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Employing cubic Bézier curve in design compliant bistable micromechanism

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ABSTRACT**:** This article reports a new compliant bistable micromechanism (BM) with force output flexibility under a space constraint. The mechanism is jointless and save energy. The mechanism has four Bézier curved beams and a shuttle mass at its center. A cubic Bézier curve with four-point controls offers high flexibility for designing beam shapes of BMs that modify the maximum and minimum force in the same design space. The finite element method is applied to predict and investigate the bistable behavior. Compared to the conventional BMs with cosine curved beams and slanted straight beams, the BM with Bézier curved beam can be designed to have smaller switching forces under the same device area and thickness. The various Bézier profiles are designed with many nonlinear characteristic. Essential design paramters affect the nonlinear characteristic of BM are examined.

KEYWORDS: Bézier curve, stable position, nonlinear behavior.

INTRODUCTION

Compliant bistable mechanisms have potential use in many applications, which reduced power consumption, especially in micro-scale. These devices are monolithic, free friction and no backlash. The mechanism has two stable equilibrium positions and does not require external axial loads. BMs have potentially used in design switches, accelerometers, gyroscope sensor, in a wide range of sizes from macro to micro scales [1-5]. Nonlinear behavior is one of the specify of the compliant BMs. Based on the buckling phenomenon of the bistable mechanism, the energy harvester was study which increases the bandwidth in operation. Various profiles of beams of BMs have been utilized to investigate the characteristic [6]. A group researhcer employed a cosine curved beam structure in their BM [7]. In order to adjust force-deflection curve of the BM under an area footprint constraint, the thickness and the width of beams should be modulated. A group researchers presented a BM with slanted straight beam structure [8].

Once a space constraint of the device is imposed, the length, the width, the thickness and the inclination angle of the slanted straight beam can be varied to meet the force output requirement besides material properties of the beam structure. A compliant bistable mechanism with many straight lines for nearly equal switching forces in the forward and backward directions is developed [9, 10]. The mechanism flexibility is taken advantage of the design of a bistable mechanism with nearly equal switching forces in the forward and backward directions. Pham and Wang demonstrated a constant-force bistable mechanism, combine two cosin curve [11]. The bistability of the mechanism originates from combined compression and bending of the beam structures. A stepped beam BM was investigated [12]. The f-d curve of their BM was varied by changing beam thickness and width. A group researcher investigated a compliant chevron-type BM [13]. The structure of the mechanism is composed of hinge springs and stepped beam. The hinge spring provides an alternative to adjust the f-d curve of their BM. Wilcox et al. introduced folded beam structures into their BMs [14]. Their design is aimed to adjust force output of their BMs. Without adding to the device complexity, width and thickness of the beam structure of the conventional BMs with cosine curved beams and straight beams can be varied.

However, design the BMs has many problems, the limitations of fabrication technology in micro-scale that challenge the researchers. The restrictions in design space require the new compliant bistable mechanism, which allows modifying the nonlinear behavior of the mechanism. In order to increase the design flexibility of the BMs, parametric curves can be adopted for the profiles and the beam structure of BMs. Bézier curve is a parametric curve frequently used in computer graphics. It is also utilized to design in many applications. Wang and Nguyen developed an ultrasonic cutting horn based on the Bézier profile [15]. This hold is exploited for high displacement amplification in order to reduce the penetration force required to enter and cut materials. A rotary ultrasonic machine with the horn shape based on the Bézier curve is demonstrated [16]. The comparison between the Bézier profile and the traditional conical profile proves the advantage of the Bézier curve in increasing the amplification and reducing stress. An application of Bezier curve in design the beams of bistable mechanism is proposed, the device generates the constant force in motion help protect the overload [17].

The compliant microgripper with geometry based on the Bezier curve is illustrated [18]. Combination with topology optimization, the design with large deflection is adopted and analyzed. In this paper, an application of Bézier curved in design the profile of beams is investigated. Design of the BM is described. The cubic Bézier with four control points is proposed to apply the BM, two control point is fixed at the two end of beam, the other two control points are modifying to achieve various characteristic. The nonlinear behavior of the BM under static loadings are analyzed by finite element analyses. The results of the presented device are compared with those of BMs with cosine beams and straight beams.

DESIGN

A typical force-displacement curve of a CBM is shown in Figure 1. At the beginning, the shuttle mass rests at its first stable equilibrium position A. When the shuttle mass in moved quasistatically in the + direction which is the forward motion, the sum of the reaction force at the fixed ends increases initially, attains its maximum value Fmax, then decreases and the shuttle mass reaches its unstable equilibrium position B. As the shuttle mass is moved further in the + direction, the reaction force turns negative, attains its minimum value F_{min}, then increases and the shuttle mass reaches its second stable equilibrium position C. While the CBM is at the unstable equilibrium position B, a light perturbation to the CBM could drive the shuttle mass into the stable equilibrium positions A or C. In the backward motion, the shuttle mass move from the second stable position C to the first stable postion A. When it operates, the force reaches the minimum force and go to the unstable postion B, it suddenlly jumps to position A. In the displacement-controlled mode of motion, the positive and negative reaction forces mean that the shuttle mass is pushed and pulled in the forward motion of the mechanism, respectively. In the load-controlled mode of motion, the load is increased quasistaticaly in the forward motion. When the position D is reached, the mechanism will snap-through toward its second equilibrium position with no appreciable change in the load. When the shuttle is moved backward, the mechanism follows the f-d curve reversely in the displacement-controlled mode of motion.

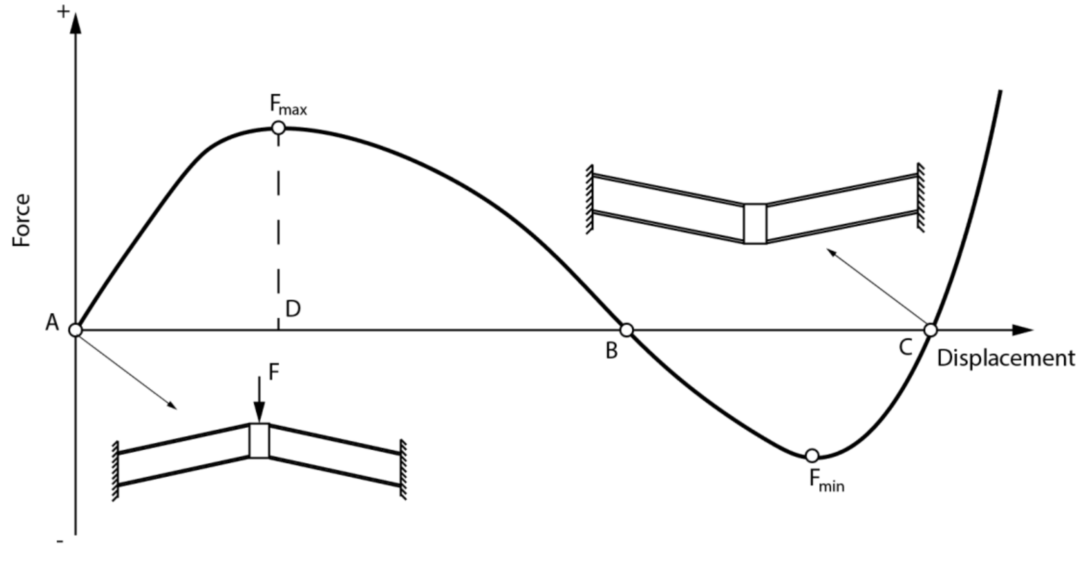


Figure 1. A typical force-displacement curve of bistable

Figure 2 shows the typical Bézier curve with 4 control points. The cubic Bézier is specified by four control points, B_1, B_2, B_3 and B_4, as shown in Figure 2. B_1 and B_4 are fixed by the end of beam. B_2 and B_3 move in design space. The parametric cubic Bézier curve is given [19].

$$P(t) = [(1-t)^3 \quad 3t(1-t)^2 \quad 3t^2(1-t) \quad t^3] \begin{bmatrix} P_{B_1} \\ P_{B_2} \\ P_{B_3} \\ P_{B_4} \end{bmatrix} \tag{1}$$

where t is the parameter, and P_{B_i} is the position vector of the point B_i.

Figure 3(a) is a schematic of a Bézier curved BM. Its shuttle mass is supported by four Bézier curved beams. A Cartesian coordinate system is also shown in the figure. Due to symmetry, only a half model of the mechanism is considered and show in figure 3(b). Dimensions of the BM and coordinates of the control points of the Bézier curve are indicated in figure 3(b). The thickness of the device is 20 μm. The anchored edges of the BM are fixed and the symmetric plane is modeled by a roller boundary condition.

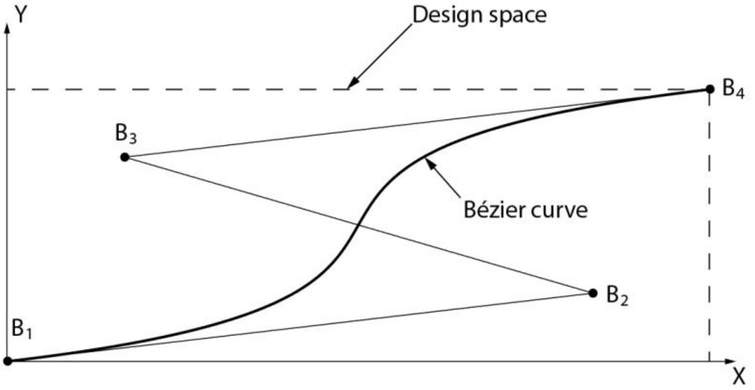


Figure 2. A schematic of cubic Bézier with 4 control points.

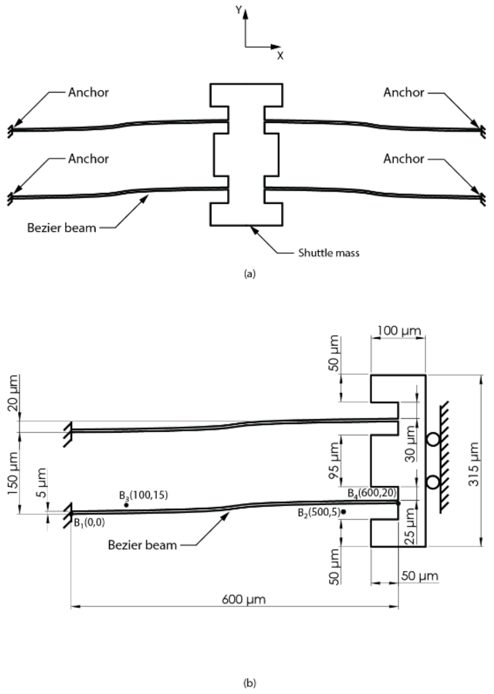


Figure 3. (a) A schematic of the BM. (b) Dimensions of a half model.

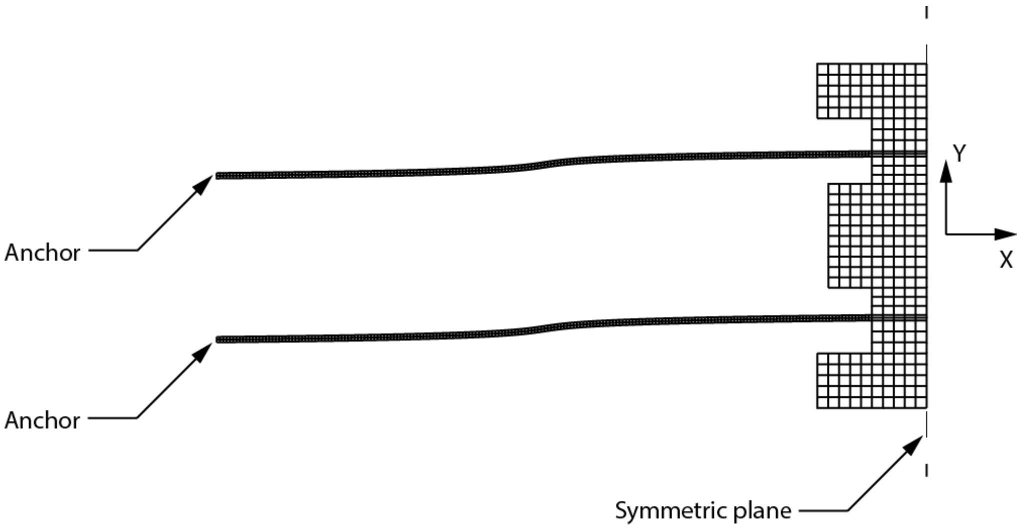


Figure 4. A mesh of the half model.

Finite element analyses are employed to obtain the relation of force-displacement of the BM. Due to the thickness of the model is uniform, a 2D model is execute in analysis. A displacement of the shuttle mass in the y-direction is applied. The reaction force in the y-direction at the fixed ends can be taken as the force applied to the BM. The x-direction displacement of the nodes at the symmetric plane is constrained to represent the symmetry condition due to the mechanism geometry and the loading conditions. For the linear elastic and isotropic model, the Young's modulus is taken as 130 Gpa and Poisson's ratio is 0.28 for typical silicon materials. Figure 4 shows a mesh for the half model. The commercial ABAQUS program is applied to solve computations. The finite element model has 1080 4-node elements.

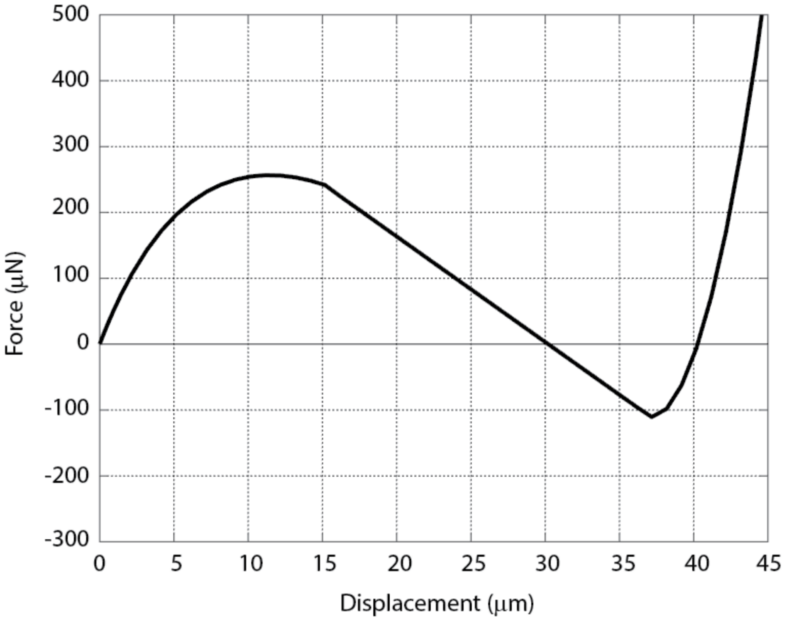


Figure 5. Force-displacemnt curve based on the finite element analysis.

A force-displacement curve based on the finite element analyses are shown in figure 5. Because of the boundary conditions, the shuttle mass do not have motion in the x-direction, reaction force in y-direction is adopted to investigate the bistable behavior. When the displacement of the shuttle mass increases from 0 (the first stable equilibrium position), the force increases initially, reaches its maximum value at 253 μN, then decreases to 0, where is an unstable equilibrium position at 30 μm. When the displacement continues increasing, the force decreases, attains its minimum value at -112 µN, then increases again and reaches 0, the second stable equilibrium position at 41 µm. The results validate the Bézier curve for design of beam profile of BMs.

RESULT AND DICUSSION

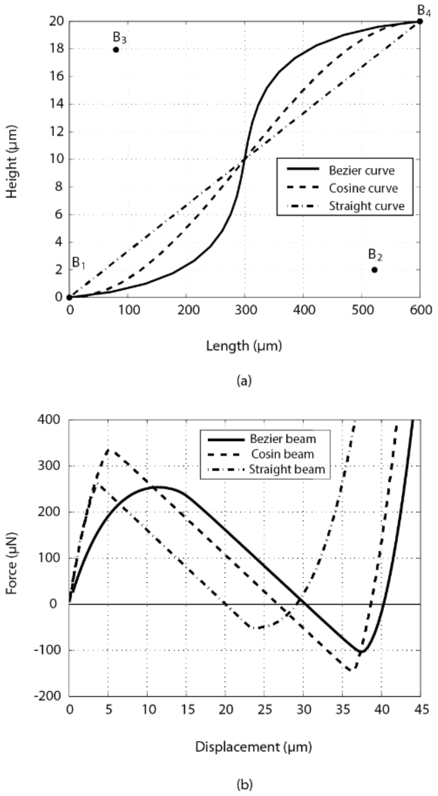
The f-d curve of the Bézier BM is compared with those of the cosine curved and slanted straight beam BMs, where the beams occupy the same area, 600 µm x 20 µm. The thickness and width of the beams are 20 µm and 5 µm, respectively. The profiles of the three types of beams are shown in Figure. 6(a). The slanted straight curve has the length is 600 µm, and the height is 20 µm. The cosine curve follows the equation (2). The four control points, B_1, B_2, B_3 and B_4, of the Bézier beam are also indicated in the figure. Both of three curve beams have the same thickness 20um. The Young's modulus and Poisson's ratio of the BMs are 130 GPa and 0.28, respectively.

$$y = \frac{H}{2}\left(1 - cos\left(\frac{\pi x}{L}\right)\right) \tag{2}$$

Figure. 6(b) shows the f-d curves of three BMs. Table 1 gives the result of the comparision. The Bézier BM with the selected coordinates of the control points has the lowest switching force among the three BMs. The straight curve give the smallest distance when the BM move from the first equilibrium position to second equilibrium position and the cosine draw the biggest force in this investigation.

Table 1. Force and displacement of slanted straight curve, cosin curve and Bézier curve.

	Maximum Force (µN)	Minimum force (µN)	Second equilibrium position (µm)
Straight curve	267	-56	29
Cosin curve	321	-136	38
Bézier curve	224	-41	41



(a)

(b)

Figure 6. (a) Profiles of and (b) force-displacement curves of various beams

The switching force of the Bézier BM can be adjusted by the selection of the control points of the Bézier curved beam. The proposed Bézier BM provides more freedom in the design of BMs. Various Bézier curve beams employed in the bistable mechanism is investigated, Table 2 shows the coordinate of B2 and B3 control points, the design space is taken over 600 µm x 20 µm. The force-displacement curves of these are performed in the Figure. 7. The biggest force obtained by case 3, which value is 407 µN. The smallest switching force is 196 µN, obtianed by case 5.

Table 2. Positions of B_2 and B_3

Cases	B2		B3	
	x	y	x	x
1	500	5	100	15
2	500	0	100	20
3	400	10	200	10
4	300	5	300	15
5	400	10	100	10

Figure. 8. provides some guidelines for the selection of the position of control points in the initial design stage for BM. The red area contains position of control point B2 and control B3 is enclosed by blue area. Almost control points B2 and B3 reflected through the line of symmetry. The nonlinear behavior plenty achieved when B2 and B3 concentrated around reflection symmetry. The biggest switching or smallest switching force can be quickly obtained by optimization solution which is introduced by Tran and Wang [20].

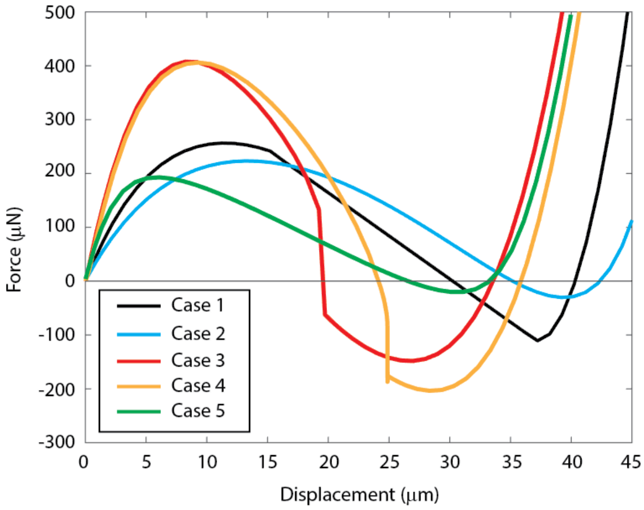


Figure 7. Force-displacement curve of various Bézier curve beams

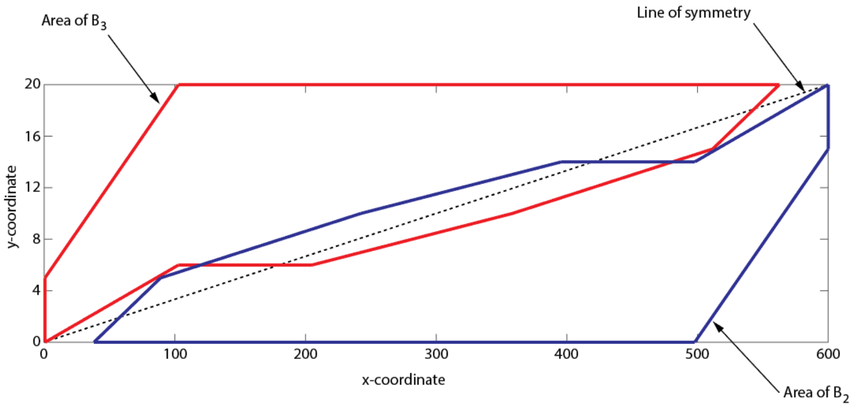


Figure 8. Position of control points B2 and B3

CONCLUSION

A new design of BM based on Bézier curved beams is investigated. The cubic Bézier with four control points is employed to design the beam profile of the mechanism. A 2D model is studied and the FEA method is applied to adopt the force-displacement relation of bistable behavior. Compared to conventional BMs based on cosine curved beams and straight slanted beams, the proposed BM has high flexibility in design for BMs. The BM with Bezier profile give the small force in backward and forward motion. The location of the control points also identifies and gives the trend for support in design. Given the device area and thickness constraints, the Bézier BM has various force outputs by moving the Bézier curved beam's control points in the design space. Based on this method, many mechanisms with bistability could be designed with the same space, help save the areas of the mechanisms. Especially, in the MEMS application, with the limited space in design, the flexible mechanism is more comfortable to design the gripper, energy harvester device, etc. Furthermore, the Bézier curve with many control points should be observed to increase the mechanism's flexibility. It provides more parameters so that the control is accessible.

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