Journal of Mechanical Engineering Research and Developments
ISSN: 1024-1752
CODEN: JERDFO
Vol. 43, No.6, pp. 299-307
Published Year 2020

Reduced Order of the Unstable System Using Balanced Truncation Algorithm Based on Continuous-Discrete Mapping and its Application to Control

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ABSTRACT: There have many researches with many order reduction algorithms to solve the problem of model order reduction. However, most of these proposed methods mainly applied to stable linear systems. In real applications, a lot of problems that require reducing model order are unstable linear systems. Therefore, algorithms for order reduction must be able for both stable and unstable linear systems. This paper introduces a balanced truncation algorithm based on continuous-discrete mapping applied to an unstable direct system, which is an extension of a balanced truncation algorithm used to a stable linear system. The illustrative examples have shown the order reduction efficiency of the algorithm.

KEYWORDS: Model order reduction (MOR), Controller order reduction (COR), Balanced truncation (B.T.) algorithm, The unstable system, Continuous-discrete mapping (C-D mapping).

INTRODUCTION

There have been many proposed algorithms for model order reduction (MOR). One of the most popular algorithms is the balanced truncation (B.T.) algorithm [1]. This algorithm is implemented by applying similar conditions to a simultaneous diagonalization process of the Gramian's control matrix and Gramian's kinetics observer matrix of the system in the open-loop system thinking. The equivalence of two diagonal matrices allows transforming the original model represented in any base system to an equivalent base system described in the coordinate system of internal balanced space. From this balanced space, low-order models can be found by eliminating specific values that have a minor contribution to the relationship between the inputs and outputs of the system, i.e., eliminating states that are less able to control and observe.

The advantage of Moore's B.T. algorithm is that a small reduction error will be obtained [1]. This B.T. algorithm is further developed and defined relationships with Hankel standards in [2,3]. Recent studies on the B.T. method focus on perfecting or adjusting the algorithm for each specific order reduction problem [4-13]. However, the B.T. algorithm is only applicable to the asymptotic stable linear systems [1-13]. This is due to the condition to determine the Gramian control matrix, and the Gramian observer matrix is that the original system must be asymptotic stable. In practice, there are many unstable high-order linear systems, as in [14-27]. Therefore, MOR algorithms, in general, and the B.T. algorithm, in particular, need to be able to reduce not only linear stability systems but also unstable linear systems. To apply Moore's B.T. algorithm for unstable systems, different approaches such as have been proposed [1, 14, 16-27]. In this paper, we are interested in Zilochian's B.T. approach applied to unstable continuous systems and in Boess's B.T. method used to unstable discrete systems [25, 26]. The idea of Zilochian's B.T. algorithm is to perform projection (displacement of coordinate origin) to convert the original system from an unstable into a stable form [25]. After that, the MOR of a stable system is applied according to the B.T. algorithm to obtain a stable order reduction system. Finally, a reverse projection is used to convert the stable order reduction system into an unstable form such as the original system. The idea of Boess's B.T. method is almost similar to Zilochian's algorithm [25, 26]. However, instead of applying the algorithm to unstable continuous systems, Zilochian's algorithm is used for unstable discrete systems. From the idea of the two algorithms above and the continuous-discrete mapping (C-D mapping), the authors introduce a B.T. algorithm based on C-D mapping, then apply the algorithm to a problem of controller order reduction (COR) of the two-wheel balancing mobile robot control system.

α STABLE DISCRETE-TIME SYSTEM

Consider a discrete linear system as follows:

$$x(k+1) = \mathbf{A}_d x(k) + \mathbf{B}_d u(k), \quad y(k) = \mathbf{C}_d x + \mathbf{D}_k u(k) \tag{1}$$

In which:

$$\left(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d, \mathbf{D}_d\right) \in \mathbb{R}^{nxn} \times \mathbb{R}^{nxm} \times \mathbb{R}^{kxn} \times \mathbb{R}^{kxm}, x(k) \in \mathbb{R}^n, \quad u(k) \in \mathbb{R}^m, \ y(k) \in \mathbb{R}^k.$$

The transfer function of the discrete system is:

$$\mathbf{G}(z) := \mathbf{C}_d \left(z\mathbf{I} - \mathbf{A}_d\right)^{-1} \mathbf{B}_d + \mathbf{D}_d, \ z \in \mathbb{C}$$

Definition 1: Discrete-time system (1) is called α-stable if the real part of the poles $\left|\lambda(\mathbf{A}_d)\right| < \alpha, \ \alpha \geq 1$. A set of α - stable discrete-time systems is denoted as D_α.

In the case of $\alpha = 1$, system (1) is called asymptotic stable discrete systems (distinct stable system – 0) as the original definition. $\mathbf{A}_d$ The matrix, in this case, has the form of the Schur matrix $\left|\lambda(\mathbf{A}_d)\right| < 1$.

β STABLE CONTINUOUS SYSTEM

Consider the continuous linear system with the following form

$$\dot{x}(t) = \mathbf{A}_c x(t) + \mathbf{B}_c u(t), \quad y(t) = \mathbf{C}_c x(t) + \mathbf{D}_c u(t) \tag{2}$$

In which:

$$\left(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c, \mathbf{D}_c\right) \in \mathbb{R}^{nxn} \times \mathbb{R}^{nxm} \times \mathbb{R}^{kxn} \times \mathbb{R}^{kxm}, x(t) \in \mathbb{R}^n, \ u(t) \in \mathbb{R}^m, \ y(t) \in \mathbb{R}^k.$$

The transfer function of a continuous system has the following form: $\mathbf{G}(s) := \mathbf{C}_c \left(s\mathbf{I} - \mathbf{A}_c\right)^{-1} \mathbf{B}_c + \mathbf{D}_c, \ s \in \mathbb{C}$.

Definition 2. The continuous system (2) is called β -stable if the real part of the poles $\mathrm{real}(\lambda(\mathbf{A})) < \beta, \ \beta \geq 0$. A set of β -stable continuous is denoted a C_β.

In the case of $\beta = 0$ a system (2) is called asymptotic stable systems (a stable continuous system - 0) as the original definition. $\mathbf{A}$ The matrix, in this case, is the Huzwitz matrix $\mathrm{real}(\lambda(\mathbf{A})) < 0$.

CONTINUOUS-DISCRETE MAPPING

Definition 3. The following mapping:

$$\Omega_{\beta,\alpha} : C_\beta \to D_\alpha$$

$$\left(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c, \mathbf{D}_c\right) \to \left(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d, \mathbf{D}_d\right)$$

In which: . $\mathbf{A}_d = \alpha\left(\mathbf{I} - \bar{\mathbf{A}}_c\right)^{-1}\left(\mathbf{I} + \bar{\mathbf{A}}_c\right)$, $\mathbf{B}_d = \sqrt{2\alpha}\left(\mathbf{I} - \bar{\mathbf{A}}_c\right)^{-1}\mathbf{B}_c$, $\mathbf{C}_d = \sqrt{2\alpha}\mathbf{C}_c\left(\mathbf{I} - \bar{\mathbf{A}}_c\right)^{-1}$, $\mathbf{D}_d = \mathbf{D}_c + \mathbf{C}_c\left(\mathbf{I} - \bar{\mathbf{A}}_c\right)^{-1}\mathbf{B}_c, \bar{\mathbf{A}}_c = \mathbf{A}_c - \beta\mathbf{I}$

It is called C-D mapping, i.e., converting the system from a β-stable continuous system into an α-stable discrete system.

There is also a reverse mapping as follows: Mapping

$$\Omega_{\beta,\alpha}^{-1} : D_\alpha \to C_\beta$$

$$\left(\mathbf{A}_d,\mathbf{B}_d,\mathbf{C}_d,\mathbf{D}_d\right)\rightarrow\left(\mathbf{A}_c,\mathbf{B}_c,\mathbf{C}_c,\mathbf{D}_c\right)$$

Where: $\mathbf{A}_c=\beta\mathbf{I}+\left(\mathbf{I}+\bar{\mathbf{A}}_d\right)^{-1}\left(\bar{\mathbf{A}}_d-\mathbf{I}\right)$ $\mathbf{B}_c=\sqrt{\frac{2}{\alpha}}\left(\mathbf{I}+\bar{\mathbf{A}}_d\right)^{-1}\mathbf{B}_d$. $\mathbf{C}_c=\sqrt{\frac{2}{\alpha}}\mathbf{C}_d\left(\mathbf{I}+\bar{\mathbf{A}}_d\right)^{-1}$. $\mathbf{D}_c=\mathbf{D}_d-\frac{1}{\alpha}\mathbf{C}_d\left(\mathbf{I}+\bar{\mathbf{A}}_d\right)^{-1},\bar{\mathbf{A}}_d=\frac{\mathbf{A}}{\alpha}$ is called reverse C-D mapping, i.e., convert the system from an α-stable discrete system into a β-stable continuous system.

In the case of β = 0 and α = 1, the mapping is called bilinear mapping.

APPLICATION OF THE BT ALGORITHM BASED ON CONTINUOUS-DISCRETE MAPPING TO CONTROL THE TWO-WHEEL BALANCING ROBOT

In [28], a model of a two-wheel balancing robot (front and rear wheels) was built, with a transfer function as follows.

$$\mathbf{S}(s)=\frac{\boldsymbol{\theta}(s)}{\mathbf{U}(s)}=\frac{-0.223s}{s^3+4.722s^2-47.2s-254}$$

Robust control is designed to control the two-wheel balancing robots so that the robot's tilt angle always approaches zero. The schematic of the robust control system is given in Figure 1.

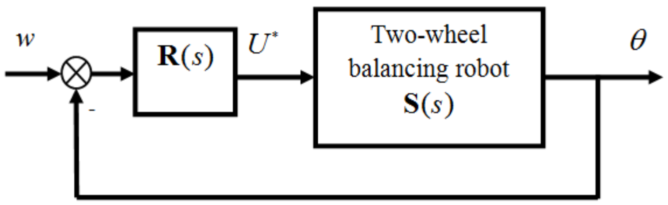


Figure 1. Schematic structure of a robust control system for two-wheel balancing robot

The robust controller in is in the form as below [28]:

$$\mathbf{R}(s)=\frac{\mathbf{H}(s)}{\mathbf{D}(s)} \quad (3)$$

with

$$\begin{aligned}
\mathbf{H}(s)=&-2.23.10^{-7}s^{30}-4.67.10^{-4}s^{29}-0.266s^{28}-22.96s^{27}-1006s^{26}-2.853.10^{4}s^{25}\\
&-5.837.10^{5}s^{24}-9.144.10^{6}s^{23}-1.139.10^{8}s^{22}-1.158.10^{9}s^{21}-9.776.10^{9}s^{20}-6.949.10^{10}s^{19}\\
&-4.199.10^{11}s^{18}-2.172.10^{12}s^{17}-9.663.10^{12}s^{16}-3.71.10^{13}s^{15}-1.231.10^{14}s^{14}-3.53.10^{14}s^{13}\\
&-8.74.10^{14}s^{12}-1.862.10^{15}s^{11}-3.398.10^{15}s^{10}-5.276.10^{15}s^{9}-6.903.10^{15}s^{8}-7.511.10^{15}s^{7}\\
&-6.676.10^{15}s^{6}-4.721.10^{15}s^{5}-2.556.10^{15}s^{4}-9.953.10^{14}s^{3}-2.482.10^{14}s^{2}-2.977.10^{13}s\\
&-0.00439
\end{aligned}$$

$$\begin{aligned}
\mathbf{D}(s)=&\,4.971.10^{-14}s^{30}+2.032.10^{-10}s^{29}+2.663.10^{-7}s^{28}+1.221.10^{-4}s^{27}+9.72.10^{-3}s^{26}\\
&+0.3918s^{25}+10.14s^{24}+187.1s^{23}+2612s^{22}+2.862.10^{4}s^{21}+1.088.10^{7}s^{18}+2.523.10^{5}s^{20}\\
&+1.82.10^{6}s^{19}+5.428.10^{7}s^{17}+2.273.10^{8}s^{16}+8.005.10^{8}s^{15}+2.372.10^{9}s^{14}+5.9.10^{9}s^{13}\\
&+1.225.10^{10}s^{12}+2.107.10^{10}s^{11}+2.962.10^{10}s^{10}+3.341.10^{10}s^{9}+2.941.10^{10}s^{8}+1.931.10^{10}s^{7}\\
&+8.743.10^{9}s^{6}+2.286.10^{9}s^{5}+1.519.10^{8}s^{4}-5.226.10^{7}s^{3}+3.6.10^{-6}s^{2}+5.32.10^{-22}s
\end{aligned}$$

In a digital control system (Adruino) is used to control the two-wheel balancing robots [28]. Therefore, a high-order controller (30th order) will lead to complexity in the coding of the control program. As a result, processing time increases, and hence the response time of the control system also increases. This may fail to meet real-time control requirements and cause the system to become unstable. Therefore, to meet the needs of real-time control, it is necessary to reduce the order of the 30th order controller. The reduced-order controller, which is chosen to replace the 30th order controller, must meet the following conditions:

Condition 1: The reduction controller still ensures that its control quality equivalent to that of the original controller (step response error, frequency response error, control quality error are small. That satisfies the requirement of robust stability of the system).

Condition 2: The order of the controller is small.

The 30th order controller is an unstable model. Therefore, when applying the B.T. algorithm based on the C-D mapping to reduce the order of the 30th order controller, we obtained results, as shown in Table 1:

Table 1. Result of the order reducing order the 30th order controller

Order	Transfer function $\mathbf{R}_r(s)$
5	$\dfrac{-4.485.10^6 s^5 - 6.804.10^7 s^4 - 4.123.10^8 s^3 - 1.235.10^9 s^2 - 1.816.10^9 s - 1.09.10^9}{s^5 + 2009s^4 + 1.833.10^4 s^3 - 1913s^2 + 1.083.10^{-9} s - 2.609.10^{-11}}$
4	$\dfrac{-4.485.10^6 s^4 - 3.063.10^7 s^3 - 1.158.10^8 s^2 - 1.824.10^8 s - 1.186.10^8}{s^4 + 2004s^3 - 204.6s^2 - 0.4293s + 0.02271}$
3	$\dfrac{-4.553.10^6 s^3 - 2.091.10^8 s^2 + 2.869.10^8 s - 4.905.10^8}{s^3 + 2140s^2 - 472s + 30.59}$

Note: We will call the reduced-order controller (order of r) is: r-order controller.

To be able to choose a reduced-order controller that satisfies the requirements of the 30th order controller, the authors evaluate a step response and a frequency response of both original and reduced-order systems. Simulation results are shown in Figure 2 and Figure 3, respectively.

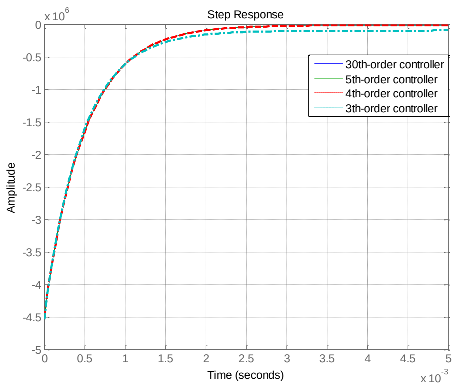


Figure 2. Step response of the 30th order controller and reduced-order controllers.

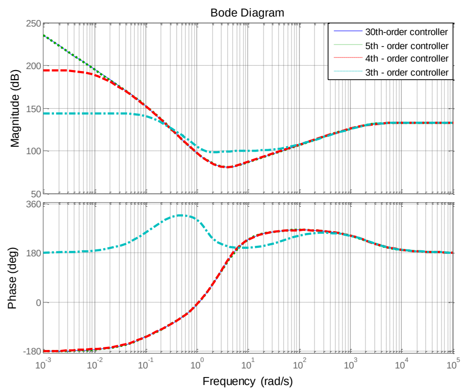
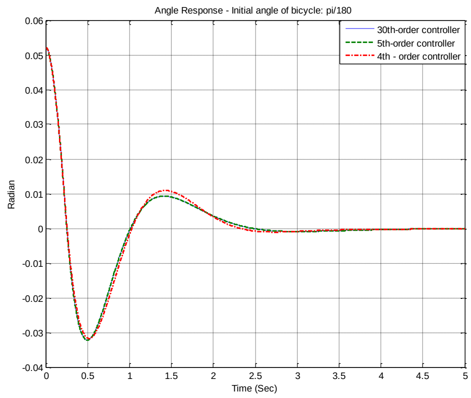


Figure 3. The frequency response of the 30th order controller and reduced-order controllers.

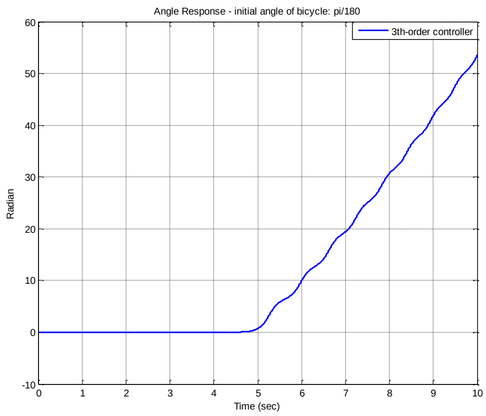
From Figure 2 and Figure 3, we can see that:

Step response of the 4th order controller and 5th order controller matches the step response of the 30th order controller accurately. There is an error between the step response of the 3rd order controller and the 30th order controller with t > 1 [s], in which the value of error increases when time increases. Frequency response (magnitude and phase) of the 5th order controller matches the frequency of the 30th order controller. With frequency $\omega > 0.0212$ [rad/s], magnitude in frequency response of the 4th order controller fits magnitude in frequency response of the 30th order controller, while there is an magnitude error with frequency $\omega < 0.0212$ [rad/s]. This error increases when the frequency decreases. The phase in the frequency response of the 4th order controller fits the phase in the frequency response of the 30th order controller. There is an error between the magnitude in the frequency response of the 3rd order controller and the magnitude in the frequency response of the 30th order controller with frequency $\omega < 69.6$ [rad/s]. Specially, when $\omega < 0.206$ [rad/s], this error increases when frequency decreases. When $\omega > 69.6$ [rad/s], this magnitude error is nearly zero. There is an error of the phase in the frequency response of the 3rd order controller and the phase in the frequency response of the 30th order controller with $\omega < 541$ [rad/s]. With $\omega > 541$ [rad/s], this phase error is closed to zero.

Matlab/Simulink is used to simulate the two-wheel balancing robot using the 30th order controller and to use reduced-order controllers (Table 1) based on [28]. Simulation results are shown in Figure 4.



(a) Tilt angle response of the two-wheel balancing robot using the 30th order controller, 5th order controller and 4th order controller



(b) Tilt angle response of the two-wheel balancing robot using the 3rd order controller

Figure 4. Tilt angle response of the two-wheel balancing robot using the 30th order controller and reduced-order controllers.

Responsive property of the two-wheel balancing robot system (referred to as control system) using the 30th order controller and 5th order controller is as below:

Maximum amplitude of the first oscillation: - 0,00653 [radian]; Amplitude of the second oscillation: + 0,00175 [radian]; Number of oscillation: 2 times; Transient time: 2,3 [s]; Steady state error: 0%.

Responsive property of the control system using the 4th order controller is as below:

Maximum amplitude of the first oscillation: - 0,0064 [radian]; Amplitude of the second oscillation: + 0,00207 [radian]; Number of oscillations: 2 times; Transient time: 2,3 [s]; Steady state error: 0 %.

Comment: It can be seen from Figure 4 that: The tilt angles of the control system using the 5th order controller and using the 30th order controller are completely matched. The response of the tilt angle of the control system using the 4th order controller follows closely the tilt angle of the control system using the 30th order controller. Control systems using a 5th order controller and 4th order controller can keep the two-wheel balancing robot at a stable equilibrium position. The control system using a 3rd order controller can keep the two-wheel balancing robot at a stable equilibrium position only within t < 5 [s].

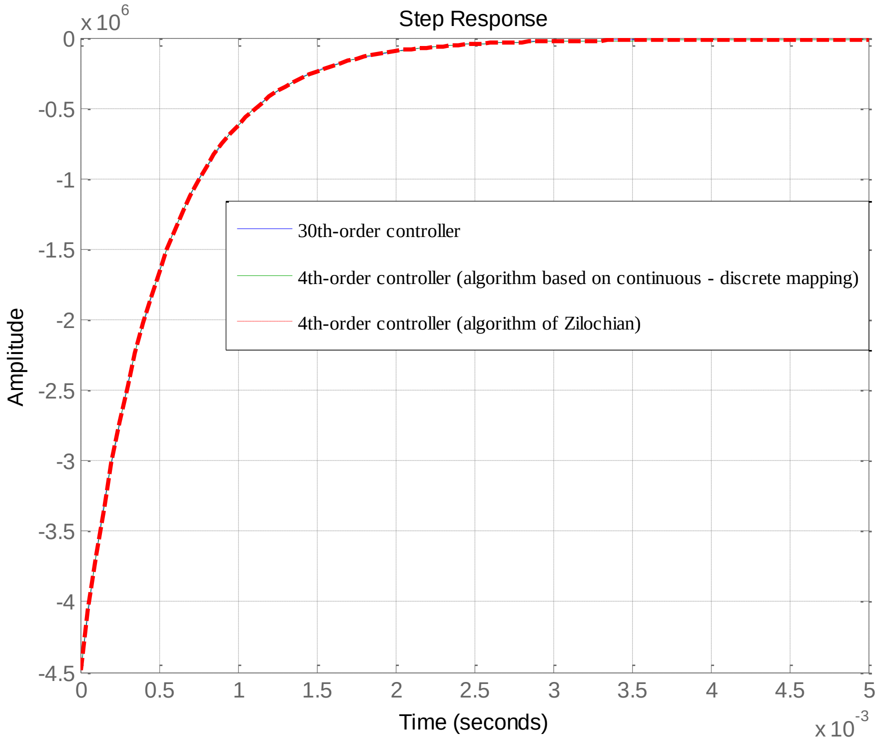
Conclusion: From the above results, we can see that: The 4th and 5th order controllers satisfy the requirements of the order reduction problem of the 30th order controller, but the 3rd order controller does not. If the priority of the order reduction problem is getting the low order controller, the 4th order controller would be selected. Otherwise, if the priority is obtaining a small step response error, small frequency response error, and small steady-state error, then the 5th order controller will be chosen to replace the 30th order controller.

COMPARING THE COR EFFICIENCY OF THE BT ALGORITHMS BASED ON C-D MAPPING AND ZILOCHIAN'S BT ALGORITHM

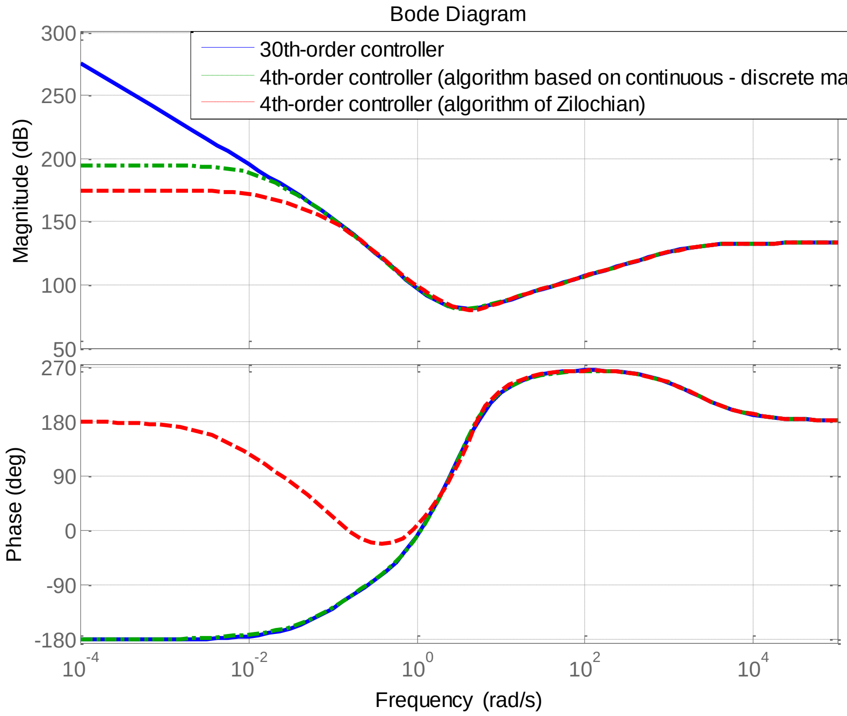
Applied Zilochian's B.T. algorithm [25] to reduce the order of the 30th order controller, we obtain a 4th order controller as bellow:

$$\mathbf{R}_4(s) = \frac{-4.485.10^6 s^4 - 2.858.10^7 s^3 - 1.285.10^8 s^2 - 2.091.10^8 s - 1.402.10^8}{s^4 + 2000s^3 + 473.9s^2 + 30.25s + 0.2572}$$

Evaluating step response, frequency response, tilt angle response of control system using 4th order controllers, we obtain results as shown in Figure 5, Figure 6.



(a) Step response of the 30th order controller and 4th order controller



(a) The frequency response of the 30th order controller and 4th order controller.

Figure 5. Step response, the frequency response of the 30th order controller, and the 4th order controller.

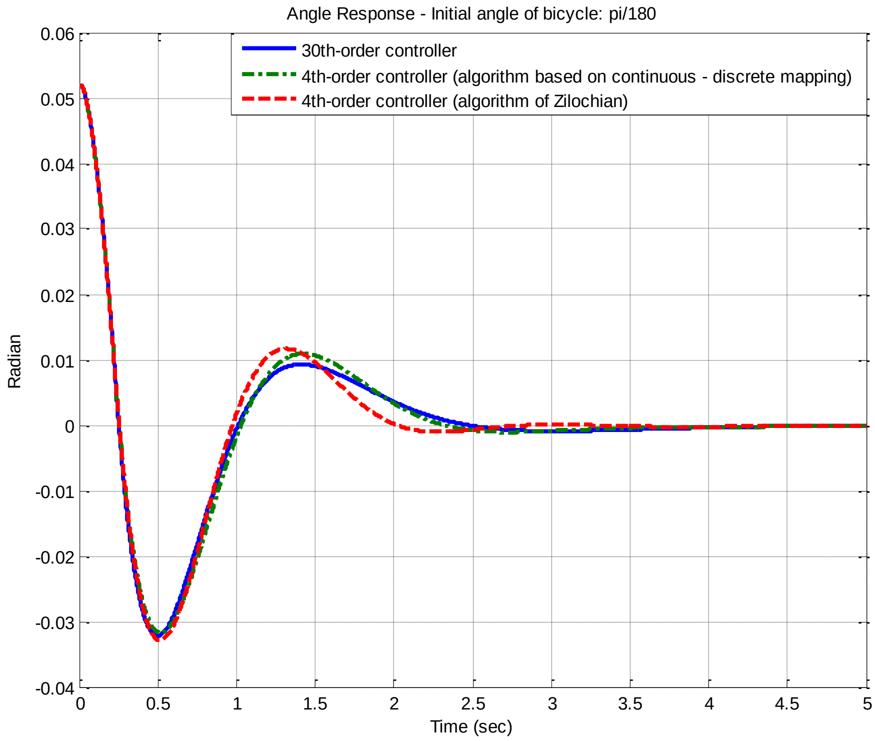


Figure 6. Tilt angle of the control system using a 30th order controller and 4th order controllers.

Responsive property of the control system using 4th order based on Zilochian's B.T. algorithm is as below:

Maximum amplitude of the first oscillation: - 0,00696 [radian]; Amplitude of the second oscillation: + 0,00237 [radian]; The amplitude of the third oscillation - 0,00019; Number of oscillations: 2 times; Transient time: 2,3 [s]; Steady state error: 0 %.

Comment: From Figure 5 and Figure 6, we see that: Step response of the 4th order controller based on both C-D mapping and Zilochian's B.T. algorithm fit the step response of the 30th order controller. Comparing the amplitude in the frequency response of the 4th order controller based on C-D mapping and the 4th order controller based on Zilochian's B.T. algorithm to the amplitude-frequency response of the 30th order controller, the former one has a smaller error range ($\omega < 0.0212$ rad/s for the former and $\omega < 0.156$ rad/s for the later) and smaller error magnitude. While there is an error between the phase of the frequency response of the 4th order controller based on Zilochian's B.T. algorithm and the phase of the frequency response of the 30th order controller (with $\omega < 67.4$ [rad/s]), there is no error between the phase of B.T. algorithm based on C-D mapping and the phase of the frequency response of the 30th order controller. Tilt angle response of the control system using the 4th order controller based on C-D mapping has an amplitude and number of the oscillation less than that of the control system using the 4th order controller based on Zilochian's B.T. algorithm.

CONCLUSION

This article has introduced a B.T. algorithm based on C-D mapping with the idea of combining Zilochian's B.T. algorithms and Boess's B.T. algorithms. Applying the B.T. algorithm based on C-D mapping to the COR problem of two-wheel balancing robot, simulation results show that both 4th order and 5th order controllers meet the requirement of the COR problem, and these controllers can replace the 30th order controller. Besides, simulation using Matlab/Simulink also shown that the quality of the control system using the 4th order controller based on C-D mapping is better than the control system using the 4th order controller based on Zilochian's B.T. algorithm.

ACKNOWLEDGMENT

This research was funded by Thai Nguyen University of Technology; No. 666; 3/2 street; Thai Nguyen City; Viet Nam.

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