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Free Vibration of Rectangular Plates with Flanged Obround Opening

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ABSTRACT: Converting an obround opening into a flange in a rectangular plate was studied in this work. The effect of various flange parameters on the plate fundamental frequency were numerically investigated. Increasing the flange length and flange width increases the frequency for all edge supporting conditions, but the sensitivity to changing the length is higher than changing the width. For fully clamped plate the frequency can be increase 1.74 times the natural frequency of the bear plate. When considering flange location and orientation, the maximum frequency is achieved at the plate center at 45° for the fully clamped and simply supported plates, while it is achieved for the cantilever plate near the free edge and 90° with the clamped edge.

KEYWORDS: Obround, Solidworks, Rectangular plate, free vibration, flanged opening.

INTRODUCTION

Plates are important products because they are used in many engineering applications. We find them in marine ships, spacecraft, structures and bridges, and they are also found in the electronics industry, home appliances and other applications in which the weight and rigidity specifications are of particular importance. Structural plates, pressure vessels, heat exchangers, and nuclear power equipment are often provided with openings to reduce weight of the structure or improve its performance. Openings may also be added to assist in assembly of structure parts or perform maintenance for these various parts. The addition of these openings helps in reducing the weight of the structure, but it changes the plate mechanical properties such as stiffness and natural frequency. To overcome this issue, the plate or these opening are provided with stiffener (most of time by welding); but this again increases the mass of the structure and another issue arises and represented by the welding defects and stress concentration. Different analytical, numerical and experimental procedures were used to estimate free vibration of rectangular plates containing central openings with different shapes (rectangular, elliptical, circular, and oval shapes) [1] or rectangular plates with circular openings distributed in rectangular pattern [2].

Factors other than the shape can affect the plate fundamental frequency; for instance, the cutout area when it is increasing it increases the plate frequency [3]. When considering the edge supporting conditions, the clamped edge results in higher plate fundamental frequency when compared with simply supported edges [4]. Finally, the cut-out position can significantly affect the value of the frequency [5]. Vibration of plates with stiffeners was investigated too with various mathematical and numerical techniques. The Mindlin theory was applied for the plate while Timoshenko beam theory was applied for stiffeners to estimate the fundamental frequency of stiffened plates having rectangular, oval and circular cutouts subjected to different edge supporting conditions [6]. For clamped and simply supported plates, the presence of small radius circular hole decreases the fundamental frequency while the later increases with large holes [7]. The hole affects the mass and stiffness of the plate and hence its fundamental frequency [8]. Temperature is another factor that affects the free vibration of stiffened composite plates, increasing temperature reduces the plate fundamental frequency [9]. Finite element method was employed to investigate the effect of stiffener parameters (type, number, and orientation) on the free vibration of stiffened plates [10].

Numerical model for rectangular plate having a flanging hyper-ellipse cutout was proposed to study the influence of different cutout parameters on the fundamental frequency of the plate [11]. Mathematical models, one for Functionally graded plate having a curvilinear stiffener and a central cutout [12], and another for a cracked stiffened flat plate [13] showed that the plate fundamental frequency increases with increasing the cutout radius and stiffener size, while it decreases with increasing the thickness of plate. For a rectangular pate with all sides clamped and having a circular hole, converting the inner edges of this hole into a flange increases the plate stiffness and hence the fundamental frequency about (27%) higher than its value for the plate with hole only [14], therefore it is preferable to add a stiffener for the plate to increase its stiffness rather than increasing its mass [15]. Since stiffened plates are widely used in various applications and are constantly exposed to dynamic loads during their operation period, they may fail due to vibration and resonance; therefore, it is necessary to know the natural frequencies of these plates accurately. The current work is concerned with shape modification of a rectangular plate containing an obround cut area and studying the effect of converting the cut area to a flange on the natural frequency behavior of the plate.

THEORY AND MATHEMATICAL MODEL

In marine, wind turbine, and aerospace structures, obround openings are commonly appears and serves as windows and doors. To conduct the vibration analysis of the structure, we can treat this opening as it represents a combination of two semicircular cut located at two opposite sides of a rectangular opening.

Modeling the Rectangular Plate with Flange

The proposed flanged model is shown in Figure 1-a, it is an obround cut (also known as stadium) at the center of a thin rectangular plate. The obround is a two-dimensional geometric shape consist of a rectangular part which have dimensions of (L * W), two semicircular parts with radius (R), and an inclination angle θ relative to x-axis.

The obround center is located at point (x, y) from the plate corner.

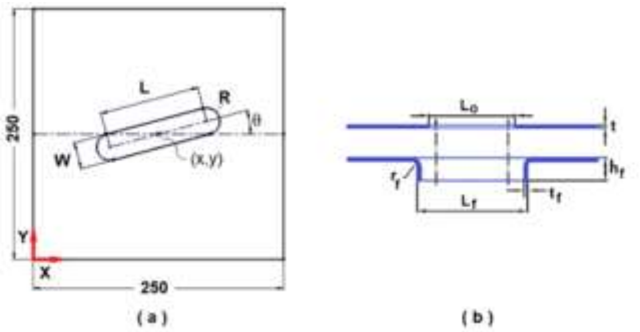

Figure 1. Schematic representation of obround flange

The obround area is given by:

$$A_o = (L * W) + \pi R^2$$

Or

$$A_o = 2LR + \pi R^2 \tag{1}$$

L_o = obround total length = L + 2R

The inner edges of the obround will be bend and stretch to form the flange with the dimensions shown in Fig. 1-b. The diameter of the obround semicircle part before flanging equals to [16]:

$$d = W_f - 2\,h_f - 0.86\,r_f - 1.43\,t \tag{2}$$

The flange wall thickness is given by:

$$t_f = t * \sqrt{\frac{d}{W_f}} \tag{3}$$

Numerical Modeling

SolidWorks 2018 was used to model an Aluminum 2024-T4 square plate as a planar surface with dimensions 250 x 250 x 1mm and mechanical properties (E= 72400N/mm^2, ρ= 2780Kg/m^3, and v= 0.33). Shell element was used to mesh the plate, the number of nodes was 9596 (each node has six degrees of freedom), and element size was

5mm which gives 4630 elements. As boundary conditions, the edges of the plate can be free (F), simply support (S), or clamped (C). Starting from the edge oriented along the x-axis and moving clockwise, we will use four letters to define three different supporting boundary conditions (CCCC, SSSS, and CFFF). Three plates were modeled for each supporting boundary condition, one plate without any cut area to validate the model and find the fundamental natural frequency. The second plate contains the obround cut to study the effect of this cut area on the fundamental frequency, and the third plate contains the flanged obround to study the effect of the flange different parameters on the fundamental frequency as shown in Figure 2.

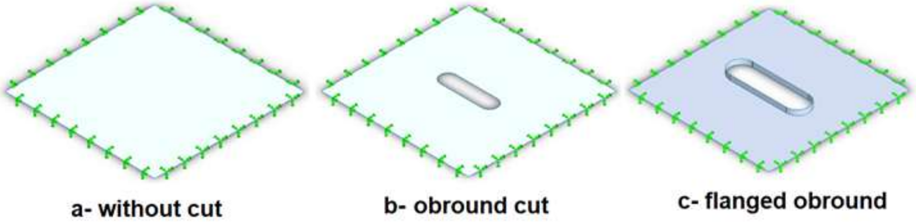
a- without cut
b- obround cut
c- flanged obround

Figure 2. Different models of plate

RESULTS AND DISCUSSION

Different parameters of the flanged obround opening were studied to observe the effect of their variation on the fundamental frequency of the rectangular plate.

Model Validation

To validate the plate model, the fundamental natural frequency for the rectangular plate without any cut was estimated first using different edge supporting conditions. Table 1 lists the results obtained from Solidworks. It is clear from the results that the model results are in good agreement with the different researchers.

Table 1. Fundamental natural frequency f_o (Hz) for the rectangular plate

	Leissa [17]	Blevins [18]	Deutsch [19]	Solidworks
CCCC	143.034	143.026	143.007	143
SSSS	78.445	78..448	78.443	78.442
CFFF	13.876	13.877	13.794	13.753

Vibration Analysis for Plates with Obround Cut

To study the effect of cutting a central obround shape on the plate fundamental frequency, we will consider different obround dimensions and investigate their effect on fundamental frequency at different edge supporting conditions.

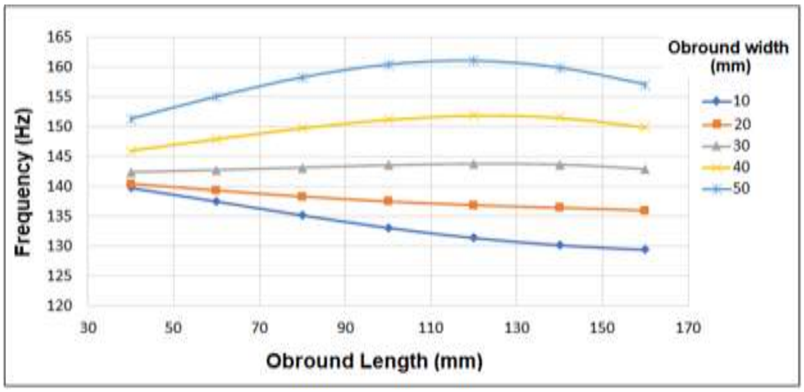

Figure 3. Fundamental frequency f_{1c} for CCCC plate with obround cut

For the CCCC plates, when the obround width is small (10mm), increasing the obround length decreases the fundamental frequency as shown in Figure 3, in this case the obround will act as a crack which decreases the plate stiffness. For higher obround width (40mm), f_{1c} is increasing with increasing the obround length and it reaches a value that is greater than the plate fundamental natural frequency (f_{1c} =143.78Hz). Increasing the obround length reduces the plate mass and this in turn increases the fundamental frequency up to a certain value then f_{1c} drops again as the effect of decreasing plate stiffness will be more dominant than the effect of decreasing mass of plate.

The maximum fundamental frequency (f_{1c} =161.041Hz) is achieved at obround dimensions (120 * 50mm), this means that f_{1c} >1.125 f_o in spite of removing (12.74 %) from the original plate area.

For SSSS plates, cutting the obround in any dimension results in decreasing the fundamental frequency and $f_{1c} < f_o$. it can be noted from Figure 4 that when the obround width is small (10mm), increasing the obround length decreases f_{1c} rapidly, and for higher obround width, increasing the slot length has a small effect on f_{1c} and the reduction in frequency is about (1.55%) for obround width 50mm.

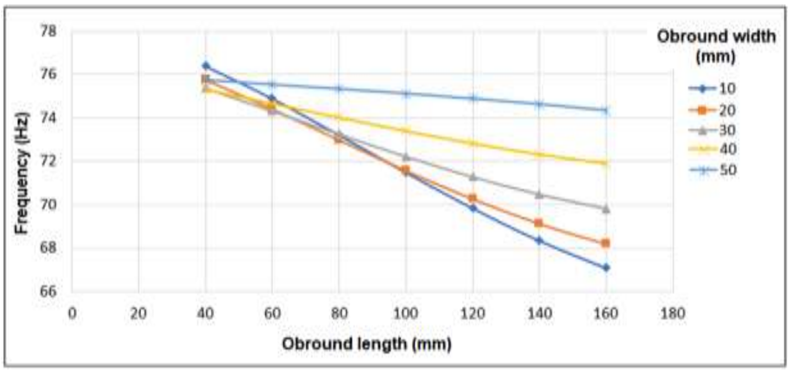

Figure 4. Fundamental frequency f_{1c} for SSSS plate with obround cut

Figure 5 shows the fundamental frequency for CFFF plate having a horizontal obround parallel to the clamped edge. Increasing the obround, length decreases the frequency for all obround dimensions; however, the reduction percentage increases as the obround width increases. For 10mm obround width, the reduction in the frequency between 10 and 160mm obround length is 10.1%; while for 50mm obround width, the reduction in frequency is 23.2%.

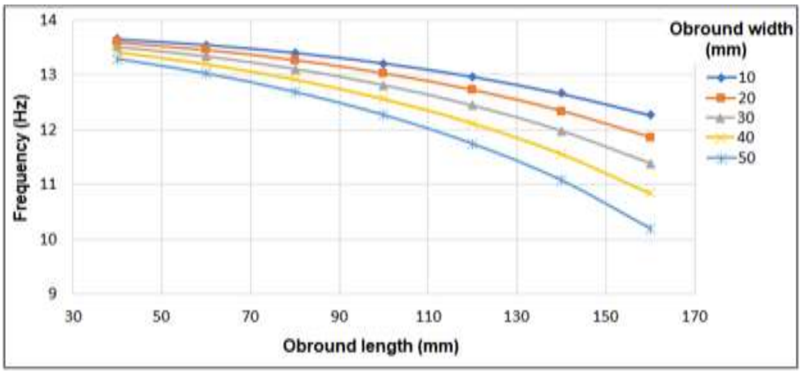

Figure 5. Fundamental frequency f_{1c} for CFFF plate with horizontal obround cut

For CFFF plate with a vertical obround cut (perpendicular to the clamped edge), again increasing the obround area decreases the fundamental frequency as shown in Figure 6, but the percentage reduction is less when compared with the horizontal obround. For obround of 50mm width and when it is longer than 140mm, the fundamental frequency starts to increase and this is due to the closeness of the obround edge from the clamped edge.

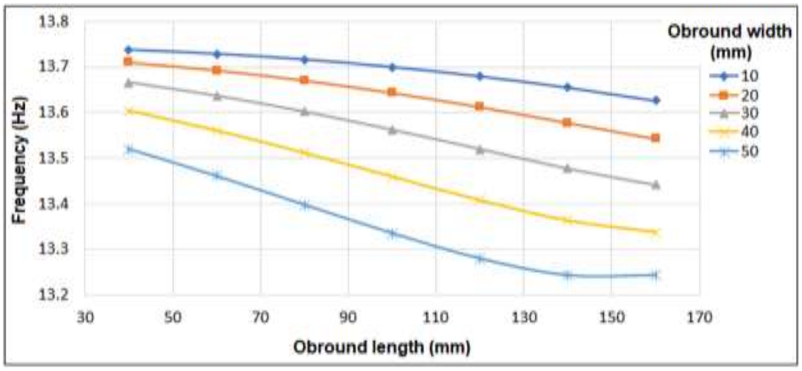

Figure 6. Fundamental frequency f_{1c} for CFFF plate with vertical obround cut

Vibration Analysis for Plates with Stiffening Flange

To study the effect of converting the obround cut into a stiffening flange on the plate fundamental frequency, a single flange will be created at the plate center. Different flange parameters (length, width, height, location, and orientation) will be considered to investigate their effect on fundamental frequency.

Taking $r_f = 3mm$, $t = 1mm$, $h_f = 0.3W_f$ and using equations 2 and 3, we can obtain the required obround diameter to produce the flange and its resulting thickness.

Effect of Flange Length

To study the effect of flange length on plate fundamental frequency, the flange length will be within the range (40 to 140mm). Considering the flange width equal to 30mm and using equation (2), the initial obround circular part diameter equals 16mm. Other flange parameters are kept constant (height=9mm, thickness= 0.73mm, orientation angle= 0°). Figure 7 shows that the fundamental frequency can be increased by converting the obround cut into a flange for all the three supporting conditions. For the CCCC and SSSS edge supporting conditions, the fundamental frequency is sensitive to the flange length and the maximum increase in this frequency occur when the flange length is maximum. The maximum fundamental frequency for CCCC is $f_{1f} = 1.74\ f_o$ and for the SSSS plate is $f_{1f} = 1.56\ f_o$ respectively. For the CFFF edge supporting condition, although the fundamental frequency decreases with increasing the length; its value ($f_{1f} = 12.91$Hz) is higher than that of the obround cut ($f_{1f} = 12.48$ Hz) and getting close to f_o (13.753 Hz).

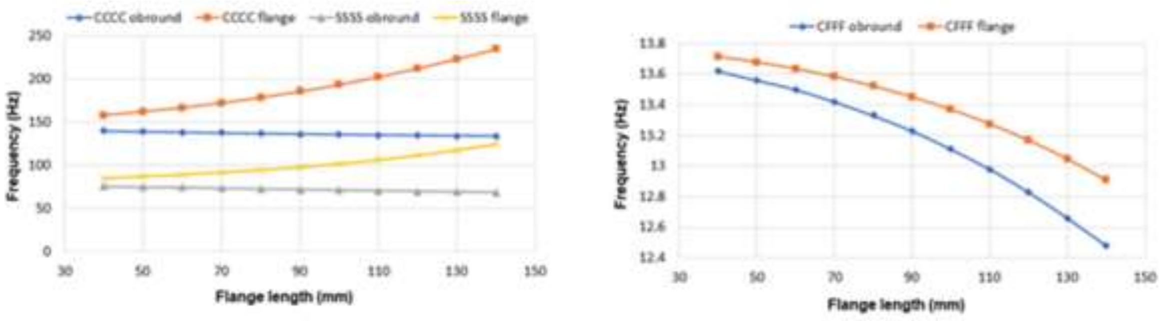

Figure 7. Fundamental frequency variation with flange length

Effect of Flange Width

This time the slot width will vary in the range (30 to 50mm) and the other flange parameters are kept constant (Length = 70mm, height = 9mm, thickness = 0.73mm, and orientation angle = 0°)

Figure 8 shows that converting the obround cut into a flange and increasing the flange width increases the fundamental frequency for all the three supporting boundary conditions and the highest increase is with the CCCC supporting condition. The maximum fundamental frequency for CCCC plate is $f_{1f} = 1.38f_o$ and for SSSS plate is $f_{1f} = 1.3f_o$ while for the CFFF plate it is getting closer to the fundamental natural frequency $f_{1f} = 0.99f_o$.

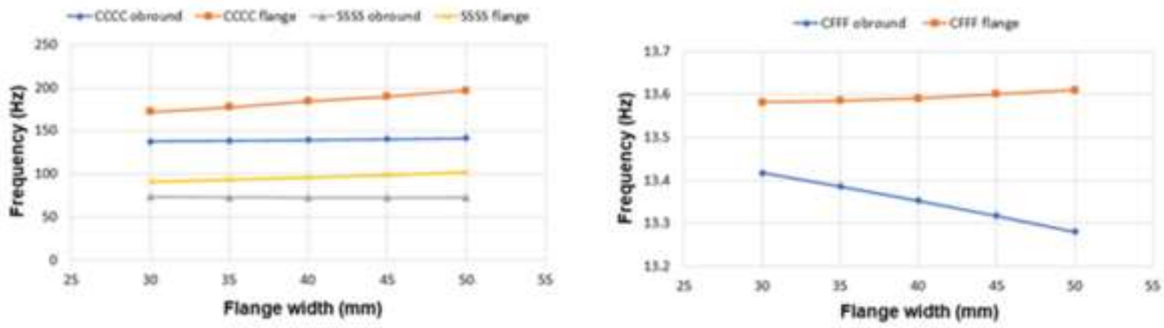

Figure 8. Fundamental frequency variation with flange width

Effect of Cutout Dimensions

To find out how changing both width and length of the flange will affect the fundamental frequency for the different edge support conditions, the cut area will be maintained constant as (2806.86mm^2). The flange width will vary between 30 to 50mm and the flange length is estimated from equation (1). The other flange constant parameters are (height=9mm, thickness= 0.73mm, orientation angle= 0^o). Increasing the flange width and decreasing its length gradually decreases the fundamental frequency for both CCCC and SSSS supporting conditions as shown in Figure 9, this indicates that the fundamental frequency is more sensitive for the change in flange length than the change in flange width. For the CFFF, it is continue increasing (f_{1f} = 13.839Hz compared with f_o = 13.753Hz).

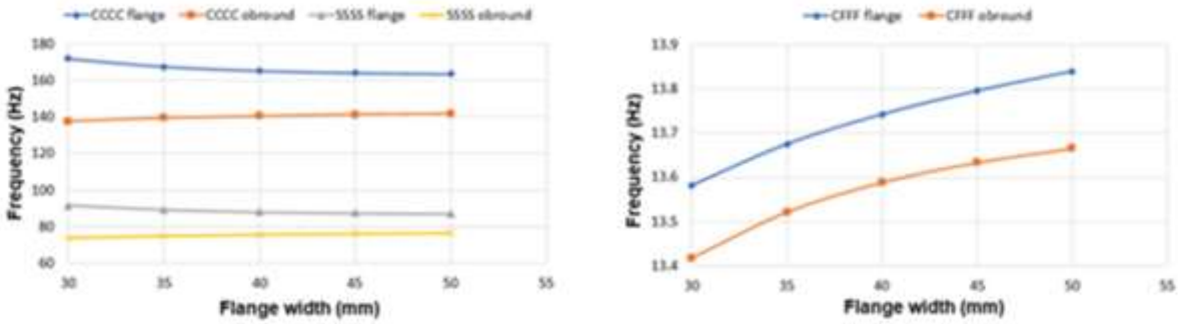

Figure 9. Effect of obround dimensions on fundamental frequency

Effect of Flange Height

The flange height will vary from 1 to 9mm, all other parameters are kept constant (length = 70mm, width = 30mm, thickness = 0.73mm, orientation angle = 0^o). Figure 10 shows that increasing the flange height up to 7mm increases the fundamental frequency for both the CCCC and SSSS conditions then it starts decreasing due to thinning the flange wall thickness, while it has no effect for the CFFF condition.

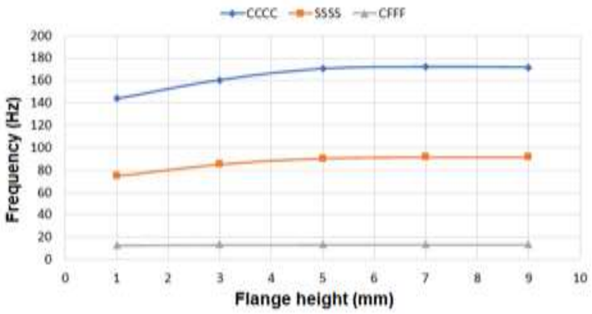

Figure 10. Fundamental frequency variation with flange height

Effect of Flange Location

To study the effect of changing flange location on plate fundamental frequency, the flange center will move along the X axis within the range (90 to 160mm) and the Y axis within the range (55 to 195mm). The other flange parameters are kept constant (length=70mm, width= 30mm, height=9mm, thickness= 0.73mm, orientation angle= 0). The variation of the fundamental frequency with the position of the flange center is shown in Figure 11. For the CCCC and SSSS plates, moving the flange center along the X-axis and Y-axis increases the fundamental frequency. The maximum frequency is achieved at the center of the plate, and for both cases it is higher than f_o. For the CFFF plate, moving the flange center along the X-axis has a neglectable effect on f_{1f} (varies between 13.56 and 13.58 Hz) which is still less than f_o. When the flange center moves along the Y-axis away from the clamped edge, f_{1f} continue increasing and becomes higher than f_o and this is due to the increase in the mass distribution near the fixed plate edge.

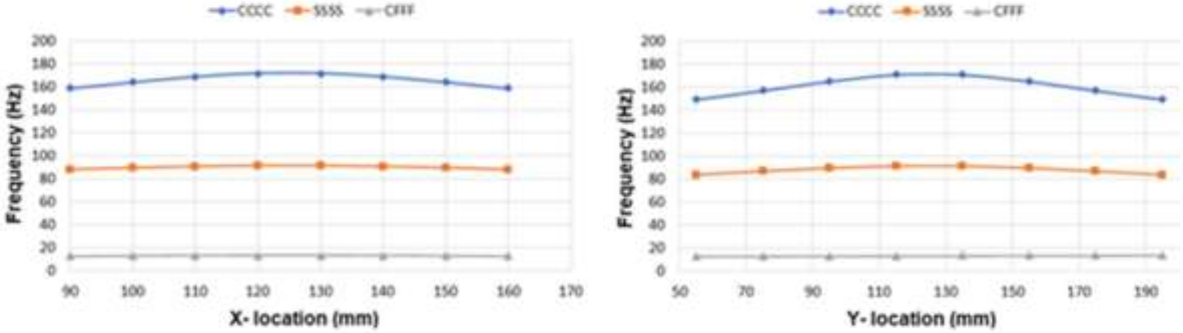

Figure 11. Effect of flange location on fundamental frequency

Effect of Flange Orientation

To study the effect of rotating the flange on plate fundamental frequency, the flange angle varies within the range (5 to 90°) in counterclockwise direction for different flange dimensions. The other parameters were kept constant (height= 9mm, thickness= 0.73mm). Depending on the flange dimensions, changing the flange angle has different effect on f_{1f} for the CCCC and SSSS plates as shown in Figures 12 and 13. For the small flange dimensions, f_{1f} reaches its maximum value when the flange angle equals zero (i.e. the flange is parallel to the x-axis), then it decreases gradually until it reach its minimum value at rotation angle equals 45°. For the larger flange dimensions, the trend is reversed, it starts increasing until it reaches the maximum frequency value at 45° then decreases up to 90°.

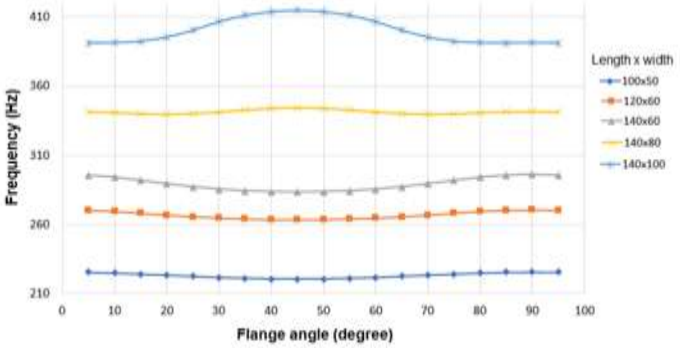

Figure 12. Frequency variation with flange angle for CCCC plate

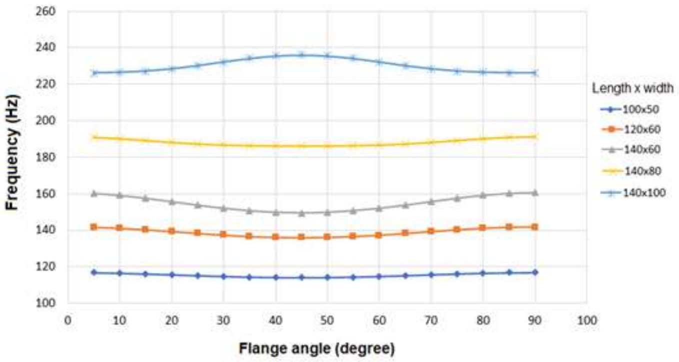

Figure 13. Frequency variation with flange angle for SSSS plate

Figure 14 shows that increasing the flange angle for CFFF plate increases f_{1f}. The maximum effect occurs when the flange is oriented perpendicular to the clamped edge (i.e. rotation angle 90°). At the lower range of angles (0 to 25°), increasing the flange area decreases f_{1f}, after that the trend reverse and increasing the flange area results in higher f_{1f}.

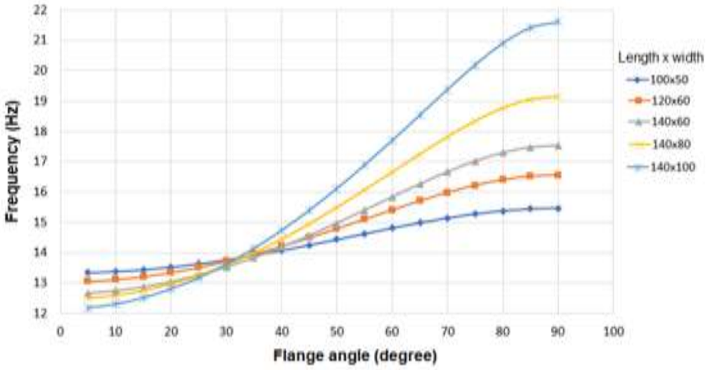

Figure 14. Frequency variation with flange angle for CFFF plate

CONCLUSIONS

Free vibration of rectangular plate having flanged obround opening was numerically investigated using Solidworks simulation add-ins. The effect of flange parameters on the plate fundamental frequency was estimated, and the following concluding results can be obtained:

- Cutting an obround area and converting it into a flange can enhance the CCCC plate fundamental frequency more than 1.7 times that of the bare plate, and more than 1.5 for the SSSS plate.
- Increasing the flange dimensions (length, width, and height) increases the plate fundamental frequency for the investigated different supporting conditions.
- The maximum increase in the fundamental frequency can be achieved when the length and width of the flange are maximum.
- Changing flange length has more effect on the fundamental frequency when compared with changing the flange width.
- The fundamental frequency for the clamped supporting conditions were higher than the simply support or free supporting conditions.
- For CFFF plates, its fundamental frequency can be enhanced by locating the flange perpendicular to the clamped edge.

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