Mixed Convection Heat Transfer Of CuO-H2O Nanofluid In A Triangular Lid-Driven Cavity With Circular Inner Body

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ABSTRACT: Combined free-forced convection heat transfer of CuO-H2O nanofluid in a triangular cavity having an insulated circular body has been numerically studied. The top base wall is isothermally heated at T_h and moved at a constant speed. While the other walls of triangle are maintained at a cold temperature T_c. Fluent 6.3 commercial program is used to solve the mass, momentum, and energy equations subjected to dimensionless boundary conditions. The present results are compared with the previous results for validation and it is found to be in an excellent agreement. The values of Richardson number taken in this study are 0.1, 1.0, 10. The temperature gradient in Grashof number is fixed at ΔT =10 K. The range of nanoparticles volume fraction extends from ϕ = 0 to 0.15. It is concluded that the overall and average heat transfer rate increase with increase in nanoparticles volume fraction and decrease in Richardson number.

KEYWORDS: mixed convection, triangular cavity, lid-driven, adiabatic body.

INTRODUCTION

Recently, mixed convection in lid-driven cavity has received significant attention in different engineering applications such as compact heat exchangers, nuclear reactors, solar collectors, cooling of electronic, and so on [1]. The fluid flow is caused by moving one or further one wall in the cavity. While, the buoyancy forces cause moving the fluid due to a temperature gradient between the hot source and cold walls. In last decades, many authors studied dominating forced, natural, and mixed convection in a different geometry of cavity with various thermal boundary conditions. Different heat transfer enhancement methods were used to study the problem of mixed convection in lid-driven cavity. Increasing the thermal conductivity of the base fluid by using nanofluids and porous medium is one of the effective of these methods and it has received a significant attention in the previous literature [1–6]. Other methods include applying the magnetic field or vibration in one direction or more with or without nanofluids [7-12], using the corrugated cavity or corrugated inner heat source, angle of inclination of cavity or heat source, position and geometry of heat source [13-16], etc. The square cavity was studied thoroughly by several researchers. The results showed that as Reynolds number increase inside the cavity, the Nusselt number is also increased. From the inlet opening of the cavity to the exit, Nusselt number seemed to decrease. Increasing the value of Richardson number at the same value of Reynolds number increases the Nusselt number within the cavity [17-18]. In the past decade, several investigations on combined convection heat transfer in lid-driven cavity have been carried out. Many geometries of the lid-driven cavities had been studied such as circle, wavy, square, rectangle, and triangle; with different thermal boundary conditions enhancement techniques. Combined convection flow in lid-driven cavities happens as a result of shear flow caused by the lid movement and buoyancy flow produced by non-homogeneity of the cavity thermal boundaries [19-21]. The natural convection in a concentric annulus between a cold outer inclined square enclosure and heated inner circular cylinder is simulated for two-dimensional steady state. It is found that both the aspect ratio and the Rayleigh number are critical to the patterns of flow and thermal fields. At all Rayleigh numbers angle of inclination has nominal effect on heat transfer [22-24].
The present work comprises a numerical study of mixed convection heat transfer in a triangular lid-driven cavity containing an insulated circular body. The cavity is filled with H$_2$O-CuO nanofluid. The base of triangle is located at the top of enclosure. It represents a lid-driven of cavity. It is heated isothermally at temperature $T_h$. While, the inclined walls of triangular cavity are cooled isothermally at $T_c$. Fluent 6.3 commercial program is used to solve the continuity, momentum, and energy equations for the present work. The values of Richardson number taken in this study are 0.1, 1.0, 10. The temperature gradient in Grashof number is fixed at $\Delta T = 10$ K. The range of nanoparticles volume fraction extends from $\phi = 0$ to 0.15. The aim of present work is to study the influences of Richardson number (Ri) and nanoparticles volume fraction ($\phi$) on the streamlines, isotherms, local and average Nusselt number on the moving hot wall, overall Nusselt number of the cavity, and skin friction factor.

MATHEMATICAL MODEL

Governing Equations

A steady 2-dimensional, laminar flow, and combined convection heat transfer of CuO-H$_2$O nanofluid in a triangular lid-driven cavity containing an adiabatic inner circular body has been studied numerically using Fluent commercial program. The horizontal top wall of cavity is heated at constant temperature $T_h$ and moving at constant speed $U$. While, the inclined walls of the triangular cavity are cooled at a constant temperature $T_c$ such that $T_h > T_c$ as schematically shown in Fig. 1. The H$_2$O-CuO nanofluid is taken as a single phase. Table 1 shows the thermo-physical properties for the CuO nanoparticles and the water at $T=300$ K. The governing equations of continuity, momentum, and energy are given below:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re \ \rho_{eff} \ \mu_{eff}} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \quad (2)
\]

\[
U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re \ \rho_{eff} \ \mu_{eff}} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + \frac{(\rho \beta)_{eff}}{\rho_{eff}} \frac{Gr \ \theta}{Re^2} \quad (3)
\]

\[
U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{1}{Re \ \Pr \ \alpha_{eff} \ \rho_{eff}} \left[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] \quad (4)
\]

The above equations (1-4) are non-dimensionalized as given below:

\[
U = \frac{u}{U_\infty}, \quad V = \frac{v}{U_\infty}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad P = \frac{p}{\rho_{eff} U_\infty^2} \quad (5)
\]

\[
Re = \frac{U_\infty L}{\mu_f}, \quad Pr = \frac{\alpha_f}{\mu_f}, \quad Gr = \frac{g \beta L^3 (T_h - T_c)}{\mu^2}, \quad Ri = \frac{Gr}{Re^2} \quad (6)
\]
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Table 1. Thermophysical properties of H2O and CuO nanoparticles [25]

<table>
<thead>
<tr>
<th>Physical properies</th>
<th>H2O</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$ (J/kg.K)</td>
<td>4179</td>
<td>535.6</td>
</tr>
<tr>
<td>$k$ (W/m².K)</td>
<td>0.613</td>
<td>20</td>
</tr>
<tr>
<td>$\mu$ (Pa.s)</td>
<td>0.000891</td>
<td>-</td>
</tr>
<tr>
<td>$\rho$ (kg/m³)</td>
<td>997.1</td>
<td>6500</td>
</tr>
<tr>
<td>$\beta$ (1/K)</td>
<td>0.00021</td>
<td>4.3×10⁻⁶</td>
</tr>
</tbody>
</table>

Boundary conditions

The dimensionless boundary conditions can be written as follows:

\[
\frac{\partial \theta}{\partial Y} = 0, \quad U = V = 0, \quad X = 0
\]

\[
\frac{\partial \theta}{\partial Y} = 0, \quad U = V = 0, \quad X = 1
\]

\[\theta = 0, \quad U = V = 0, \quad Y = 0,\]

\[\theta = 1, \quad U = 1, V = 0, \quad Y = 1\]  (7)

Thermophysical Properties of Nanofluid

The H2O-CuO nanofluid is regarded as a single phase in the present work. Therefore; the water-based fluid and the Cu nanoparticles are together in thermal equilibrium. As a result, the effective thermophysical properties are adopted in this study. The general equations of the effective thermophysical properties have been developed and implemented in Fluent as follows [26].

\[\rho_{eff} = (1 - \varphi_p) \rho_f + \varphi_p \rho_s\]  (8)

\[\mu_{eff} = (1.125 - 0.0007 \times T) \mu_f,\]

\[\varphi_p = 1\% \quad 20 \leq T[\degree C] \leq 70\]  (9)

\[\mu_{eff} = (2.1275 - 0.0215 \times T + 0.0002 \times T^2) \mu_f,\]

\[\varphi_p = 4\% \quad 20 \leq T[\degree C] \leq 70,\]  (10)

\[\frac{k_{eff}}{k_f} = 1.0 + 1.0112\varphi_p + 2.437\varphi_p \left(\frac{d_p}{\eta_p(nm)}\right) - 0.0248\varphi_p \left(\frac{k_p}{0.613}\right),\]

\[\beta_{eff} = (-0.479\varphi_p + 9.3158 \times 10^{-3}T - \frac{4.7211}{T^2}) \times 10^{-3}\]
\[ 0 \leq \varphi_p \leq 0.04 \quad 10 \leq T[^\circ C] \leq 40, \]  
\[ \rho_{\text{eff}} = \frac{(1-\varphi_p)\rho_f + \varphi_p\rho_s\rho_p}{\rho_{\text{eff}}}, \]  
\[ (12) \]

Numerical Solution

The governing equations subject to considered boundary conditions are solved by using Fluent 6.3 commercial program. The grid independency is shown in Fig. 2. The number of grids used in the present study is 8,281. Laminar model and time independent solver are used in this study. The coupling of pressure-velocity equations is resulted from using simple scheme. A second order upwind scheme is appropriate to use for the mixed convective heat transfer. The thermo-physical properties given above are existing in Fluent software. The simulation is terminated as the residuals for continuity and momentum equations attain $10^{-6}$ and the residual for energy equation attains $10^{-8}$. The local Nusselt number based on the length of the top wall of triangular cavity is written as:

\[ Nu_L = \frac{k_{\text{eff}}}{k_f} \left. \frac{\partial \theta}{\partial y} \right|_{y=1} \]  
\[ (14) \]

The average Nusselt number of the hot horizontal lid-driven wall is given as follows:

\[ Nu_m = \int_0^1 Nu_L(X) \, dX \]  
\[ (15) \]

The local friction factor \( f_x \) on the moving lid related to the local shear stress \( \tau_x \) is calculated as follows:

\[ C_f = \frac{\tau_x}{\rho u_s^2} = \left. \frac{\mu}{\rho u_s^2} \frac{\partial u}{\partial y} \right|_{\text{moving lid}} \]  
\[ (16) \]

**Figure 2.** Grid independency

**VALIDATION**

The present results are validated by compared the present numerical results with the numerical results worked by Billah et al. [27] as shown in Fig. 3. The figure shows the variation of average Nusselt number with Grashof number.
for different nanoparticles volume fractions. It describes a case study of mixed convection heat transfer in a triangular lid-driven cavity with inner circular body. The solid line represents the present results while the dashed line represents Billah et al. work [27]. It can be seen that the results of two works are similar to each other with a very small difference.

Figure 3 Comparison the present results with work of Billah et al. [27]

RESULTS AND DISCUSSION

Streamlines and isotherms

The present study is carried out for H$_2$O-CuO nanofluid as working fluid. Prandtl number is 6.2. The aim is to investigate the effects of controlling parameters such as and Richardson number (Ri) and the nanoparticles volume fraction ($\varphi$). The range of nanoparticles volume fractions extends from 0 to 0.15. The effect of Richardson number on streamlines and isotherms in a triangular lid-driven cavity filled with water only (solid black line $\varphi = 0.0$) or H$_2$O-CuO nanofluid (dashed red line $\varphi = 0.15$) and containing an adiabatic circular body is shown in Fig.4. Three Richardson numbers are used which are: Ri=0.1, 1.0, and 10; respectively.

It is shown that, the circulating cells are generated due to the lid dragging near the working fluid. It is noticed that, small vortices are formed near the left side of circular body at Ri=0.01. Then it begins to pulled toward the right side of the upper region of enclosure as Richardson number Ri increases to 10. There is no doubt that the heated nanofluid moves down from the hot top wall to enclosure space. The lid-driven generates vortices rotating in clock wise direction and two small vortices are formed on the left circular body and on the right corner of triangular enclosure. These two small vortices emerge together and deviates towards the upper right region of enclosure as Richardson number increases to 10. Also, this figure indicates that the isotherms move from the heat source to left side of the enclosure. The strength of stream function increases with decrease in Richardson number to 0.1 because the dominating forced convection in the heat transfer process. The values of stream functions for Ri=0.1, 1, and 10 are 0.392, 0.064, and 0.021 for the water working fluid; respectively. The maximum values of stream functions for the CuO-water nanofluid are the same as water-based fluid except at high Richardson number (Ri=10) at which $\psi_{max}$ increases to 0.037.

The isotherms represent the lines with equal intervals between hot horizontal wall and the inclined cold walls. The isotherms are regulated near the inclined walls causing a steep temperature gradient at these regions. The inner circular body causes retarding of flow and disturbing the thermal fields. The isotherm lines converge to each other on the left side and diverge on the right side of cavity as Richardson number decreases because od dominating forced convection in the heat transfer process on the right-hand side and dominating conduction heat transfer on the left-hand side. Moreover, the isotherm lines deviate towards right side at Ri=10 with using of nanofluid because increase in thermal
conductivity of the based fluid after adding the CuO-nanoparticles, and dominating natural convection on the heat transfer process.

![Streamline and temperature contour for lid driven cavity with nanofluid with adiabatic circular body](image)

Figure 4. Streamline (left) and temperature contour (right) for lid driven cavity with nanofluid with adiabatic circular body (solid black line $\varphi=0$, and dashed red line $\varphi = 0.15$).

Nusselt numbers

The local Nusselt number variation along the hot horizontal top wall length of triangular cavity at different nanoparticles volume fractions ($\varphi = 0, 0.04, 0.1$) for $Ri=0.1$, 1, and 10 are shown in Figures 5-7; respectively. It is shown that the local Nusselt number increases at a thin region near the left inclined wall for $Ri=0.01$ because of the dominating forced convection in the heat transfer process. Then, it begins to decrease sharply until reaches a minimum value near the right end of moving wall after which it returns to increase because of decreasing the thermal boundary layer thickness towards the direction of motion. The increasing of local Nusselt number near the left inclined wall diminishes with increase in Richardson number. In this case, the maximum local Nusselt number occurs at left upper corner of cavity. Moreover, it is observed that the local Nusselt number increases as the nanoparticles volume fraction increase. This increasing of local Nusselt number is reduced as Richardson number increases. The local Nusselt number begins to increase again close to right end of moving wall. The last increasing is reduced with decrease in Richardson number.
The variation of overall Nusselt number in the triangular cavity and the average Nusselt number for the moving hot top wall versus the volume fraction of nanoparticles for different Richardson number are shown in Figures 8 and 9, respectively. It is obvious that the overall and average heat transfer rate increase with increase in nanoparticles volume fraction and decrease in Richardson number because of dominating forced convection in the heat transfer process. The average Nusselt number values on the moving hot wall are more than twice the overall Nusselt number in the triangular cavity.

Figure 5. Variation of local Nusselt number along the moving wall for $Ri=0.01$.

Figure 6. Variation of local Nusselt number along the moving wall for $Ri=1$. 
Figure 7. Variation of local Nusselt number along the moving wall for $Ri=10$.

Figure 8. Overall Nusselt number variation in the triangular cavity with variation of nanoparticles volume fraction.
Figure 9. Average Nusselt number variation on hot wall in the triangular cavity with variation of nanoparticles volume fraction.

Skin friction factor

The average skin friction factor on the moving hot wall in the triangular cavity is varied with changing the nanoparticles volume fraction for all values of Richardson number as shown in Fig. 10. It is noticed that, the skin friction factor increases slightly with increase in nanoparticles volume fraction because increasing the thermal conductivity of the based fluid by adding nanoparticles which retard the flow. Moreover, the skin friction factor increases with decrease in Richardson number in the cavity because of increasing the shear force on the moving hot wall resulted from increasing the forced convection relative to natural convection.

Figure 10. Variation of average skin friction factor on moving hot wall in the triangular cavity versus nanoparticles volume fraction.
CONCLUSIONS

1. The strength of stream function increases with decrease in Richardson number.

2. The isotherms are regulated near the inclined walls causing a steep temperature gradient at these regions.

3. The local Nusselt number increases at a thin region near the left inclined wall for $Ri=0.01$. The local Nusselt number increases as the nanoparticles volume fraction increases. This increasing of local Nusselt number is reduced as Richardson number increases.

4. The average Nusselt number values on the moving hot wall are more than twice the overall Nusselt number in the triangular cavity.

5. The skin friction factor increases slightly with increase in nanoparticles volume fraction and decrease in Richardson number.

Nomenclature

- $u, v$: velocity components in x- and y-directions, respectively
- $g$: gravitational acceleration
- $L$: reference length
- $Gr$: Grashof number
- $h$: overall heat transfer coefficient
- $h_x$: local heat transfer coefficient
- $J$: Jacobian of coordinate transformation
- $k$: thermal conductivity of fluid
- $N$: number of corrugations
- $Nu_L$: Local Nusselt number
- $Nu_m$: mean Nusselt number
- $Pr$: Prandtl number
- $Re$: Reynolds number
- $Ri$: Richardson number ($Ri = Gr/Re^2$)
- $T$: fluid temperature
- $T_h$: higher temperature at side walls
- $T_l$: lower temperature at base wall
- $\alpha$: fluid thermal diffusivity
- $\beta$: coefficient of volume expansion for fluid
- $\lambda$: amplitude
- $\phi$: Solid volume fraction
- $\mu$: dynamic viscosity of fluid
- $\nu$: kinematic viscosity of fluid
- $\tau_x$: local shear stress
- $\text{Subscript}$
- $\text{f}$: fluid
- $\text{p}$: nanoparticles
- $\text{eff}$: effective

REFERENCES


