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Numerical Method for Finding the Balancing and Unbalancing Forces of Single Piston Engine

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ABSTRACT: Mathematical equations are derived in this research with the use of a binomial theorem for finding balancing forces acting on a single piston engine. This method does not reflect the exact mathematical model because of its approximate solution. In order to achieve high accuracy of calculations, the Newton-Raphson method has been employed to calculate the response of the system. The moment exerted on the crank is found out in two cases, the effect of gas pressure and without gas pressure on the piston. Results revealed similar behavior generated in the two theorems of the resultant of all external forces in x-direction, y-direction, and the moment exerted on the crank in case of without gas pressure effect. The obtained results of the response, with the help of MATLAB code, confirm the applicability of the Newton-Raphson method to give more accurate results than the binomial theorem. The results improvement of the moment values without pressure effect is 5% and with pressure is 2.9%. The improvement of all external forces in x-direction values is 10% without pressure and 9.8% with pressure effect, however for the external forces in y-direction values is 9.98% where is no pressure effect.

KEYWARDS: Mathematical model; Binomial theorem; Newton-Raphson; Crank shaft

INTRODUCTION

Any of the motor's crankshaft must be balanced to be operated without damage. Balancing forces must be evaluated to ensure the study response of the engine. Different analyses have been carried out to explain the dynamic response of the engine mechanism of reciprocating movement. The binomial theorem has been used as a tool for simplifying the mathematical relation and equations in research studies [1,2]. The linear analysis used a combination of static forces and inertia forces with ignoration of the friction force for simplification, studied, and analyzed the engine mechanism of reciprocating movement [3]. A new methodology suggested for gauge dynamic balancing of motorcycle crankshaft. The motorcycle comprises of the crankshaft is a rigid rotor, connecting the rod and piston. The crankshaft that generated the centrifugal force made the engine vibrate while the engine was moving. Experimental validation was accomplished and compared with mathematical modeling. There was a good agreement between the results [4]. Several parameters were considered for the optimal design, like how forces acted on the mountain and velocity. A comparison between the simulation and theoretical arithmetic results were confirmed with a static analysis [5,6]. The number of cylinders and the crankshaft arrangement are parameters that significantly affect the torque forces and then on the engine balancing, these parameters studied for design optimization purpose [7].

A model of reciprocating single engine with a new feature of expressing the moment as a function of crank motion instead of crank rotation with the use of Hamilton's principle application [8]. An original probabilistic model of the balance of internal combustion engines with computer simulation was used to validate the probabilistic balancing model. The Rayleigh law for the distribution of probability, distributed statically, was used for the analytical modeling of the assumed first order, forces, and moment [9]. In this research, a mathematical model developed by using a numerical method instead of the Binomial expansion. Newton Raphson is a root-finding algorithm, has been used as a comparison method to the Binomial method as a solution for finding balancing and unbalancing forces of a single piston engine. Results revealed similar behavioral responses generated in the two theorems of the resultant of all external forces in x-direction, y-direction, and the moment exerted on the crankshaft. The obtained response with the help of MATLAB code confirms the applicability of the Newton Raphson method to give more accurate results than the binomial theorem. For design optimization, the solution with the Newton Raphson method should be considered in the case of finding the balancing force for the crankshaft. Based on our knowledge no one has been used the Newtown Raphson method in the calculation to find the balancing and unbalancing forces that affect the crankshaft.

MATHMATICAL MODELING

An illustration of a slider mechanism system of a single cylinder is in Figure 1, which is classical of 4-bar used in many machines. The crank length in the mechanism is denoted by R_1 and the mass distance of the crank by r_1. The figure illustrates that the length of the connecting rod is R_2 and the distance to the center of gravity of the connecting rod is r_2. The piston is presented by a box sliding along a fixed surface. As the crank rotates in the direction shown by β, the instantaneous displacement of the piston x changes in a reciprocating manner. The angle of rotation of the connecting rod is α for the case where $R_2 >> R_1$. The applied moment on the crank for a single cylinder is denoted by the symbol M_z. In this figure, the mass of the crank, connecting rod, and piston are shown by the symbols m_k, m_r, and m_p respectively. I_k and I_r represent the mass moment inertia for the crank and the connecting rod about their mass centers, respectively.

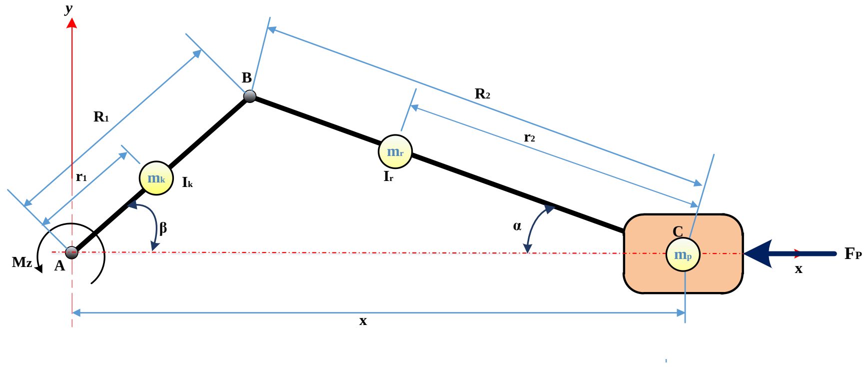


Figure 1. A schematic of a slider mechanism system

Dynamic analysis of the slider crank mechanism without gas pressure (F_p), from Figure 1,

$$x = R_1 cos\beta + R_2 cos\alpha \tag{1}$$

And

$$R_1 sin\beta = R_2 sin\alpha \tag{2}$$

where

R_1 is the length of the crank shaft, R_2 is the length of the connecting rod, β is the crank shaft angle and α is the connecting rod angle.

But $cos\alpha = \sqrt{1-\sin^2\alpha}$, then x will be

$$x = R_1 cos\beta + R_2\sqrt{1-\sin^2\alpha} \tag{3}$$

From Equation (2), $sin\alpha = \frac{R_1}{R_2} sin\beta$, then

$$x = R_1 cos\beta + R_2\sqrt{1-\left(\frac{R_1}{R_2} sin\beta\right)^2}$$

(4)

Let $\lambda = \frac{R_1}{R_2}$ then Equation (4) can be written as,

$$x = R_1 cos\beta + R_2 \sqrt{1-\lambda^2 \sin^2 \beta} \quad (5)$$

Or

$$x = R_1 \left(cos\beta + \frac{1}{\lambda}\sqrt{1-\lambda^2 \sin^2 \beta} \right) \quad (6)$$

Using the binomial theorem,

$$x = R_1 \left(\frac{1}{\lambda} + cos\beta - \frac{\lambda}{2} sin\beta - \frac{\lambda^2}{8}\sin^2 \beta - \frac{\lambda^5}{16}\sin^6 \beta - \dots \right) \quad (7)$$

Observing that,

$$\left\{ \begin{array}{c} \sin^2 \beta = \frac{1}{2}(1 - cos2\beta) \\ \sin^4 \beta = \frac{1}{6}(3 - 4cos2\beta + cos4\beta) \\ \sin^6 \beta = \frac{1}{32}(10 - 15cos2\beta + 6cos4\beta - cos6\beta) \end{array} \right\} \quad (8)$$

Substituting Equation (8) into Equation (7) yields,

$$x = R_1 \left(A_0 + cos\beta - \frac{1}{2}A_2 cos2\beta - \frac{1}{16}A_4 \cos 4\beta + \frac{1}{36}A_6 \cos 6\beta - \dots \right) \quad (9)$$

Where

$$\left\{ \begin{array}{l} A_0 = \frac{1}{\lambda} - \frac{1}{4}\lambda - \frac{3}{64}\lambda^3 - \frac{5}{256}\lambda^5 - .... \\ A_2 = \lambda - \frac{1}{4}\lambda^3 + \frac{15}{128}\lambda^5 + ... \\ A_4 = \frac{1}{4}\lambda^3 + \frac{3}{16}\lambda^5 + ... \\ A_6 = \frac{9}{126}\lambda^5 + ... \end{array} \right\} \quad (10)$$

Now, using Newton-Raphson method, x can be determined numerically as follows, From Equations (1) and (2),

$$q_1 = x - R_2 cos\alpha - R_1 cos\beta \quad (11)$$

$$q_2 = R_2 sin\alpha - R_1 sin\beta \quad (12)$$

Equations (11) and (12) can be solved numerically using Newton-Raphson method as follows,

$$\begin{Bmatrix} x \\ \alpha \end{Bmatrix} = \begin{Bmatrix} x_0 \\ \alpha_0 \end{Bmatrix} - \begin{bmatrix} \frac{dq_1}{dx} & \frac{dq_1}{d\alpha} \\ \frac{dq_2}{dx} & \frac{dq_2}{d\alpha} \end{bmatrix} \begin{bmatrix} q_{01} \\ q_{02} \end{bmatrix} \quad (13)$$

Where

$$\frac{dq_1}{dx} = 1, \quad \frac{dq_1}{d\alpha} = R_2 sin\alpha, \quad \frac{dq_2}{dx} = 0, \quad \frac{dq_2}{d\alpha} = R_2 cos\alpha \quad \text{and} \quad J = \begin{bmatrix} 1 & R_2 sin\alpha \\ 0 & R_2 cos\alpha \end{bmatrix} \quad (14)$$

$$S = \begin{Bmatrix} x \\ \alpha \end{Bmatrix} - J \backslash \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, x = S(1), \alpha = S(2) \quad (15)$$

Dynamics of system of particles

For each particle, two equations of motion can be written as follows,

$$\frac{d}{dt}\left(m\dot{x}\right) = F_{xe} + F_{xi}$$

(16)

$$\frac{d}{dt}\left(m\dot{y}\right) = F_{ye} + F_{yi} \quad (17)$$

where F_{xe} and F_{ye} denote the projections of the resultant external force acting on any particle and F_{xi} with F_{yi} denote the projections of the resultant internal force acting on that particle. Taking the summation of these equations of motions for all the particles of the system, we obtain,

$$\frac{d}{dt}\Sigma\left(m\dot{x}\right) = \Sigma F_{xe} \quad (18)$$

$$\frac{d}{dt}\Sigma\left(m\dot{y}\right) = \Sigma F_{ye} \quad (19)$$

The summation of internal forces vanishes in each case since these forces always appear as pairs of balanced collinear forces which cancel each other. From Figure 1,

$$F_x = -\Sigma F_{xe} \quad (20)$$

$$F_y = -\Sigma F_{ye} \quad (21)$$

$$M_z = \Sigma\left(F_y x - F_x y\right) \quad (22)$$

Connecting-rod mass can be approximated as an “equivalent” System as follows,

$$m_r = m_B + m_C \quad , \quad m_B(R_2 - r_2) = m_C\, r_2 \quad , \quad I_r = m_B(R_2 - r_2)^2 + m_C\, r_2^2 \quad (23)$$

$$m_B = m_r \frac{r_2}{R_2} \quad \text{and} \quad m_C = m_r\left(1 - \frac{r_2}{R_2}\right), \quad (24)$$

$$I_r = m_r (R_2 - r_2)\, r_2 \quad (25)$$

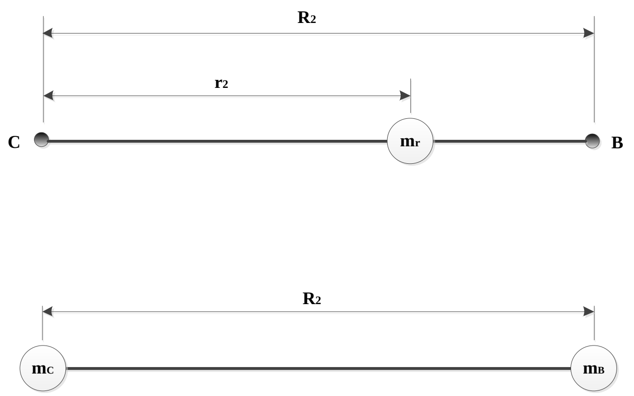


Figure 2. Shows the equivalent masses for connecting rod

From Equation (20),

$$\Sigma F_{xe} = -F_x = \frac{d}{dt}\left(-m_k r_1 \dot{\beta} sin\beta - m_B L_1 \dot{\beta} sin\beta + m_C \dot{x}\right)$$

(26)

Differentiating eqn. (26) successively to obtain the summation of forces in *x*-direction gives

$$-F_x = -m_k r_1 \ddot{\beta} sin\beta - m_k r_1 \dot{\beta}^2 cos\beta - m_B R_1 \ddot{\beta} sin\beta - m_B R_1 \dot{\beta}^2 cos\beta + m_C \ddot{x}$$

$$F_{x_N} = \left(m_k r_1 + m_B R_1\right)\left(\ddot{\beta} sin\beta + \dot{\beta}^2 cos\beta\right) - m_C \ddot{x} \quad (27)$$

From Equation (21),

$$\Sigma F_{ye} = -F_y = \frac{d}{dt}\left(-m_k r_1 \dot{\beta} cos\theta + m_B R_1 \dot{\beta} cos\beta\right) \quad (28)$$

Differentiating Equation (28) successively to obtain the summation of forces in *y*-direction gives

$$-F_y = -m_k r_1 \ddot{\beta} cos\beta - m_k r_1 \dot{\beta}^2 sin\beta + m_B R_1 \ddot{\beta} cos\beta - m_B R_1 \dot{\beta}^2 sin\beta$$

Or

$$F_{y_N} = \left(m_k r_1 + m_B R_1\right)\left(\dot{\beta}^2 sin\beta - \ddot{\beta} cos\beta\right) \quad (29)$$

$$M_{z_N} = F_x R_1 \sin\left(\beta\right) - F_y R_1 \cos\left(\beta\right) \quad (30)$$

Gas Pressure

In this research, the effect of gas forces on inertia moment with no assumption of friction forces act on the for simplification. The relationship between the pressure and the gas force can be described from reference [10] in Equation (31) as:

$$P = 1.4 e^{2.603\beta} - 4.9 e^{0.4} \quad (31)$$

Where $F_P = P\left(\frac{\pi}{4} d^2\right)$ (32)

$$F_{x_{NP}} = F_P + \left(m_k r_1 + m_B R_1\right)\left(\ddot{\beta} sin\beta + \dot{\beta}^2 cos\beta\right) - m_C \ddot{x} \quad (33)$$

$$F_{y_{NP}} = \left(m_k r_1 + m_B R_1\right)\left(\dot{\beta}^2 sin\beta - \ddot{\beta} cos\beta\right) \quad (34)$$

$$M_{z_{NP}} = F_{xNP} R_1 \sin\left(\beta\right) - F_{yNP} R_1 \cos\left(\beta\right) \quad (35)$$

Using the binomial theorem, we obtain,

$$\begin{aligned} F_{x_B} &= \left(m_k r_1 + m_B R_1\right)\left(\ddot{\beta} sin\beta + \dot{\beta}^2 cos\beta\right) + m_C R_1 \dot{\beta}^2 \left(cos\beta + A_2 cos2\beta - A_4 cos4\beta + A_6 cos6\beta - \ldots\right) \\ &+ m_C R_1 \ddot{\theta}\left(sin\beta + \frac{1}{2} A_2 sin2\beta - \frac{1}{4} A_4 sin4\beta + \frac{1}{6} A_6 sin6\beta - \ldots\right) \end{aligned} \quad (36)$$

Assuming uniform angular velocity ($\dot{\beta} = constant$), the terms with the angular acceleration, $\ddot{\beta}$, disappear from the equation, then,

$$F_{x_B} = \left(m_k r_1 + m_B R_1\right)\dot{\beta}^2 cos\beta + m_C R_1 cos\beta + m_C R_1 \dot{\beta}^2 \left(A_2 cos2\beta - A_4 cos4\beta + A_6 cos6\beta - ...\right) \quad (37)$$

Since $m_r = m_B + m_C$, then,

$$F_{x_B} = \dot{\beta}^2 \left[\left(m_k r_1 + m_r R_1\right) cos\beta + m_r \left(1 - \frac{r_2}{R_2}\right) R_1 \left(A_2 cos2\beta - A_4 cos4\beta + A_6 cos6\beta - \ldots\right) \right] \quad (38)$$

$$F_{y_B} = \dot{\beta}^2 \left(m_k r_1 + m_r \frac{r_2}{R_2} L_1 \right) sin\beta \quad (39)$$

For moment with respect to the z-axis

$$M_{z_B} = F_{xB} R_1 \sin\left(\beta\right) - F_{yB} R_1 \cos\left(\beta\right) \quad (40)$$

Using the gas forces effect the Equations (38), (39), and (40) will be modify as:

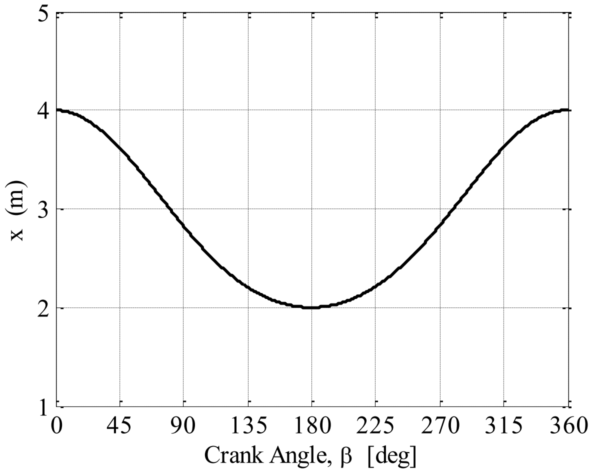
$$F_{x_P} = F_P + \dot{\beta}^2 \left[\left(m_k r_1 + m_r R_1\right) cos\beta + m_r \left(1 - \frac{r_2}{R_2}\right) R_1 \left(A_2 cos2\beta - A_4 cos4\beta + A_6 cos6\beta - \ldots\right) \right] \quad (41)$$

$$F_{y_P} = \dot{\beta}^2 \left(m_k r_1 + m_r \frac{r_2}{R_2} L_1 \right) sin\beta \; \ldots \text{ No effect to gas pressure on this force} \quad (42)$$

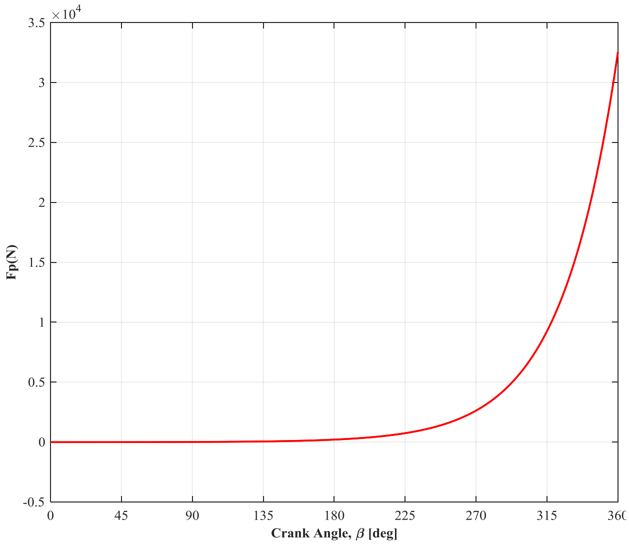
$$M_{z_P} = F_{x_P} R_1 \sin\left(\beta\right) - F_{y_P} R_1 \cos\left(\beta\right) \quad (43)$$

RESULTS AND DISCUSSION

The results of this research are done with the help of the MATLAB program. The optional parameters that be used in the calculation are: $R_1 = 60mm, R_2 = 220mm, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 10kg, m_r = 1kg, m_p = 0.6\,kg.$ The results in all figures are conducted in one revelation. In order to evaluate the forgoing analysis, Equations (11) and (12) were used to calculate the instantaneous displacement of the piston $\left(x\right)$ versus the angular position of the crank with the use of the Newton-Raphson method as shown in Figure 3 (a). It may be seen that the largest instantaneous displacement of the piston depends on the length of the crank and the length of the connecting rod. Figure 3 (b) shows the relation between the piston force and the angle of the crank shaft, it illustrates that the force increases exponentially to reach a maximum value at 360^o w.r.t Equation (32).



(a)



(b)

Figure 3: Crank angle with displacement and with piston force (a) Instantaneous displacement of the piston x , (b) piston force with crank angle

Figure 4 shows the angular displacement of the connecting rod. Equation (11) was used to calculate the angular displacement of the connecting rod versus the angular position of the crank with the use of the Newton-Raphson method. The largest angular displacement of the connecting rod also depends on the length of the crank and the length of the connecting rod.

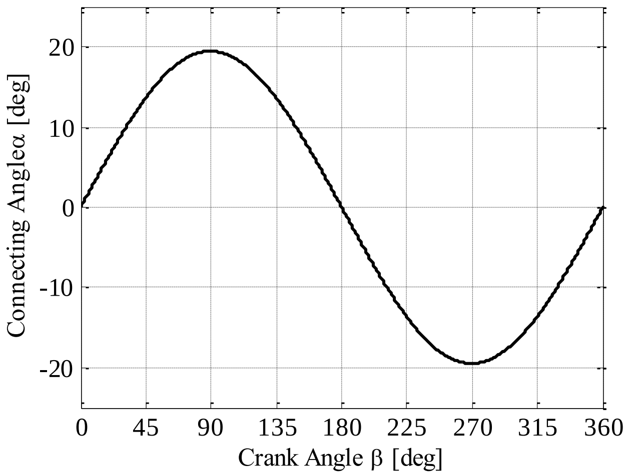


Figure 4: Angular displacement of the connecting rod (α)

$$R_1 = 1\,m, R_2 = 3\,m, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 1kg, m_r = 1kg, m_p = 1\,kg.$$

The result of the computed magnitude of the moment exerted on the crank without pressure is displayed in Figure 5. The plotted results in one revolution of a single-piston engine. The solid red line represents the Newton-Raphson method to Equation (30). The dotted black line represents the Binomial theorem based on Equation (40) as shown in the figure. From the figure, the maximum value of the moment with the Newton-Raphson method is higher than the maximum value of the moment of the Binomial theorem. The maximum instantaneous resultants of the moments for both thermoses happen at crank angles $\beta = 67.5^{\circ},\ 202.5^{\circ}\ and\ 375.5^{\circ}$. The improvement of the moment values is 5% when using the Newton -Raphson method instead Binomial method.

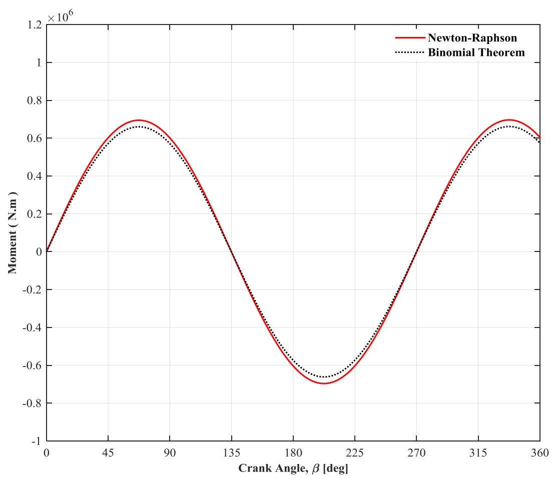


Figure 5. The moment without the gas pressure on the crank for a single cylinder engine (M_z)

For $R_1 = 60\,mm, R_2 = 220\,mm, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 10\,kg, m_r = 1kg, m_p = 0.6\,kg.$

Figure 6 displays the moment with the gas pressure on the crank for a single-cylinder engine (M_z). The solid red line represents the Newton-Raphson method with respect to Equation (35). The dotted black line represents the Binomial theorem based on Equation (43) as shown in the figure. The maximum values for both theorems occur in the position of the crank angle $\beta = 360^{\circ}$. The improvement of the moment values with pressure effect is 2.9% when using the Newton-Raphson method instead Binomial method. The response behavior shape is not sinusoidal because of the effect of gas pressure on the system; however, it is a periodic one. The gas pressure in one cylinder produces a force on the piston that does not need to be balanced. The gas pressure inside the cylinder produces a large moment. The instantaneous moment produced by the engine is the sum of the moment due to the gas pressure and the moment due to the inertia forces. However, the moment exerts on the crank, without gas pressure, depends only on the one that comes from the inertia moment due to the inertia forces.

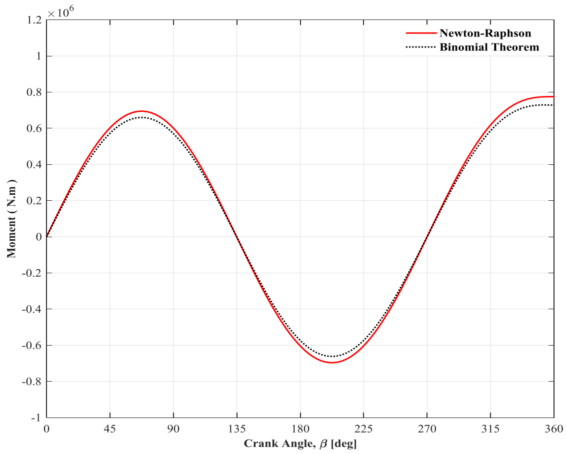


Figure 6: The moment with the gas pressure on the crank for a single cylinder engine $\left(M_z\right)$

For $R_1 = 60\,mm, R_2 = 220\,mm, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 10\,kg, m_r = 1\,kg, m_p = 0.6\,kg.$

Figures 7 and 8 show a comparison between the results of the Binomial theorem and the Newton-Raphson method w.r.t resultants of external force in the x- direction. Figure 7 shows the resultant of all external forces in the *x*-direction without pressure effect. The instantaneous resultants of all external forces in the x-direction for the Newton-Raphson method (solid-red line) based on Equation (27), while the instantaneous resultants of all external forces in the x-direction for the Binomial theorem (dotted light black line) with respect to Equation (38). The maximum values for both theorems occur in two positions of the crank angle at $\beta = 0^{\circ}\, and\, 270^{\circ}$ in the figure. In Figure 8 The maximum values for both theorems occur in three positions of crank angle $\beta = 0^{\circ}, 270^{\circ}\, and\, 360^{\circ}$ in the figure. The instantaneous resultants of all external forces in x-direction for the Newton-Raphson method (solid-red line) based on Equation (33) while the instantaneous resultants of all external forces in x-direction for the Binomial theorem (dotted light black line) with respect to Equation (41) in the figure. It is noticed that the maximum value of the oscillating force for the Newton-Raphson method is a little higher than the maximum value of the instantaneous resultants of all external forces in x-direction. The improvement of all external forces in x-direction values without pressure is 10% and 9.8% with pressure when using Newton-Raphson method instead Binomial method. Thus, the Binomial theorem is approximately method since the λ^2 term is neglected because of its small value as in Equation (7) while the Newton-Raphson method does not neglect any term. For this reason, the researcher needs to consider taking the Newton-Raphson method instead of the Binominal method for its accuracy.

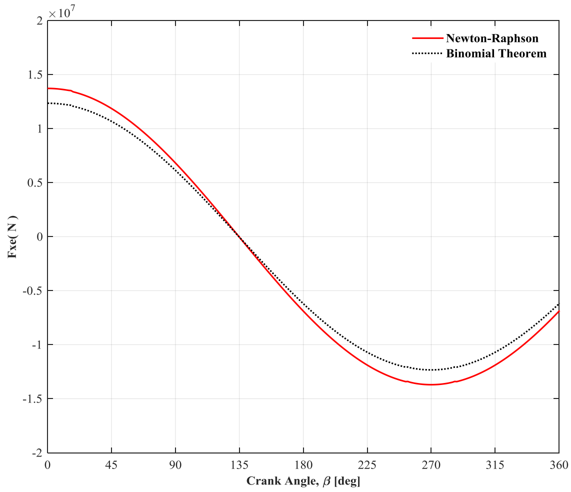


Figure 7: The resultant of all external forces in *x*-direction without pressure

For $R_1 = 60\,mm, R_2 = 220\,mm, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 10\,kg, m_r = 1\,kg, m_p = 0.6\,kg.$

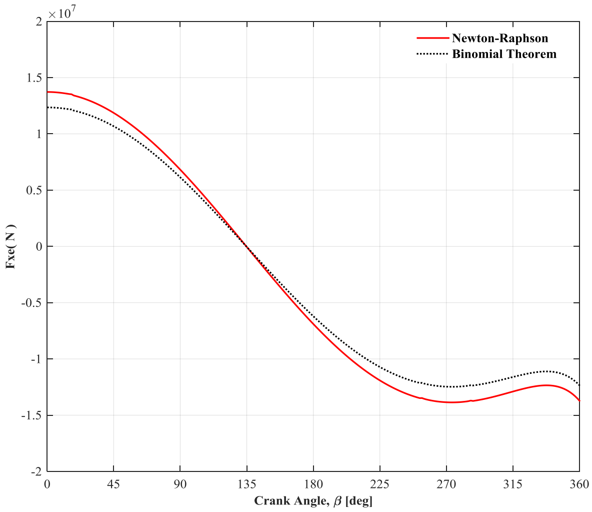


Figure 8: The resultant of all external forces in x-direction with pressure

For $R_1 = 60\,mm, R_2 = 220\,mm, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 10\,kg, m_r = 1\,kg, m_p = 0.6\,kg.$

Figure 9 shows the results for the Binomial theorem and the Newton-Raphson method in the y-direction. However, the instantaneous resultants of all external forces in y-direction are the opposite of a sinusoidal for the instantaneous resultants of all external forces in the x-direction. The instantaneous resultants of all external forces in the *y*-direction for the Newton-Raphson method to Equation (29) is larger than the instantaneous resultants of all external forces in the y-direction for Binomial theorem based on Equation (39). Therefore, the Newton-Raphson method is more accurate than the Binomial theorem. The designers need to consider taking the values that are more accurate to the Newton-Raphson method. It is also important to note that the largest resultant of all external forces in the *y*-direction is happens at $\beta = 135^{\circ}$ for both theorems. It is also seen that the instantaneous resultants of all external forces in the x-direction are larger than the instantaneous resultants of all external forces in the y-direction. The improvement of all external forces in the y-direction values is 9.98% without pressure when using Newton -Raphson method instead Binomial method. Under gas pressure, there is no effect to resultants of all external forces in the y-direction.

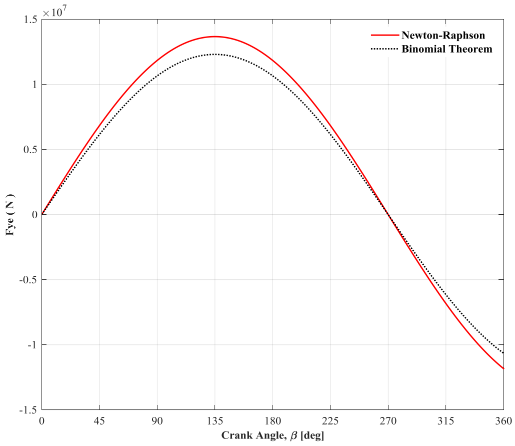


Figure 9: The resultant of all external forces in *y*-direction without pressure

For $R_1 = 60\,mm, R_2 = 220\,mm, r_1 = \frac{R_1}{2}, r_2 = \frac{R_2}{2}, m_k = 10\,kg, m_r = 1\,kg, m_p = 0.6\,kg.$

CONCLUSION AND RECOMMENDATION

The following conclusions are supported by the analysis, results, and discussion of this paper:

1. The gas pressure in one cylinder produces a force on the piston that does not need to be balanced. The gas pressure inside the cylinder produces a large moment. The instantaneous moment produced by the engine is the sum of the moments due to the gas pressure and the moment due to the inertia forces.
2. That the inertia moment exerts on the crank without effect the gas pressure depends on only the inertia moment due to the inertia forces.
3. The improvement of the moment values without pressure is 5% and the improvement of the moment values with pressure effect is 2.9% when using Newton -Raphson method instead of Binomial method.
4. The maximum instantaneous resultants of the moments for both thermoses happen at crank angles $\beta = 67.5^{\circ}$, 202.5° *and* 375.5° in case no pressure effect. With the pressure effect the maximum instantaneous resultants of the moments for both thermoses located at a crank angle $\beta = 360^{\circ}$.
5. The maximum values of instantaneous resultants of all external forces in x-direction in case no pressure for both theorems occur in positions of the crank angle $\beta = 0^{\circ}$ *and* 270° while in case of insert pressure $\beta = 0^{\circ}, 270^{\circ}$ *and* 360°.
6. The improvement of all external forces in x-direction values is 10% without pressure and 9.8% with pressure effect when using Newton -Raphson method instead Binomial method.
7. The largest resultant of all external forces in the *y*-direction is happens at $\beta = 135^{\circ}$ for both theorems.
8. The improvement of all external forces in the y-direction values is 9.98% without pressure when using the Newton -Raphson method instead Binomial method. There is no action to pressure force in the y-direction.
9. Similar behavior is generated from the two theorems of the resultant of all external forces in *x*-direction, y-direction, and the moment exerted on the crank.
10. It is safe to consider that the Newton-Raphson method is more accurate than the Binomial theorem.

This work is done considering modeling the inline reciprocating engine. The work can be expanded for future work to include the procedure of V-engine and Boxer engine designs in the internal combustion engine.

NOMENCLATURE

$\dot{x}$ instantaneous velocity of the system in the *x*-direction

$\dot{y}$	instantaneous velocity of the system in the *y*-direction
F_x	the resultant of all external forces in the *x*-direction
F_y	the resultant of all external forces in the *y*-direction
M_z	moment on the crank from a single cylinder
P	gas pressure acting on the piston
F_P	pressure force acting on the piston
d	diameter of the piston
I_k	mass moment-of-inertia of the crank about its mass center
I_r	mass moment-of-inertia of the connecting rod about its mass center
R_1	length of the crank
R_2	length of the connecting rod
m_c	mass of the crank
m_r	mass of the connecting rod
m_p	mass of the piston
r_1	location of the mass center for the crank
r_2	location of the mass center for the connecting rod
x	instantaneous displacement of the piston
β	angular displacement of the crank
α	angular displacement of the connecting rod
$\dot{\beta}$	angular velocity of the crank

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