Natural Convection Heat Transfer In An Inclind Elliptic Enclosure With Circular Heat Source

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ABSTRACT: The present paper deals with a numerical study of natural convection heat transfer in an inclined annular enclosure filled with Al₂O₃/water hybrid nanofluid. The cold outer elliptic surface of enclosure is kept at constant temperature T_c and the hot inner circular cylinder wall is kept at constant temperature T_h. The stream function-vorticity method is used to resolve the governing equations. The prevailing equations are discretized utilizing the way of finite volume and resolved via code of FORTRAN. Validation was completed by comparison the present work with previous results and found to be in excellent agreement. The range of Rayleigh number and volume fraction were 10⁴ ≤ Ra ≤ 10⁶ and 0 ≤ φ ≤ 0.1. The angles of inclination were φ= 0°, 45°, and 90°. Results were presented in terms of streamlines, isotherms, and average Nusselt numbers. The angular distribution of local Nusselt number for the inner and outer cylinders depends on Rayleigh number, angles of inclination, and nanoparticles volume fraction.

KEYWORDS: natural convection, heat, elliptic, enclosure, cavity.

INTRODUCTION

Nanofluids are colloidal suspensions of fine nanomaterials with size range of 1-100 nm in carrier fluids and have higher thermal conductivity than the base fluid [1]. Using nanofluids in practical applications is to enhance the heat transfer rates in these equipment’s such as thermal storage systems, cooling of electronic devices, solar collectors, and heat exchangers. Different nanofluids were used in enclosures and cavities to improve the heat transfer process such as Al₂O₃, Ag, Au, AgO, Cu, CuO, and TiO₂ [2-17].

Khalil et al. (2003), [2] studied the natural convection heat transfer in an enclosure filled with nanofluids for different pertinent parameters. They deduced a correlation of the average Nusselt number for different Grashof numbers and volume fractions. Hakan and Eiyad (2008), [3] used different types of nanoparticles in a partially hot enclosure to conclude that the enhancement of heat transfer process using nanofluids, is more pronounced at low aspect ratio than at high aspect ratio. Eiyad et al. (2008), [4] studied the heat transfer process by natural convection in horizontal annuli using Cu, Ag, Al₂O₃ and TiO₂ nanoparticles based on water. They concluded that using of the various kinds and different volume fractions of nanoparticles gives adverse effects on heat transfer characteristics. Elif (2009), [5] used water based-Cu, Ag, CuO, Al₂O₃, and TiO₂ nanofluids to search free convection heat transfer in an inclined square enclosure. He concluded that the average Nusselt number rises significantly as particle volume fraction and Rayleigh number rise. Eiyad and Hakan (2009), [6] used Cu-nanofluid to investigate the effects of inclination angle on the characteristics of thermal and fluid fields in a two-dimensional enclosure. It was concluded that inclination angle can be a control parameter for nanofluid filled enclosure. Sheikholeslami et al. (2012), [7] used Cu–water nanofluid and magnetic field inside a circular enclosure containing a hot inner sinusoidal circular cylinder. They found that the heat transfer rate increases with increase of nanoparticle volume fraction, the number of undulations. They (2014), [8] performed another study for the magnetohydrodynamic flow in a concentric annulus filled with Cu–water nanofluid. They showed that increase of enhancement ratio depends on decrease of Rayleigh number augment of Hartmann number. Sivasankaran and Pan (2014), [9] studied the influence of amplitude and phase deviance of sinusoidal...
temperature spreading on the heat transfer procedure of nanofluids in a square cavity. It is found that the heat transfer rate enhances with increase of the amplitude ratio and volume fraction of nanoparticles. Mansour and Bakier (2015), [10] used Cu–water nanofluid to study the natural convection inside enclosure subjected to changeable thermal boundary conditions and inclined magnetic field. They found that the convection heat transfer is dominated inclination as the angle of inclination increases and the magnetic force pointed to horizontal trend. Ravnik and Škerget (2015), [11] used Al$_2$O$_3$; Cu and TiO$_2$ nanofluids with pure water and air for validation purposes inside a hot circular and elliptical cylinder placed in a cooled cubic enclosure. They concluded that the conduction dominated flow regime gives highest heat transfer enhancement, and the convection dominated flow regime gives a smaller increase in heat transfer efficiency. Tahar and Ali (2017), [12] used Al$_2$O$_3$/water nanofluid and Cu-Al$_2$O$_3$/water hybrid nanofluid based on water to study the thermal and flow characteristics by natural convection in an enclosure. It was shown that the use of Cu-Al$_2$O$_3$/water hybrid nanofluid gives better thermal and dynamic performance compared to Al$_2$O$_3$/water nanofluid. Yang Hu et al. (2017), [13] studied natural convection in an eccentric annular enclosure filled with a Cu–water nanofluid. It is concluded that the addition of the nanoparticles into pure fluid enhances the flow pattern. Mebarek (2018), [14] studied the effect of different base fluids including ethylene glycol, engine oil, and water with Titanium nanofluids on the heat transfer process in a cylindrical annulus. He found that the thermal efficiency depends on the Rayleigh number and the volume fraction of the nanoparticles and the average heat transfer process depends on the type of the base fluid. Ishrat et al. (2018), [15] investigated the magnetohydrodynamics (MHD) conjugate natural convection flow inside a rectangular enclosure filled with CO-H$_2$O nanofluid. They found that the rate of heat transfer rises as Rayleigh number rises and drops with a rise of the Hartmann number. Tayebia et al. (2019), [16] studied the natural convection heat transfer in a confocal elliptic annulus filled with CNT-water nanofluid with hot inner cylinder and cold outer cylinder. They showed that the average Nusselt number increases as the volume fraction of the nanoparticles increases particularly at high Rayleigh numbers. Suhail and Altamush (2020), [17] studied natural convective heat transfer of H$_2$O-Al$_2$O$_3$ nanofluid inside a partially heated vertical annulus with high aspect ratio. For all models of nanofluids, Rayleigh number decreases with nanoparticle concentration.

The present work comprises a numerical study of the hydrodynamic and thermal characteristics resulted from natural convection heat transfer in an inclined elliptic annular enclosure filled with Cu-Al$_2$O$_3$/water hybrid nanofluid. The cold outer surface of enclosure is kept at constant temperature $T_C$ and the hot inner cylinder wall is kept at constant temperature $T_h$. The range of Rayleigh number and volume fraction were $10^4 \leq Ra \leq 10^6$ and $0 \leq \varphi \leq 0.1$. The angles of inclination were $\phi = 0^\circ$, $45^\circ$, and $90^\circ$. The stream function–vorticity method is used to solve the governing equations. The prevailing equations are discretized by utilizing the way of finite volume and resolved via FORTRAN code.

MATHEMATICAL MODEL

Consider an inclined annular enclosure with hot inner circular cylinder and cold outer elliptic cylinder, as shown in Figure 1-a. The enclosure is tilted at $\phi$ from the vertical. The outer cylinder is maintained at a constant lower temperature ($T_o$), whereas the inner cylinder is maintained at a constant higher temperature ($T_h$). The angles of inclination are $0^\circ$, $45^\circ$, and $90^\circ$, respectively. The calculated formulating for the combined convection in an inclined arc-shape bore was projected by [18]. The mathematical model of this work was utilized and prolonged to study characteristics of transfer of heat and the form of flow in an elliptic enclosure with circular heat source, and is briefly reviewed below.
Figure 1. A representation view and geometry explanation of the physical field.

Figure 1 shows a schematic diagram of elliptic annulus enclosure with inner circular cylinder. The fluid in the attachment is a water-based nano-fluid comprising with various volume fraction of nanoparticles. The features of thermo-physical of the nano-fluid are specified as exposed in Table 1 [18]:

<table>
<thead>
<tr>
<th>Material</th>
<th>( C_p ) (( \frac{J}{kg.K} ))</th>
<th>( \rho ) (( \frac{kg}{m^3} ))</th>
<th>( k ) (( \frac{W}{m.K} ))</th>
<th>( \beta ) (( \frac{1}{K} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>4179</td>
<td>997.1</td>
<td>0.613</td>
<td>21\text{\times}10^{-5}</td>
</tr>
<tr>
<td>( Al_2O_2 )</td>
<td>765</td>
<td>3970</td>
<td>40</td>
<td>0.85\text{\times}10^{-5}</td>
</tr>
</tbody>
</table>

In this regard, the density \( \rho_{nf} \) , specific heat energy \( (\rho C_p)_{nf} \), and thermal expansion coefficient \( (\rho \beta)_{nf} \) for the nano-fluid are valued as [18]:

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi \rho_s \tag{1}
\]

\[
\beta_{nf} = (1 - \phi)\beta_f + \phi \beta_s \tag{2}
\]

\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{3}
\]

Model of Maxwell-Garnets was measured to compute the nano-fluid thermal conductivity [18].

\[
k_{nf} = k_f \frac{(k_f + 2k_s) - 2\phi(k_f - k_s)}{(k_f + 2k_s) + \phi(k_f - k_s)} \tag{4}
\]

The nano-fluid dynamic viscosity signified as model of Brinkman [18] as shows:

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \tag{5}
\]

The viscous incompressible two-dimensional laminar air flow in the cavity is governed by continuity, momentum, and energy equations. The fluid properties are assumed to be constant except the density variation in the buoyant force according to Boussinesq approximation. The way of stream function–vorticity is utilized to resolve the prevailing calculations and the coordinate’s conversion, as exposed in Figure 1b, it is made for plotting the wavy shape in a rectangular computational field. The prevailing calculations are given in Equations (1–3) by utilizing dimensionless temperature (\( \Theta \)), stream function (\( \Psi \)), and dimensionless vorticity (\( \Omega \)), which are founded on the body-fitted curvilinear coordinate (\( \xi, \eta \)). These terms include together buoyant force and the inertial [19, 20]
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\[
c_{1} \frac{\partial^{2} \psi}{\partial \eta^{2}} + 2c_{2} \frac{\partial^{2} \psi}{\partial \xi \partial \eta} + c_{3} \frac{\partial^{2} \psi}{\partial \xi^{2}} + c_{4} \frac{\partial \psi}{\partial \eta} + c_{5} \frac{\partial \psi}{\partial \xi} = J\Omega
\]  
(6)

\[
\frac{\partial \alpha \psi}{\partial \xi} \frac{\partial \alpha \psi}{\partial \eta} = Pr \frac{\eta}{f} \left( c_{1} \frac{\partial^{2} \psi}{\partial \eta^{2}} + 2c_{2} \frac{\partial^{2} \psi}{\partial \xi \partial \eta} + c_{3} \frac{\partial^{2} \psi}{\partial \xi^{2}} + c_{4} \frac{\partial \psi}{\partial \eta} + c_{5} \frac{\partial \psi}{\partial \xi} \right) - Pr \frac{\eta}{f} Pr Ra \sin \phi \left( \frac{\partial \psi}{\partial \eta} - \frac{\partial \psi}{\partial \xi} \right) - 
\cos \phi \left( \frac{\partial \psi}{\partial \eta} - \frac{\partial \psi}{\partial \xi} \right)
\]  
(7)

\[
\frac{\partial \alpha \psi}{\partial \xi} \frac{\partial \alpha \psi}{\partial \eta} = \frac{k_{nf}}{f} \left( \frac{\partial \psi}{\partial \eta} + 2c_{2} \frac{\partial \psi}{\partial \xi \partial \eta} + c_{3} \frac{\partial^{2} \psi}{\partial \xi^{2}} + c_{4} \frac{\partial \psi}{\partial \eta} + c_{5} \frac{\partial \psi}{\partial \xi} \right)
\]  
(8)

We can express the equations (6–8) in terms of the Jacobian of the conversion of coordinate to the curvilinear coordinates \((\xi, \eta)\) from the rectangular coordinates \((X,Y)\) as shown in Eq. (9); and is stated in Eq. (10). Here, 6.2 is the value of number of prandtl for water.

\[
\Omega = \frac{\omega f^{2}}{a f}; \quad \psi = \frac{\psi}{a f}; \quad \theta = \frac{(T - T_{c})}{(T_{h} - T_{c})}
\]  
(9)

\[
J = \frac{\partial \psi}{\partial \xi} \frac{\partial \psi}{\partial \eta} - \frac{\partial \psi}{\partial \xi} \frac{\partial \psi}{\partial \eta}
\]  
(10)

\[
c_{1} = \frac{1}{f} \left[ \left( \frac{\partial \psi}{\partial \eta} \right)^{2} + \left( \frac{\partial \psi}{\partial \xi} \right)^{2} \right]; \quad c_{2} = -\frac{1}{f} \left( \frac{\partial \psi}{\partial \xi} \frac{\partial \psi}{\partial \eta} + \frac{\partial \psi}{\partial \eta} \frac{\partial \psi}{\partial \xi} \right); \quad c_{3} = \frac{1}{f} \left( \left( \frac{\partial \psi}{\partial \xi} \right)^{2} + \left( \frac{\partial \psi}{\partial \eta} \right)^{2} \right)
\]  
(11)

\[
c_{4} = \frac{1}{f} \left[ \left( \frac{\partial \phi}{\partial \eta} \right) \left( \frac{\partial \phi}{\partial \xi} \right) + \left( \frac{\partial \phi}{\partial \xi} \right) \left( \frac{\partial \phi}{\partial \eta} \right) \right] - \frac{1}{f} \left[ \frac{\partial \phi}{\partial \xi} \left( \frac{\partial \phi}{\partial \eta} \right) + \frac{\partial \phi}{\partial \eta} \left( \frac{\partial \phi}{\partial \xi} \right) \right]
\]  
(12)

\[
c_{5} = \frac{1}{f} \left[ \frac{\partial \phi}{\partial \eta} \left( \frac{\partial \phi}{\partial \xi} \right) - \frac{\partial \phi}{\partial \xi} \left( \frac{\partial \phi}{\partial \eta} \right) \right]
\]  
(13)

\[
X = \frac{x}{L}; \quad Y = \frac{y}{L}
\]  
(14)

\[
Ra = \frac{\theta_{f} (T_{h} - T_{c}) L^{3}}{\theta_{f, nt} f^{2}} \frac{\nu f^{2}}{\alpha f}
\]  
(15)

The standardized vorticity and stream function are associated to the velocities without dimension, as stated in Eqs. (16) and (17):

\[
\Omega = \frac{1}{f} \left[ \left( \frac{\partial \psi}{\partial \xi} \frac{\partial \psi}{\partial \eta} + \frac{\partial \psi}{\partial \eta} \frac{\partial \psi}{\partial \xi} \right) + \frac{\partial \psi}{\partial \xi} \frac{\partial \psi}{\partial \eta} \right]
\]  
(16)

\[
U = \frac{1}{f} \left( - \frac{\partial \psi}{\partial \xi} + \frac{\partial \psi}{\partial \eta} \right); \quad V = \frac{1}{f} \left( - \frac{\partial \psi}{\partial \xi} - \frac{\partial \psi}{\partial \eta} \right)
\]  
(17)

Boundary Conditions

The boundary conditions on the cold moving lid and the hot wall are given in Eqs. (18) and (19), respectively:

\[
U = 0; \quad V = 0; \quad \psi = 0; \quad \theta = 0; \quad \Omega = -\frac{1}{f} \left( \frac{\partial \psi}{\partial \eta} \right)
\]  
(18)
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\[ U = 0; \quad V = 0; \quad \Psi = 0; \quad \theta = 1; \quad \Omega = -\frac{1}{f} \left( \frac{\partial V}{\partial \eta} \frac{\partial \xi}{\partial \alpha} + \frac{\partial V}{\partial \alpha} \frac{\partial \xi}{\partial \eta} \right) \]  

(19)

Generation of Grid

In this research, an analytical appearance could derive for the conversion of coordinate to the computational from the physical field; all-geometrical factors could calculate accurately. The great elliptic function could be shown as [20]:

\[ \left( \frac{x}{a} \right)^{2n} + \left( \frac{y}{b} \right)^{2n} = 1 \]  

(20)

Where \( a \) and \( b \) are the elliptic lengths in the \( x \) and \( y \) path respectively, \( n \) is a positive integer.

The conversion of coordinate for the current problematic can be accurately arrangement, which is shown as:

\[ x = -\sin \sin \xi \cdot [r_i + (r_o - r_i)\eta] \]  

(21)

\[ y = \cos \cos \xi \cdot [r_i + (r_o - r_i)\eta] \]  

(22)

Where \( r_i \) is the radius of the inner circular cylinder, and \( r_o \) is the equivalent radius of the outer elliptic cylinder and it is computed as in reference one:

\[ r_o = \frac{b}{\cos \cos (\xi) \times \sin \sin (\xi)^{2n+1}/2n} \]  

(23)

The transformed computational domain to (\( \square \),\( \square \)) plane is \( 0 \leq \eta \leq 1 \) and \( 0 \leq \xi \leq 2\pi \). A typical generated grid is shown in Figure (1-b), for 41 \( \square \)41 nodes.

The enforced boundary settings aren’t slipup and isothermal on both internal circular cylinder and external square attachment surfaces. Thus, the boundary conditions could be definite as:

\[ U|_{\eta=0,1} = 0, \quad V|_{\eta=0,1} = 0 \]  

(24a)

\[ \psi|_{x=0} = 0, \quad \psi|_{x=1} = 0 \]  

(24b)

\[ \theta|_{x=0} = 1, \quad \theta|_{x=1} = 0 \]  

(24c)

\[ \Omega|_{\eta=0,1} = c \frac{\partial^2 \psi}{\partial \eta^2} \big|_{\eta=0,1} = c \frac{\partial U}{\partial \eta} \big|_{\eta=0,1} \]  

(24d)

Solution Procedure

The equations (6-8), and the boundary conditions (18 and 19), are discretized via the way of finite-volume. The calculating of producing of grid is founded on the system of curvilinear coordinate (\( \xi, \eta \)). The functions of conversion of coordinate (i.e., \( \xi = \xi(X,Y) \) and \( \eta = \eta(X,Y) \)) were proposed by Thompson et al. [21] and adopted by Chen, et al. [22]. The terms of finite-volume for \( \psi, \theta \) and \( \Omega \) can be conjointly resolved by utilizing the way of iteration. The way of central difference is utilized for the process of discretization. The functions of conversion \( \eta = \eta(X,Y) \) and \( \xi = \xi(X,Y) \) are attained individually via resolving the subsequent two elliptic Poisson calculations, as specified in Eq. (25):

\[ \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = P(X,Y) \]  

(25a)

\[ \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} = Q(X,Y) \]  

(25b)
Where \( Q \) and \( P \) are two random functions definite to regulate the grids local density. The way of succeeding over-relaxation was approved to raise computational accurateness. The approved relaxation elements for energy equations, vorticity, and stream function are 1.0, 0.1, and 1.5 respectively. This resolution method has complete explanation in Cheng and Chen [23].

System Characteristics

The local Nusselt number \( N_u_x \) and the overall Nusselt number \( N_u_{ave} \) are stated in Eqs. (21) and (22):

\[
N_u_x = \frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial n}\bigg|_{moving\ lid}
\]

\[
N_u_{ave} = \frac{1}{s} \int_0^s N_u_x \, ds
\]

Where:

\( k_f \): The thermal conductivity of fluid.

\( n \): The external coordinate usual to the wall.

Code validation

The numerical solution methodology used in the present code was validated by comparing its results for local Nusselt number with the analytical results reported by F.M. Mahfouz [24] as shown in Figure 2 for the problem of natural convection heat transfer in case of \( Mr=2.25, \, Ar_i=0.436, \, Ra=3.72 \times 10^5 \) and \( Pr=0.7 \). It can be seen that the numerical model is in a good agreement with work [24].

\[\text{Figure 2. Local Nusselt number variation along inner and outer walls for Mr=2.25, Ra=3.72 \times 10^5, Pr=0.7, and Ar_i=0.436 and assessment with mathematical outcomes of Mahfouz [24].}\]

RESULTS AND DISCUSSION

Streamlines and isotherms
Numerical investigation using finite volume method and FORTRAN code is applied to study the natural convection transfer of heat in an tending annulus formed by outer elliptic cylinder and inner circular cylinder filled with Al\textsubscript{2}O\textsubscript{3}–water nanofluid. The ratio amid the two main axes is near to 3. The range of Rayleigh number and volume fraction were $10^4 \leq \text{Ra} \leq 10^6$ and $0 \leq \varphi \leq 0.1$ with Prandtl number equals to 6.2. The angles of inclination were $\phi = 0^\circ$, $45^\circ$, and $90^\circ$. Figure 3 shows streamline (left) and isotherm counter (right) at Ra=10\textsuperscript{6}, and three angles of inclination $\phi=0^\circ$, $45^\circ$, $90^\circ$. It shows that the flow field is formed of two circulating eddies, one rotates clockwise on the right side and the other rotates counter clockwise on the left side. These eddies are consisted of circulating of flow down lengthwise the external elliptic cylinder wall and rising lengthwise the internal circular cylinder wall. The two eddies are disconnected via the line of function of zero-stream (line above circular cylinder) which crosses with the external elliptic wall at two points; the top point denotes the point which is stagnation of flow on the wall whereas the bottom point denotes the point of separation of flow from the wall. In horizontal position ($\phi = 0^\circ$), this line vertically distributes the attachment to two alike eddies with no overall circulation of flow($\Psi_{max,min} = 192, -192$). The center of vorticity lies on the horizontal axis of enclosure. While in case of inclined enclosure ($\phi = 45^\circ$), the zero-stream function line is removed far from the wall, representing the increasing of streamline function and circulation of flow ($\Psi_{max,min} = 210, -153$) because of increase the convection currents. The value of wall stream function indicates intensity of circulation which depends on different parameters such as the inclination angle, volume fraction, and Rayleigh number. The global flow circulation depending on controlling parameters, for this particular case, is not generated. In vertical position ($\phi = 90^\circ$), the center of vorticity has creeped towards the upper part of enclosure on each side, and the stream function is weakened($\Psi_{max,min} = 167, -167$). The isotherms contour displays the thermal plume formation at the upper of the internal circular cylinder wall due to the heated rising currents lengthwise the two sides of wall of internal cylinder. The thickness of thermal boundary-layer close to the internal hot wall increases due to raise the grades of temperature and thus indicates higher heat transfer rates. The isotherms become distorted and a plume deviates towards left side as the angle of inclination moves from horizontal($\phi = 0^\circ$), to inclined position($\phi = 45^\circ$), which shows the dominance of convective transfer of heat and rises the buoyancy force and overwhelms the viscous force. The thermal plume is symmetric about the vertical axis for both vertical and horizontal position and becomes asymmetric at inclined position in the path of the buoyancy-driven flow in the direction of the higher portion of the external elliptic wall.

There is no doubt that, the presence of nanoparticles affects the physical behavior of isotherms and streamlines. Figure 4 shows the effect of nanoparticle concentration on streamline (left) and isotherm counter (right) for Ra=10\textsuperscript{6}, and three angles of inclination $\varphi=0^\circ$, $45^\circ$, $90^\circ$. As can be shown from this figure that the flow strength raises as the volume fraction of the nanoparticles rises for all angles of inclination. The dynamic domain is fashioned via a couple of symmetrical vortices at vertical and horizontal positions with clockwise circulation in the right side and anticlockwise circulation in the left side for all volume fraction values. The existence of nanoparticles aids to accumulate of the isotherms close to the hot circular wall, which means enhancement in the rate of transfer of heat. It is concluded that using a nanofluid is more active paralleled to pure water case.

![Graph showing streamline and isotherm counter](image)

$\Psi_{max,min} = 192, -192, \ \phi = 0^\circ$ (horizontal position)
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\[ \Psi_{\text{max, min}} = 210, -153, \phi = 45^\circ \text{ (inclined position)} \]

\[ \Psi_{\text{max, min}} = 167, -167, \phi = 90^\circ \text{ (vertical position)} \]

**Figure 3.** Streamline (left) and isotherm counter (right) for base fluid water at Ra=10^6, and \( \phi = 0^\circ, 45^\circ, 90^\circ \).

\[ \Psi_{\text{max, min}} (\varphi = 0.) = 192, -192, \Psi_{\text{max, min}} (\varphi = 0.05) = 203, -203 \]

\[ \Psi_{\text{max, min}} (\varphi = 0.1) = 206, -206, \phi = 0^\circ \text{ (horizontal position)} \]
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Local Nusselt number

The features of thermal domain close to the walls are estimated via finding local Nusselt number spreading at the walls. The angular spreading of local Nusselt number for the inner and outer cylinders at Ra=10^6 and different angles of inclination and nanoparticles volume fraction is shown in Figure 5 and 6; respectively. It can be observed that the values of local Nusselt number are higher on the outer wall of inner circular cylinder than on the inner surface of outer elliptic cylinder. In the area of impingement of flow on the wall. Such flow is formerly heated through its rising movement lengthwise the two sides of hot internal wall and as it knockouts the cold external wall great negative grade of temperature is formed. Figure 4 shows that the maximum value of local Nusselt number of the outer surface of inner hot circular cylinder occurs at the point of stagnation of hot cylinder (at angular position, \( \gamma = 180^\circ \), bottom of hot cylinder) for \( \phi = 0^\circ \) (horizontal position). The increasing of local Nusselt number can be attributed to smallest resistance to upper gradient of velocity and conduction (and thus heat convection coefficient) nears the walls as well. While the minimum values occur at the top of hot cylinder (angular position \( \phi = 0^\circ \) and 360°, the plume region). The maximum value of local Nusselt number deviates towards the left-hand side of hot cylinder near the horizontal axis as the angle of inclination moves from horizontal to vertical for all values of nanoparticles volume fraction of fluid. It is observed also that, the rise nanoparticles volume fraction lead to raise angular distribution of Nusselt number and

\[ \Psi_{\text{max,min}}(\varphi = 0.) = 210, -153, \Psi_{\text{max,min}}(\varphi = 0.05) = 219, -165 \]

\[ \Psi_{\text{max,min}}(\varphi = 0.1) = 222, -170, \phi = 45^\circ \text{ (inclined position)} \]

\[ \Psi_{\text{max,min}}(\varphi = 0.) = 167, -167, \Psi_{\text{max,min}}(\varphi = 0.05) = 177, -176 \]

\[ \Psi_{\text{max,min}}(\varphi = 0.1) = 179, -178, \phi = 90^\circ \text{ (vertical position)} \]

**Figure 4.** Effect of nanoparticle concentration on Streamline (left) and isotherm counter (right) for Ra=10^6, and inclined angle=0°, 45°, 90°.
close to each other at the minimum local Nusselt number region. Figure 5 shows that the maximum value of local Nusselt number of internal surface of external cold elliptic cylinder occurs at first and fourth octant of enclosure (i.e. at angular position $\gamma = 45^\circ$ and $135^\circ$) for $\phi = 0^\circ$ (horizontal position). While the minimum values occur at the second and third octants of elliptic cylinder ($\gamma = 135^\circ$ and $225^\circ$). The maximum value of local Nusselt number stays only at the first octant of elliptic cylinder at the angle of inclination $\phi = 45^\circ$ (inclined position) and $\phi = 90^\circ$ (horizontal position) for all values of nanoparticles volume fraction of fluid. Also, the local Nusselt number value increases with increase nanoparticles volume fraction and close to each other at the minimum local Nusselt number region. The convection currents get much faster with increase of volume fraction, causing active movement of flow and more advanced thermal plume. The strong fluid movement close to the walls causes great gradient of temperature at the walls and gives great local Nusselt number.

(a) $\phi = 0^\circ$ (horizontal position)

(b) $\phi = 45^\circ$ (inclined position)
Figure 5. Local Nusselt number on hot cylinder (Ra=10^6) for inclined angle 0°, 45°, 90°
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Figure 6. Local Nusselt number on cold cylinder (Ra=10^6) for inclined angle 0, 45, 90°

Average Nusselt number

Figure 7 shows the variation of average Nusselt number with Rayleigh number for the inner hot circular cylinder (left) and outer cold elliptic cylinder (right) at different fluid nanoparticles volume fraction and inclination angles of 0°, 45°, 90°. The results show that the values of average Nusselt number increases as Rayleigh number and volume fraction increase. This rise in the average Nusselt number is recognized to the rises of the nanofluid thermal conductivity with the rise in the nanoparticles volume fraction. At low Rayleigh number and a fixed value of φ, the convection strength is weak and increases as Rayleigh number increases for each angles of inclination. For a specified Rayleigh number, the extreme rate of transfer of heat is attained via the nanofluid and it can be noticed that the influence of fluid nanoparticles is more distinct at high Rayleigh numbers. Figure 7 also shows the Nu rises with rise of the inclination angle. The reduction of resistance to the flow occurs since the estimated area of internal hot cylinder standard to the flow of buoyancy vertical drops. As a result, the flow velocity adjacent to the internal hot wall increases. This causes a drop in the thickness of thermal boundary layers and velocity and great heat transfer rate (great average Nusselt number).
CONCLUSIONS

The present study includes numerical study for buoyancy-driven fluid flow and heat transfer of Al₂O₃/water nanofluid in an annular space between inner hot circular cylinder and outer cold elliptic cylinder with constant temperatures \( T_h \) and \( T_c \); respectively. The results are attained for wide ranges of Rayleigh numbers and fractions of volume of the fluid nanoparticles at three angles of inclination. It was concluded that:

1. The increasing of streamline function and flow circulation as angle of inclination moves from vertical to horizontal because of increase the convection currents.

2. The isotherms contour displays the details of thermal plume at the upper of the internal circular cylinder wall due to the heated rising currents lengthwise the two sides of internal cylinder wall.
3. The flow strength rises as the fraction of volume of the nanoparticles rises for all angles of inclination.

4. The angular distribution of local Nusselt number for the inner and outer cylinders depends on Rayleigh number, angles of inclination, and nanoparticles volume fraction.

5. The extreme rate of transfer of heat is attained via the nanofluid which its effect is more pronounced at high Rayleigh numbers.

Nomenclature

\( \text{Ar}_1 \) Axis ratio of inner cylinder
\( c_1, c_2, c_3 \) Coordinate transformation coefficients
\( c_4, c_5 \) Coordinate transformation coefficients
\( f_x \) Local factor of friction
\( g \) Gravity acceleration
\( Gr \) Number of Grashof
\( h \) Total transfer of heat coefficient
\( h_x \) Local transfer of heat coefficient
\( J \) Jacobian of conversion of coordinate
\( \kappa \) Fluid thermal conductivity
\( N \) Number of corrugations
\( Nu_{ave} \) Total Nusselt number
\( Nu_x \) Local Nusselt number
\( Mr \) Major axis ratio
\( Pr \) Number of Prandtl
\( Re \) Number of Reynolds
\( Ri \) Number of Richardson (\( Ri = \frac{Gr}{Re^2} \))
\( T \) Fluid temperature
\( \theta \) Dimensionless temperature
\( T_h \) Hot temperature of inner cylinder
\( T_c \) Cold temperature of outer cylinder
\( u, v \) Components of velocity in x- and Y-directions, respectively
\( U, V \) Dimensionless velocity components in X- and y-directions, respectively
\( x, y \) Rectangular coordinates
\( X, Y \) Rectangular coordinates without dimension
\( \alpha \) Thermal diffusivity of fluid
\( \beta \) Coefficient of extension of fluid volume
\( \lambda \) Amplitude
\( \phi \) Angle of inclination
\( \varphi \) Volume fraction=PHI
\( \mu \) Fluid dynamic viscosity
\( \vartheta \) Fluid kinematic viscosity
\( \xi, \eta \) Curvilinear coordinates for body-fitted
\( \tau_x \) Shear stress of local
\( \psi \) Function of stream
\( \omega \) Vorticity
\( \Omega \) Vorticity without dimension

REFERENCES


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