Journal of Mechanical Engineering Research and Developments
ISSN: 1024-1752
CODEN: JERDFO
Vol. 43, No. 7, pp. 271-278
Published Year 2020

Effects of Internal Connecting Layer Properties on the Vibrations of Double Beams at Different Boundary Conditions

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ABSTRACT: Double beams are important engineering and practical applications that are used in the construction of structures, aircraft, and construction. The properties of the inner layer between the two beams affect the frequencies of these double beams. In this work, the effect of linear elasticity and non-linear elasticity on frequencies of double beams has been calculated at different boundary conditions like simply-supported to simply-supported, clamped to clamped, free to free, clamped to free, clamped to simply supported, and simply-supported to free. The energy balance method based on Galerkin-Petrov (EGP) and Homotopy Perturbation Method (HPM) are used to compute the effect of the non-linearity stiffness of the inner layer on those frequencies. There is no significant difference in the frequencies computed by both methods with very little difference between them. Increasing the linear elasticity values of the inner layer causes an increase in frequencies and increasing the values of the non-linear elasticity of the inner layer causes a very slight increase in the values of the frequencies of the double beams. The increase in the maximum amplitude values affected the increase in the first frequency of the double beams, its effect increases by increasing the values of the non-linear elasticity of the inner layer, and its effect is limited on the second frequency and upwards.

KEYWORDS: double beams, non-linear stiffness, and vibration properties.

NOTATIONS

A: Cross-sectional area of each beams.
C: the maximum amplitude between two beams.
E: Modulus of elasticity of each beam.
I: Second moment of area of each beam.
K_L: Stiffness of linear inner layer.
K_{NL}: Stiffness of non-linear inner layer.
L: Length of each beam.
$T_1(t)$: Unknown time function of upper beam.
$T_2(t)$: Unknown time function of lower beam.
$\phi(x)$: Shape function of beam.
ρ: Mass density of each beams.
ω_{EGP}: Natural frequency for the double beams using energy balance method based on Galerkan-Petrov approach.
ω_{HPM}: Natural frequency for the double beams using Homotopy perturbation method.
Ω_i: Dimensionless natural frequency for ith mode.
y_1: Transverse deflection of upper beam.
y_2: Transverse deflection of lower beam.

INTRODUCTION

Due to the unique properties of double beams such as high strength and low weight which enabled them to be used in important engineering applications, especially in the field of aeronautical engineering. Another important application of a double-beam system is in vibration attenuation. The dynamic characteristics of the double-beam system have been studied by several researchers. Alborz et al. [1] studied the free transverse vibration of the double beam and presented analytical formulation for the natural frequencies and normalized mode shapes of the system. Y.Q. Zhang et al. [2] investigated the properties of vibration and buckling of a double beam system under compressive axial loading. The natural frequencies derived from explicit expressions and the analytical solution for the critical buckling load of the system is derived. Alborz Mirzabeigy et al. **[3]** investigated the free transverse vibration of two parallel beams. The differential transform method is utilized and a semi-analytical solution is developed to obtain the natural frequencies and mode shape functions. Y. X. Li et al. [4] developed a semi-analytical method to compute the mode shapes and frequencies and studied the effects of the Winkler layer and viscoelastic layer damping on the vibration of double-beam systems. Md. Saifur Rahman et al. [5] developed the geometric nonlinear formulations model. to study the vibrations of the double beam and modified multi-level residue harmonic balance method to investigate the forced nonlinear vibrations. S. Graham and C. Nicely **[6]** presented an exact solution of the free vibrations of an arbitrary number of beams connected by viscoelastic layers of the Kelvin-Voigt type. Alborz Mirzabeigy et all. [7] found the solution of equations of motion and explicit expression using the Bernoulli-Fourier method and investigated analytically the dynamic response of the double-beam system connected by an elastic layer. Alborz M. and Reza M. [8] investigated the vibration characteristics of double beams and applied the Winkler model and the Euler–Bernoulli hypothesis respectively for the beam at different boundary conditions. Z. Oniszczuk **[9]** studied the dynamic response of double beams joined by a Winkler elastic layer at S-S boundary conditions. The free vibrations of a double- beam are realized by synchronous and asynchronous deflections. Vladimir Stojanovic et al. [10] studied a general procedure for the determination of the buckling load and natural frequencies of beams under the compressive load using shear deformation and Timoshenko theory. S. G. Kelly and S. Srinivas **[11]** developed a general theory to determine the natural frequencies and mode shapes for a set of elastically beams. The Rayleigh-Ritz method is used to develop mode shapes and natural frequencies for sets of non-identical beams. P. G. Kessel [12] investigated the vibration properties for the double-beam systems in which one of the beams is under a moving point load. Z. Oniszczuk [13] presented a theoretical vibration analysis of double-beams connected by the elastic layer, the vibrations are recognized by two types of motions: synchronous and asynchronous frequencies with lower and higher frequencies respectively.

METHODOLOGY

In this work, consider two beams are connected by the elastic layer. Both beams have the same length (L), the cross-section area (A), modulus of elasticity (E), and the density (ρ) as shown in figure (1). The elastic layer may have linear properties (K_L) and non-linear properties (K_{NL}). The equations of motion are used by the Bernoulli-Euler beam theory for free vibrations of a system. The equations are illustrated as following [14]:

$$EI\frac{\partial^4 y_1(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_1(x,t)}{\partial t^2} + K_L[y_1(x,t) - y_2(x,t)] + K_{NL}[y_1(x,t) - y_2(x,t)]^3 = 0 \quad (1)$$

$$EI\frac{\partial^4 y_2(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_2(x,t)}{\partial t^2} + K_L[y_2(x,t) - y_1(x,t)] + K_{NL}[y_2(x,t) - y_1(x,t)]^3 = 0 \quad (2)$$

Where:

$y_1(x,t) = \emptyset(x)T_1(x,t)$

$y_2(x,t) = \emptyset(x)T_2(x,t)$

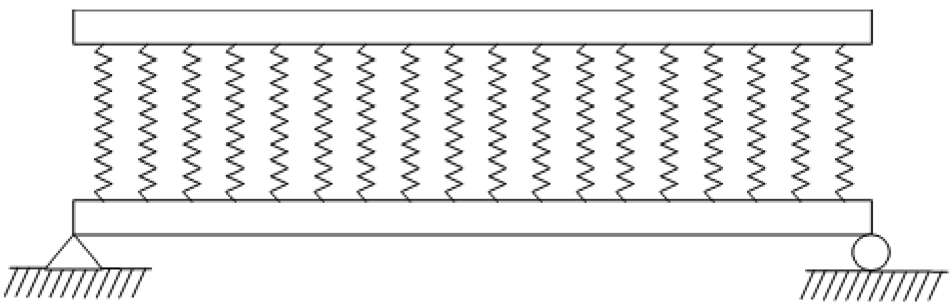


Figure 1. A schematic of double-beam with the elastic inner layer

The non-linear equations (1) and (2) can be derived as follow [14]:

$$\ddot{T}_1 + \alpha T_1 + \beta(T_1 - T_2) + \lambda(T_1 - T_2)^3 = 0 \quad (3)$$

$$\ddot{T}_2 + \alpha T_2 + \beta(T_2 - T_1) + \lambda(T_2 - T_1)^3 = 0 \quad (4)$$

Where:

$$\alpha = \frac{\int_0^L EI \frac{d^4\phi(x)}{dx^4}\phi(x)dx}{\int_0^L \rho A\phi^2(x)dx} = \frac{\Omega^4 EI}{\rho A L^4} \quad (5)$$

$$\beta = \frac{\int_0^L K_L\phi^2(x)dx}{\int_0^L \rho A\phi^2(x)dx} = \frac{K_L}{\rho A} \quad (6)$$

$$\lambda = \frac{\int_0^L K_{NL}\phi^4(x)dx}{\int_0^L \rho A\phi^2(x)dx} \quad (7)$$

The equations (3) and (4) are solved by two different methods. The first method is named, the energy balance method based on Galerkin-Petrov (EGP) approach as following [14]:

$$\omega_{EGP} = \sqrt{\alpha + 2\beta + \frac{7}{5}\lambda C^2} \quad (8)$$

A second method is named the Homotopy Perturbation Method (HPM) as following [14]:

$$\omega_{HPM} = \sqrt{\alpha + 2\beta + \frac{3}{2}\lambda C^2} \quad (9)$$

To study the non-linear effect of the inner layer on the frequencies, the increase in frequency due to inner layer nonlinearity (IFLN) is defined as following [14]:

$$IFNL = \frac{\omega - \omega_{C=0}}{\omega_{C=0}} \times 100\% \quad (10)$$

The increase in frequency ratio (IFR) is defined as follow:

$$IFR = \frac{\omega_n - \omega_1}{\omega_1} \times 100\% \quad (11)$$

In this work, six different boundary conditions for double beams like SS-SS, C-C, F-F, C-F, C-SS, and SS-F. The shape functions of that above boundary conditions for the beams are shown in table (1).

Table 1. Shape functions of the beam for different boundary conditions.

Boundary conditions	$\boldsymbol{\phi(x)}$	$\boldsymbol{\alpha_n}$
Clamped to Clamped	$C_n[sinh\beta_n x - sin\beta_n x + \alpha_n(cosh\beta_n x - cos\beta_n x)]$	$\frac{sinh\beta_n l - sin\beta_n l}{cos\beta_n l - cosh\beta_n l}$
Free to Free	$C_n[sin\beta_n x + sinh\beta_n x - \alpha_n(cos\beta_n x + cosh\beta_n x)]$	$\frac{sin\beta_n l - sinh\beta_n l}{cosh\beta_n l - cos\beta_n l}$
Simply-Support to Simply-Support	$C_n sin\beta_n x$	−
Clamped-Free	$C_n[sin\beta_n x - sinh\beta_n x - \alpha_n(cos\beta_n x - cosh\beta_n x)]$	$\frac{sin\beta_n l + sinh\beta_n l}{cos\beta_n l + cosh\beta_n l}$
Simply-Support to Free	$C_n[sin\beta_n x + \alpha_n sinh\beta_n x]$	$\frac{sin\beta_n l}{sinh\beta_n l}$
Clamped to Simply-Support	$C_n[sin\beta_n x - sin\beta_n x + \alpha_n(cosh\beta_n x - cosh\beta_n x)]$	$\frac{sin\beta_n l - sinh\beta_n l}{cos\beta_n l - cosh\beta_n l}$

The first five dimensionless frequencies of a beam from J. Xing [15] as shown below in table (2).

Table 2. The dimensionless frequencies of a beam.

No. of	**F-F**	**S-S**	**C-C**	**C-S**	**C-F**	**S-F**

Frequency						
Ω_1	4.730041	3.141593	4.730039	3.926601	1.875104	3.926602
Ω_2	7.853205	6.283185	7.853195	7.068577	4.69409	7.068581
Ω_3	10.995608	9.424778	10.995581	10.21016	7.854753	10.210171
Ω_4	14.137165	12.566371	14.137109	13.351733	10.995527	13.351757
Ω_5	17.27876	15.707963	17.278656	16.493294	14.13714	16.493339

RESULTS AND DISCUSSIONS

To study the effects of inner layer properties on the vibration of a double beam, the following properties of both beams are used to investigate that effects. The length is 2 m, the width is 10 cm, the thickness is 5 cm, the density is 2700 Kg/m^3, and the modulus of elasticity is 70 Gpa. To estimate the values of frequencies in equations (8) and (9), the values of the parameter (λ) are computed for five modes at six boundary conditions which are utilized in this work as shown in table (3). Table (4) shows the difference between the values of the first five frequencies by using EGP and that by using HPM for six different boundary conditions when the maximum amplitude is 0.2. In that table observed the large difference between the frequencies by the EGP method and by the HPM method not exceed 0.04% and the maximum difference occurred in first frequency for all six boundary conditions.

Table 3. Values of (λ) parameter at different boundary conditions.

No. of Mode	Values of (λ) parameter					
	SS-SS	C-C	F-F	C-F	SS-F	SS-C
1	0.75	1.91844	1.91837	4.35825	0.84389	1.68649
2	0.75	1.68649	1.68649	1.71646	0.80299	1.60607
3	0.75	1.63660	1.63661	1.69540	0.78669	1.57344
4	0.75	1.60610	1.60604	1.63618	0.77806	1.55616
5	0.75	1.58681	1.58677	1.60606	0.77271	1.54545

Table 4. The difference between EGP and HPM methods.

a- Free to Free

No. of mode	ω_{EGP} rad/sec	ω_{HPM} rad/sec	Difference%
1	495.0176	495.1612	0.028991
2	1172.444	1172.497	0.004544
3	2244.990	2245.017	0.001203
4	3688.400	3688.416	0.000437
5	5497.732	5497.743	0.000194

b- simply-supported to Simply-supported

No. of mode	ω_{EGP} rad/sec	ω_{HPM} rad/sec	Difference%
1	426.3907	426.4558	0.015277
2	830.5841	830.6176	0.004026
3	1685.849	1685.865	0.000977
4	2934.526	2934.535	0.000323
5	4556.327	4556.333	0.000134

c- Clamped to Clamped

No. of mode	ω_{EGP} rad/sec	ω_{HPM} rad/sec	Difference%
1	627.049	627.1623	0.018069
2	1234.004	1234.055	0.004102
3	2277.735	2277.762	0.001168
4	3708.400	3708.416	0.000433
5	5511.124	5511.134	0.000193

d- Clamped to Simply-supported

No. of mode	ω_{EGP} rad/sec	ω_{HPM} rad/sec	Difference%
1	615.0564	615.1579	0.01651
2	1074.956	1075.012	0.005148
3	1999.045	1999.074	0.001458
4	3327.232	3327.250	0.000521
5	5033.661	5033.673	0.000226

e- Clamped to Free

No. of mode	ω_{EGP} rad/sec	ω_{HPM} rad/sec	Difference%
1	615.6812	615.9433	0.042574
2	742.2029	742.2885	0.011540
3	1298.753	1298.801	0.003723

f- Simply-supported to Free

No. of mode	ω_{EGP} rad/sec	ω_{HPM} rad/sec	Difference%
1	724.9612	725.0043	0.005947
2	1141.424	1141.45	0.002283
3	2035.566	2035.581	0.000703

4	2313.211	2313.237	0.001132
5	3730.323	3730.339	0.000427

4	3349.312	3349.321	0.000257
5	5048.303	5048.309	0.000112

Therefore, there is no appreciable difference between the values using the two methods, and either of the two methods can be used to calculate the frequencies in the presence of the non-linear elastic layer and with a non-influential error rate. Table (5) shows the effect of the maximum amplitude on the first five frequencies of double beams with a nonlinear elastic layer at two values of C equal to 0.1 and 0.2 for six different boundary conditions. It is observed in this table, that the largest difference occurred in the first frequency for all that boundary conditions when the maximum amplitude increased from 0.1 to 0.2. Thus, the maximum amplitude does not significantly affect the high frequencies of the beams with a non-linear elastic layer. The variation of the values of frequencies with linear stiffness of the inner layer for the double beam at six different boundary conditions is shown in figure (2). In this figure, it was observed that when linear stiffness increased, the frequencies increased in general as well, but this increase is greater for the first frequency, and the increasing decreases relatively with the second frequency and with a lower percentage for the third frequency and so on. Figure (3) shows the variation of increasing frequencies ratio (IFR) with the non-linear stiffness of the inner layer at six different boundary conditions.

Table 5. The difference between the frequencies at different maximum amplitude values.

a- Free to Free

No. of mode	Frequency at C= 0.1	Frequency at C= 0.2	Difference%
1	493.5083	495.0176	0.305848
2	1171.885	1172.444	0.047746
3	2244.706	2244.99	0.012631
4	3688.231	3688.4	0.004591
5	5497.62	5497.732	0.002042

b- simply-supported to Simply-supported

No. of mode	Frequency at C= 0.1	Frequency at C= 0.2	Difference%
1	425.7061	426.3907	0.160812
2	830.2329	830.5841	0.042305
3	1685.676	1685.849	0.010264
4	2934.426	2934.526	0.003387
5	4556.263	4556.327	0.001405

c- Clamped to Clamped

No. of mode	Frequency at C= 0.1	Frequency at C= 0.2	Difference%
1	625.858	627.049	0.190287
2	1233.473	1234.004	0.043098
3	2277.456	2277.735	0.012270
4	3708.232	3708.400	0.004542
5	5511.012	5511.124	0.002032

d- Clamped to Simply-supported

No. of mode	Frequency at C= 0.1	Frequency at C= 0.2	Difference%
1	613.9891	615.0564	0.173824
2	1074.375	1074.956	0.054095
3	1998.739	1999.045	0.015315
4	3327.050	3327.232	0.005467
5	5033.542	5033.661	0.002372

e- Clamped to Free

No. of mode	Frequency at C= 0.1	Frequency at C= 0.2	Difference%
1	612.9221	615.6812	0.450143
2	741.303	742.2029	0.121396
3	1298.245	1298.753	0.039111
4	2312.936	2313.211	0.011893
5	3730.156	3730.323	0.004489

f- Simply-supported to Free

No. of mode	Frequency at C= 0.1	Frequency at C= 0.2	Difference%
1	724.5084	724.9612	0.062501
2	1141.15	1141.424	0.023977
3	2035.416	2035.566	0.007384
4	3349.222	3349.312	0.002697
5	5048.244	5048.303	0.001179

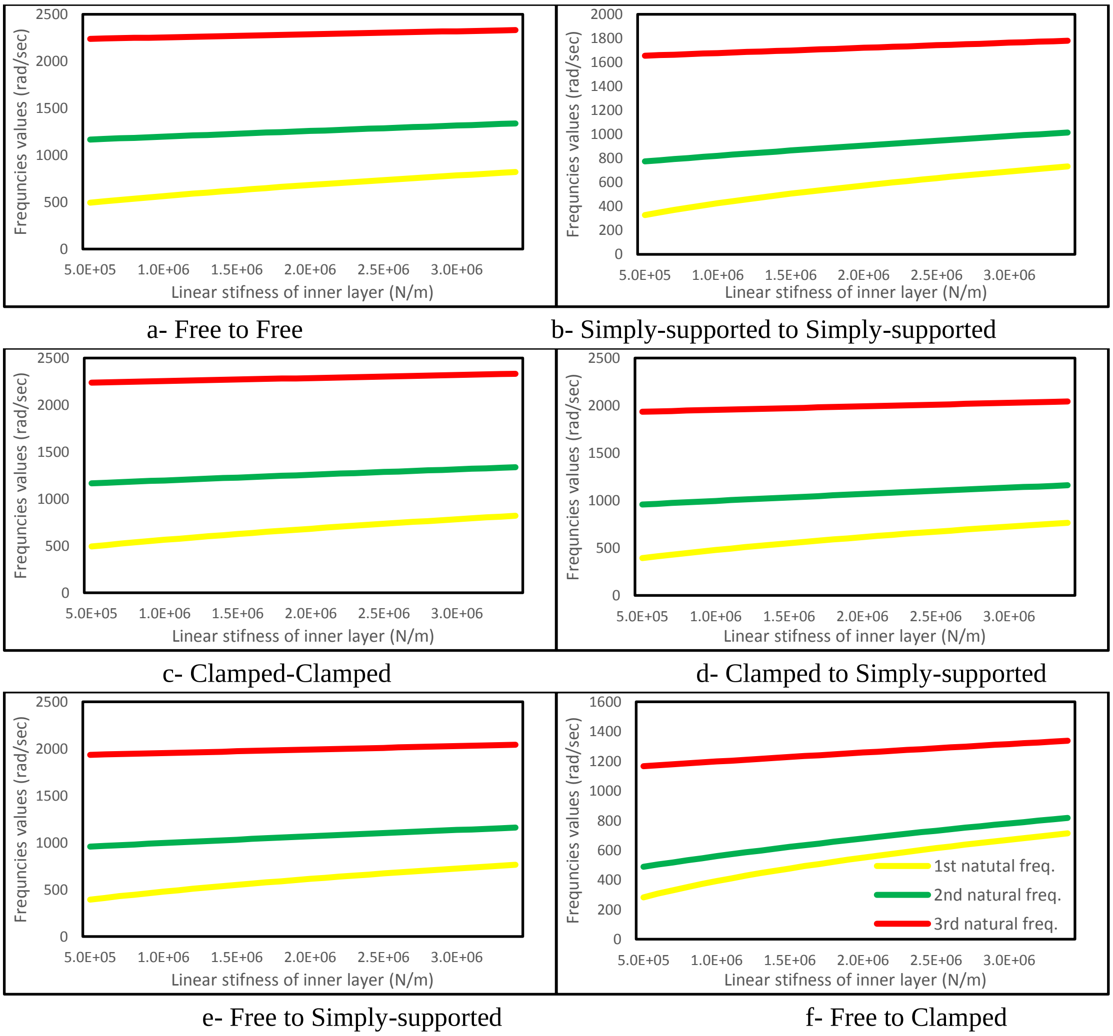
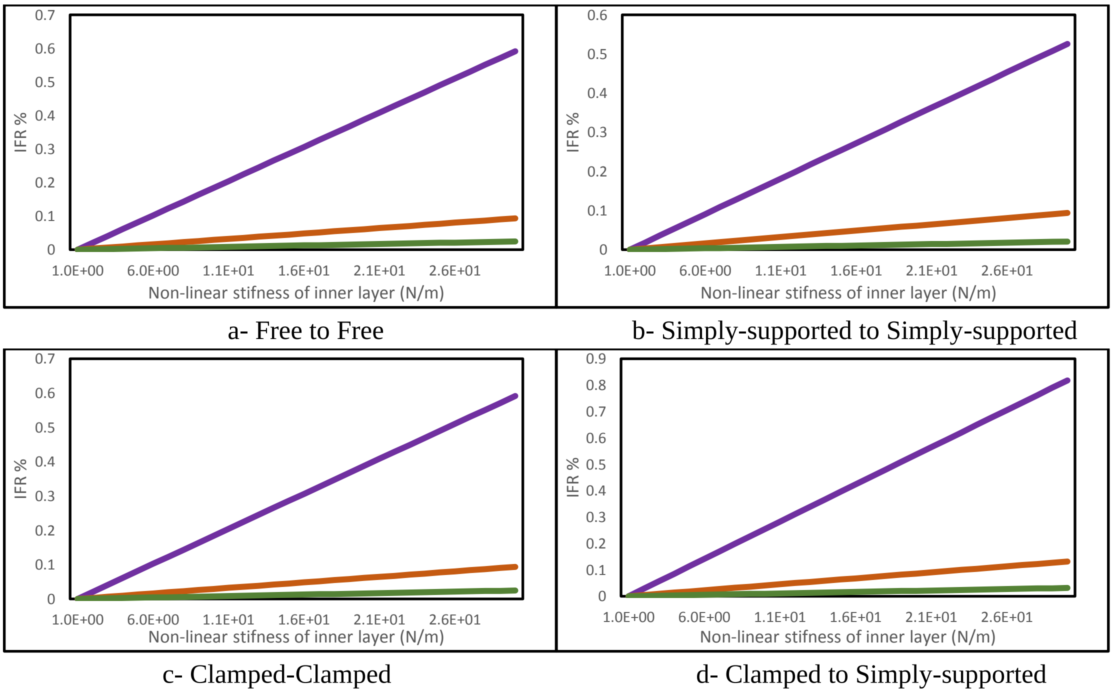
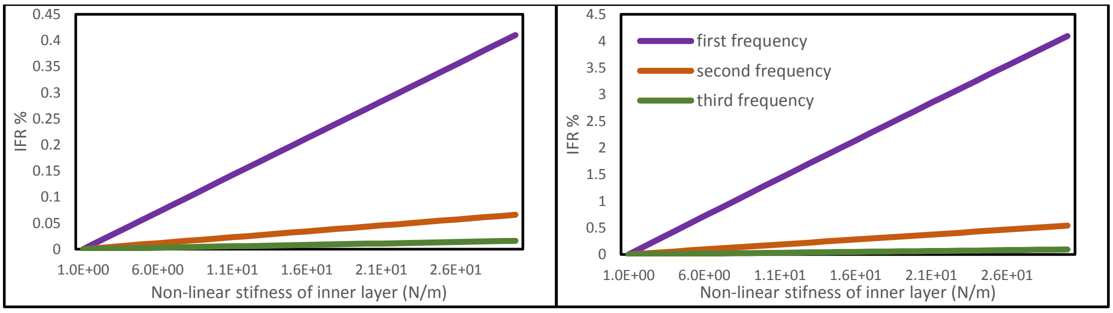


Figure 2. The variation of the natural frequencies with linear stiffness of the inner layer.

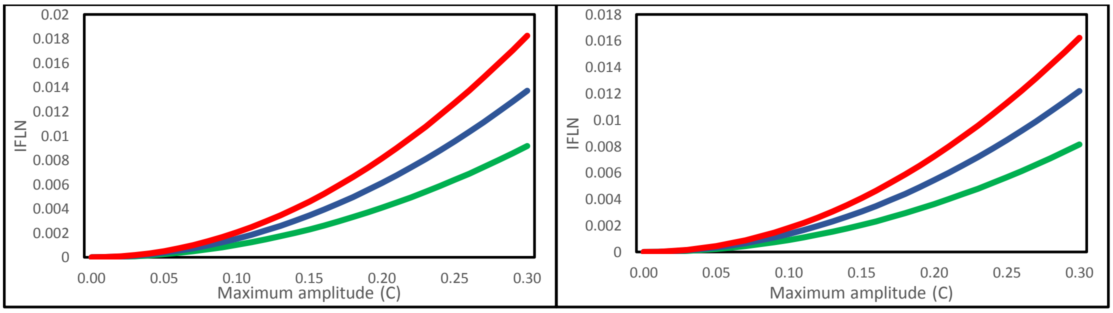




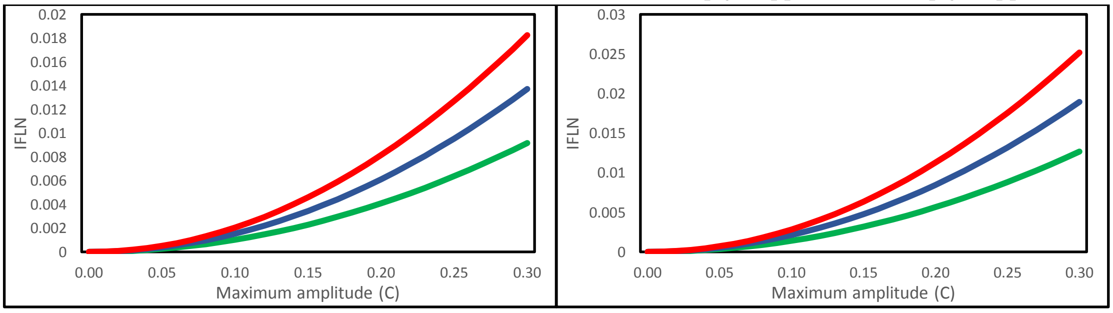
e- Free to Simply-supported　　f- Free to Clamped

Figure 3. The variation of the frequencies with increasing in non-linear stiffness values of the elastic layer.

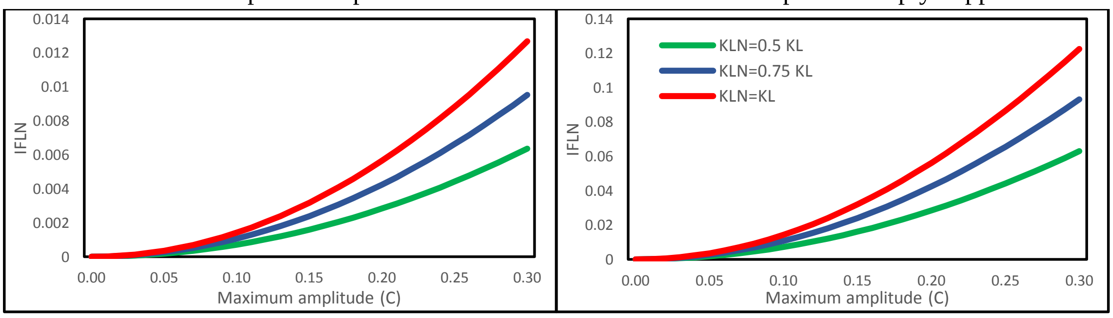
In this figure, noticed at the boundary conditions SS-SS, F-F, C-C, C-SS, and F-F, when the stiffness increased from $5 \times 10^4\ N/m$ to $150 \times 10^4 N/m$, The first frequency increased by less than 1% at the above boundary conditions. While the second frequency increased by less than 0.1%, and for the third frequency the increase rate was much less. Also noticed at boundary condition free to clamped, when the stiffness increased from $5 \times 10^4\ N/m$ to $150 \times 10^4 N/m$, the increase in the first frequency was 4%, the second frequency was 0.4%, and the third frequency was less than 0.1%. Generally, no significant effect was observed for the increase in the value of non-linear elasticity at high frequencies from the second upwards. Also, its effect was relatively small on the first frequency values under different boundary conditions. Figure (5) shows the variation of IFNL with the increase in values of the maximum amplitude for three values of the stiffness of the non-linear layer at six different boundary conditions. In this figure, it is observed that when increasing the values of the maximum amplitude, the IFNL values increase exponentially, and the tendency of this exponential increase increases with the increase in the values of the non-linear elasticity of the inner layer and of all boundary conditions.



a- Free to Free　　b- Simply-supported to Simply-supported



c- Clamped-Clamped　　d- Clamped to Simply-supported



e- Free to Simply-supported f- Free to Clamped

Figure 4. The variation of the

CONCLUSIONS

The effect of the non-linearity stiffness of the inner layer on frequencies of double beams are computed in this work by using the energy balance method based on Galerkin-Petrov (EGP) and Homotopy Perturbation Method (HPM), There is no significant difference in the frequencies computed by both methods, with very little difference between them. Increasing the linear elasticity values of the inner layer causes an increase in frequencies, especially for the first frequency, and this effect is less than the second frequency and upwards. While increasing the values of the nonlinear elasticity of the inner layer causes a very slight increase in the values of the frequencies of the double beams. The increase in the maximum amplitude values affected the increase in the first frequency of the double beams, and its effect increases by increasing the values of the non-linear elasticity of the inner layer, and its effect is limited on the second frequency and upwards. The above conclusions are obtained at different ambient conditions for double beams like SS-SS, C-C, F-F, C-F, C-SS, and SS-F.

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