is the time error when the missile reaches the engagement point.

r_{td} is a relative distance between the missile and target at the final time.

To eliminate slippage, this compensation is given:

$$\sin\psi_d = -\frac{V_t}{V_m}\sin(\Delta\psi - \Delta\psi_t) \tag{41}$$

A wind turbulence model

It can be noted that wind turbulence can be modeled in several forms such as disturbance models according to the horizontal wind field and harmonized wind field. In this paper, we investigate the influence of the harmonized wind field in the horizontal plane to the quality of the proposed guidance law.

The harmonized wind model is given as follows:

$$\begin{cases} V_{Wx_0} = 10\sin\left(2\pi t / T_W\right) \\ V_{Wz_0} = 10\sin\left(2\pi t / T_W + \varphi\right) \end{cases} \tag{42}$$

SIMULATION RESULTS AND DISCUSSION

In this work, numerical simulations were designed for evaluating the quality of effectiveness as well as the applicability of this proposed guidance law.

In the simulation experiments, the parameters of a missile and target are given in Table 1.

Table 1. The parameters of a missile and target point

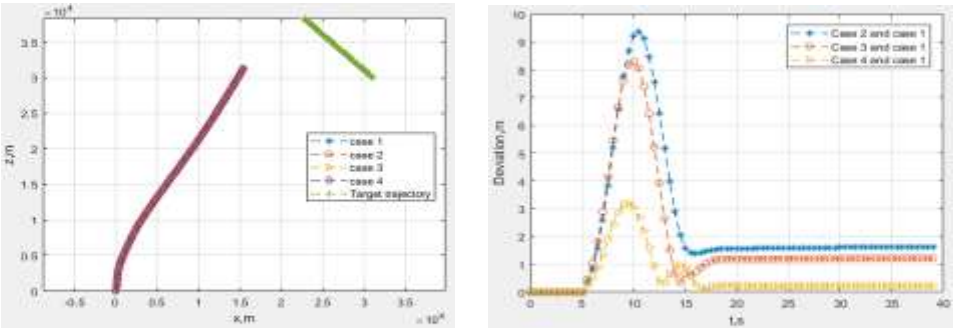
Parameter		Value
Velocity of missile	V_m	900 m/s
Overload factor	$\lvert n_m \rvert$	≤ 30
Line-of-sight angle	σ_0	atan(30/31)

Initial missile position	(x_{m0}, z_{m0})	(0, 0) m
Initial target position	(x_{t0}, z_{t0})	(31.000, 31.0000) m
Relative distance between the missile and target at the final time	r_{td}	10.000 m
Velocity of target	V_t	300 m/s
Path angle of target	ψ_t	135^0
Velocity of wind	W_0	10 m/s

In order to investigate the impact of wind on the proposed guidance law, we design four cases to do numerical simulation. In this study, we assume that the wind is harmonic wind with amplitude W_0 = 10m/s, period T_w = 10s, and the time of occurrence $T \in [5; 15]$s. The direction angle of wind compared to OZ axis are π/6, π/3, π/2 corresponding to case 2, 3, and 4. The mathematical model of wind in four cases is given in the table 2.

Table 2. The mathematical model of wind

Case 1	Case 2	Case 3	Case 4
No wind	$\begin{cases} V_{Wx_0} = 10\sin\left(2\pi t / T_W\right) \\ V_{Wz_0} = 10\sin\left(\dfrac{2\pi t}{T_W} + \dfrac{\pi}{6}\right) \end{cases}$	$\begin{cases} V_{Wx_0} = 10\sin\left(2\pi t / T_W\right) \\ V_{Wz_0} = 10\sin\left(\dfrac{2\pi t}{T_W} + \dfrac{\pi}{3}\right) \end{cases}$	$\begin{cases} V_{Wx_0} = 10\sin\left(2\pi t / T_W\right) \\ V_{Wz_0} = 10\sin\left(\dfrac{2\pi t}{T_W} + \dfrac{\pi}{2}\right) \end{cases}$



a) **b)**

Figure 3. The missile trajectory and target trajectory (a) and the difference of missile trajectories between four cases (b).

The Figure 3 shows the missile trajectory and target trajectory in four cases. It can be seen that the missile trajectories in four cases are the same. However, when the harmonic wind is introduced, there is a bias compared with the case that does not consider the wind.

The overload factor and its difference in the four cases are shown in Fig. 4. When the wind occurs, there was a bias of overload factor ($\Delta n_{tl} = 1.3$). However, this bias will be disappeared when the wind is finished.

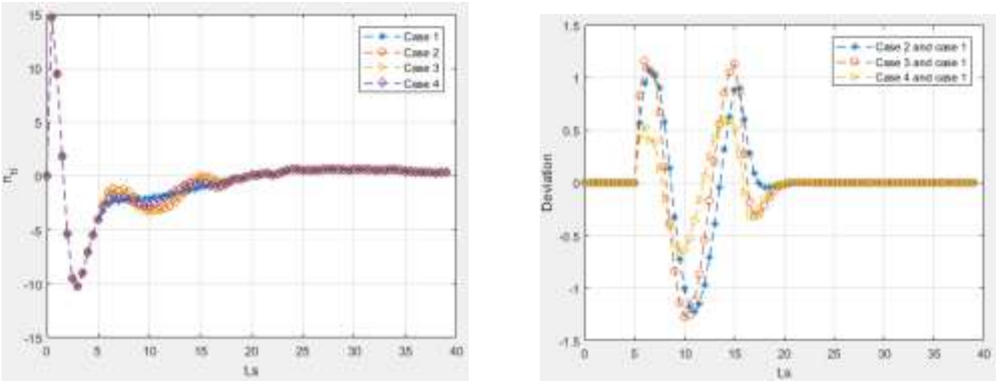


Figure 4. The overload factor and its difference between four cases

Figure 5 represents the instant deviation and its bias. In Fig. 5, as expected, the instant deviation goes to zero. Also, it becomes significant in the period that wind is introduced, and quickly reduces to instantaneous value over time as wind is disappeared.

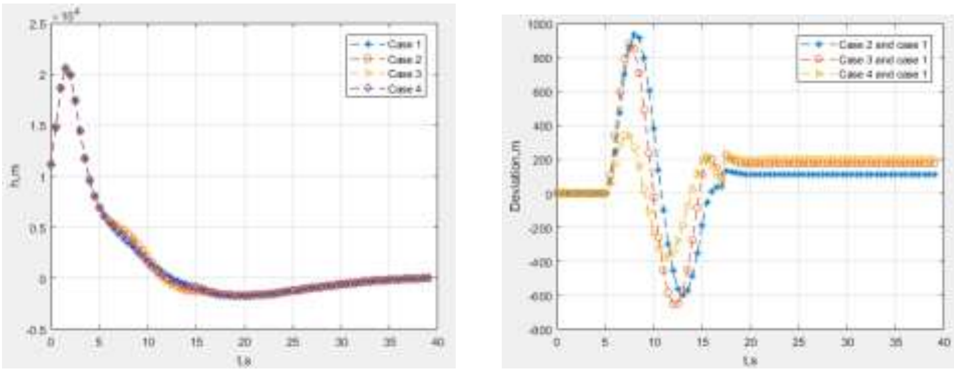


Figure 5. The instant deviation and its bias.

The LOS rate and its difference between four cases are shown in Fig. 6. There is a deviation of the LOS rate at the final time between two cases: one considers a harmonic wind, whereas the other does not consider it. However, it should be noted that this deviation is so small, $\Delta\dot{\sigma} \approx -0,005$ deg/s.

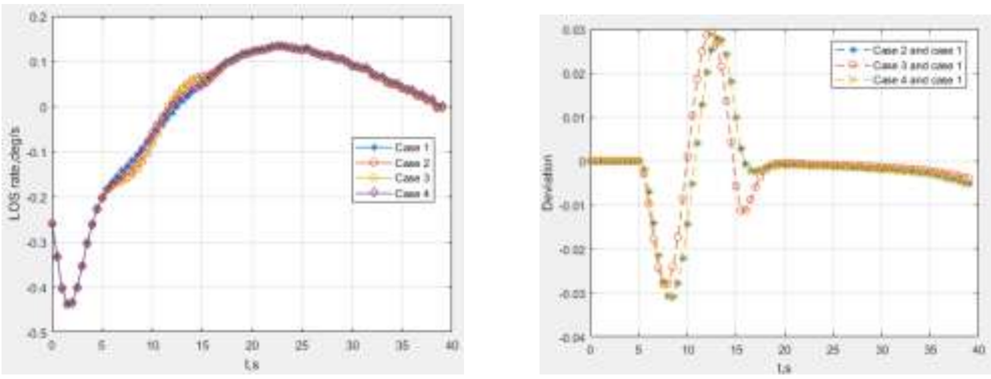


Figure 6. The line-of-sight rate and its difference between four cases.

The heading angle and its difference between four cases are shown in Fig. 7. It can be seen that the heading angle reaches the desired one even there is a disturbance wind.

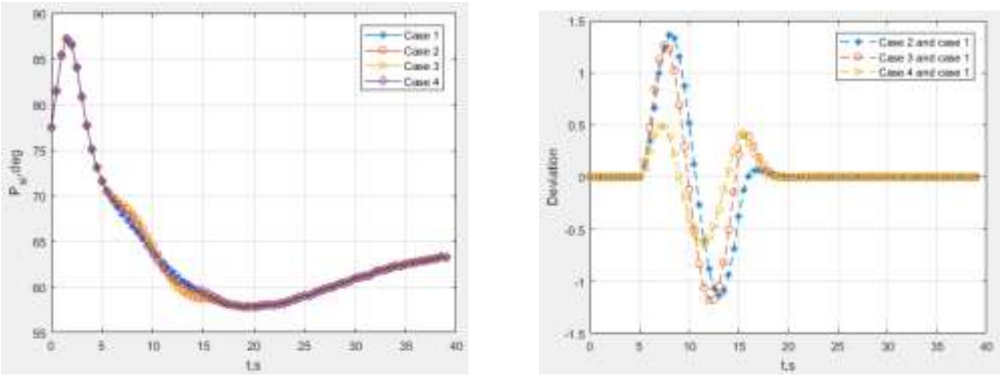


Figure 7. The heading angle and its difference between four cases.

CONCLUSION

An optimal midcourse guidance law is presented in this paper, which is based on the optimal control theory that minimizes a range of weighted control energy with initial boundary conditions. The simulation results show that the constraint requirements are satisfied at the final time, such as small LOS rate, overload factor, and instant deviation. By introducing the harmonic wind, the simulation results provide the evaluations of the quality of effectiveness as well as the applicability of this proposed guidance law.

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Nomenclature

h = instant deviation

g = gravity acceleration

n_m = overload factor of the missile

Δn_m = bias of overload factor of the missile

n_{zm} = normal overload factor

n_{zmCP} = allowed normal overload factor

T_{CP} = allowed time constant

T_w = period of harmonic wind

t_f = final time

t_{sl} = time error when missile reaches the engagement point

V_m = velocity of the missile

V_t = velocity of the target

R = relative distance between a missile and a target

r_{mt} = relative distance between a missile and a target at the final time

r_0 = relative distance between a missile and a target at the initial time

W_0 = Velocity of wind

x = X component of the position of the missile

z = Z component of the position of the missile

x_m* = X component of the desired position of the missile

z_m* = Z component of the desired position of the missile

ψ_t = flight path angle of the target

ψ_m = flight path angle of the missile

ψ_m* = desired flight path angle of the missile

σ_0 = initial line-of-sight angle