Contact Mechanics and Nonlinear Contacts Stiffness for Hemi-elliptical Soft Fingertip in Grasping and Manipulation

Hassan Dawood Salman†, Sadeq Hussein Bakhy‡, Mohsin Noori Hamzah‡

† Ministry of Education / Department of Vocational Education of Babylon, Iraq
‡ Mechanical Engineering Department, University of Technology, Baghdad, Iraq

*Corresponding Author Email: 20093@uotechnology.edu.iq

ABSTRACT: Soft finger is commonly used as fingertip in robot and prosthesis hand applications, to provide the stability in grasping and manipulation. The study of contact mechanics for the soft finger in previous researches has been carried on the hemispherical and hemi-cylindrical structures, whereas this study concentrates on the use of a hemi-elliptical structure, which represents more realistic soft fingertips. In this study, a nonlinear contact mechanics model has been established to relate the vertical depression of a hemi-elliptical soft fingertip to the apply load as power-law equation. Then, the nonlinear contact stiffness of the hemi-elliptical soft fingertips under applying a normal force was derived. The influence of hemi-elliptical geometry on the vertical depression equation and stiffness was analyzed by introducing a new dimensionless parameter factor (κ). Stiffness relationship of Hertzian contact for linear elastic materials is shown to be a special case of the general model presented in this paper. Experimental results are presented to confirm the theoretical analysis using three different silicone rubbers. The results clearly indicated that the nonlinear contact stiffness increases with the increase of curvature ratio (R/R).

KEYWORDS: Soft fingertip, Nonlinear Contact stiffness, Hertzian contacts, Power-law model, Soft contact

INTRODUCTION

The softness of fingertips is considered as one of the important mechanical characteristics that conform to the geometry of the target object due to the fast expansion of contact area at the initial contact, which makes the resulting grasping more stable in the dexterous manipulation [1-3]. For the design and development of the effective artificial soft robotic fingertips, the depth knowledge of the realistic contact mechanics is required. Estimation of the contact mechanics model, including the determination of contact parameters associated with the normal load and the vertical depression profile is an essential issue in the general robotic applications [4]. Many researchers have been interested in the nonlinear contact mechanics of the soft fingertip in grasping and manipulating tasks as an active research field. I. Kao and N. Xydas presented an accurate modeling of contact mechanics for hemispherical soft fingertips as a power-law equation by extending the Hertzian contact theory for linear elastic materials to include nonlinear materials [5].

N. Elango and R. Marappan developed a nonlinear contact model of a cylindrical soft finger for power grasping, where the geometrical relationship among the contact parameters, such as the contact width and the deformation related to apply load was derived [6]. In the same concept, S. Bakhy et al. proposed the nonlinear contact-mechanics of the hemi-cylindrical soft fingertips as a power-law equation through theoretical modeling and validation the results experimentally using three different soft materials [7]. T. Watanabe and Y. Fujihira investigated experimentally the effect of fingertip stiffness on the stable grasping that related with friction force using spherical soft fingertips with varying stiffness [8]. Also, Y. Fujihira et al. studied experimentally the effect of the soft fingertip deformation and the curvature of contact surface on the maximum resistible force during the grasping of an object by using four different silicones of cylindrical fingertips [9]. K. Raja and R. Malayalamurthi derived the contact parameters of a sandwich structure of soft fingertip with an internal rigid core and analyzed the influence of the soft layer stratum and the rigid core material on the a power-law equation based on the experimental investigations and asserted by finite element simulations to obtain the optimal design of sandwich soft fingers [10].

S. Yuvaraja et al. proposed a power-law model of a hemispherical soft fingertip pressed on various curved surfaces using two-dimensional finite element models, and the effects of the contact parameters, such as vertical displacement, contact radius and contact pressure using wide range of normal force were presented for ten
different fingertip combinations [11]. H. Dong et al. analyzed the mathematical models of contact mechanics with a constructed grasping stiffness matrix to the developed evaluation approach of the quality of stability grasping [12]. H. Salman et al. established a new analytical model of the nonlinear contact-mechanics for a hemi-elliptical soft fingertip to have a relationship between the normal load and the radii of contact as a power-law equation [13]. Also, several works investigated the mechanistic bases of contact for developing the mechanical models of soft fingertips [14-16]. Most of the previous models and experimental researches considered the contact parameters of the hemi-cylindrical and hemispherical soft fingertips. However, most of the practical application used the hemi-elliptical shape, which represents more realistic soft fingertips. In this paper, a nonlinear contact modeling of hemi-elliptical soft fingertips which relates the vertical depression with respect to the normal load is derived theoretically and experimentally validated using three grades of silicon. In addition, the equations that describe the nonlinear contact stiffness behavior are presented. However, this aspect has not been confirmed by any previous research.

THEORETICAL DEVELOPMENT OF CONTACT MECHANICS

In these sections, the linear elastic contact model is firstly summarized, and then the nonlinear contact mechanics model for a hemi-elliptical soft-finger, including the relationship between the vertical depression and the normal load is derived.

The Linear Elastic Model

The first satisfactory analysis of contact mechanics was provided by Hertz based on the contact between two linear elastic materials (hemispherical against a planar) and found that the deflection δ between the two elastic solids is proportional to the normal load N raised to the power of 2/3 as the in the following equation [17]:

\[ \delta = C_s N^{2/3} \]  \hspace{1cm} (1)

Where, \( C_s \) is a proportional constant that depends on the geometry of hemispherical as well as the material properties. If hemi-elliptical elastic solids press against planar elastic solids, the deflection equation can be writing as:

\[ \delta = \kappa C_s N^{2/3} \]  \hspace{1cm} (2)

Where, \( \kappa \) is the contact coefficient, which represents the influence of hemi-elliptical geometry on the vertical depression is related to the curvature ratio of hemi-elliptical shape.

The Nonlinear Elastic Model

In this section, the elliptical linear elastic contact model in “Equation (2)” will be extended to include nonlinear materials. The power-law equation for the hemi-elliptical soft fingers in contact with a rigid surface was developed by H. Salman et al. to define the relationship between the effective radius of contact area \( \rho \) and the normal load \( N \) as [13]:

\[ \rho = C N^\gamma \] \hspace{1cm} (3)

Where, \( C \) is a proportional constant that depends on the geometry and material properties, and the exponent \( \gamma \) is related to the strain hardening factor \( n \) of fingertip materials, where \( \gamma = n / 2n+1 \). Since \( 0 \leq n \leq 1 \), the exponent range is \( 0 \leq \gamma \leq 1/3 \). As shown in Figure 1.a, the hemi-elliptical soft fingertip with radii \( R_x \) and \( R_y \) is pushed onto a rigid plane. In this case, the contact area is ellipse, and the effective radius contact can be written as \( \rho = \sqrt{\delta} \).

In Figure 1.b, the geometric constrain equation is given by:

\[ (R - \delta)^2 + \rho^2 = R^2 \] \hspace{1cm} (4)

Where, \( \delta \) is the vertical approach (deformation) of the hemi-elliptical soft fingertips, and \( R \) is the reduced radius of curvature, which is defined as [18]:

\[ 1/R = 1/R_x + 1/R_y \] \hspace{1cm} (5)
Expanding “Equation (4)” and substituting into “Equation (3)” with assumption $\delta \ll R$ leads to:

$$\delta = \frac{C^2}{2R} N^{2\gamma} \rightarrow \delta = C_s N^\beta$$

(6)

Where, the exponent $\beta$ is related by the following definition: $\beta = 2\gamma$, and $C_s$ is a proportional constant that depends on the size and curvature ratio of hemi-elliptical geometry, which can be writing as:

$$C_s = \kappa C_e$$

(7)

Where, $\kappa$ is the dimensionless parameter factor, which represents the influence of the curvature ratio ($R_x/R_y$) of the soft hemi-elliptical geometry on the vertical depression equation ($\kappa = 1$ for hemispherical fingertips $R_x/R_y = 1$) and $C_e$ is a proportional constant for the hemispherical soft fingertips ($R_x/R_y = 1$).

Substituting “Equation (7)” into “Equation (6)” gives:

$$\delta = \kappa C_s N^\beta$$

(8)

Equation (8) indicates that the vertical deformation $\delta$ of the soft fingertip is proportional to the apply normal load raised to the power of $\beta$, with a range from (0) to (2/3) that corresponding to the range of $\gamma$ from (0) to (1/3). For a linear elastic contact, $\beta = 3/2$ that associated with $\gamma = 1/3$. Thus, one has the Hertzian contact model:

$$\delta = \kappa C_s N^{2/3}$$

(9)

Clearly, the Hertzian theory of a linear elastic contact in “Equation (2)” is a special case of the general modeling that is proposed in this work.

EXPERIMENTAL VALIDATION

The Fingertip Specimens

To investigate the deformation of a soft fingertip under apply loads, specimens of simplified purposely-shaped fingertips have been tested. The simplified fingertips “in Figure 2.b” have a hemi-elliptical geometry with six different curvature radius ratios ($R_x/R_y = 1, 1.5, 2, 3, 4, \text{ and } 5$). The value of $R_x$ and $R_y$ was calculated from “Equation (5)” with the same reduced radius of curvature $R = 12 \text{ mm}$. Where, it is important to locate the 'x’ and 'y’ coordinates so that the following condition is fulfilled $R/R_y \geq 1$. The experimental activity allowed investigating the influence of the curvature ratio of soft pads using three different softness silicon rubbers ($G-828$, $G-815$, and $G801$).

Experimental Setup

The tensile testing machine shown “in Figure 2.a” was used as an experimental setup for the measurements of apply force and deformation. Where, each fingertip was mounted on a linear stage through the vertical moving jaw of the machine and using the tensile tester to press the fingertip on a flat solid surface (Plexiglas used as a smooth surface for all experiments to avoid the distortion of fingertip imprints). The normal force was applied...
between the fingertip and the flat solid surface at the range (0 - 130 N), and the vertical depression (deformation) was measured for each selected force by electronic scale in computer screen.

**Figure 2.** (a) Experimental Setup and (b) Hemi-elliptical soft fingertip with different curvature radii

**Experimental Results**

Experiments were conducted for various hemi-elliptical fingertip materials with different curvature ratios of \( R_x/R_y \) using the apparatus described “in Figure 2”. The fingertip presses on the flat solid surface with a range of normal force (0–130 N). The vertical approach versus the normal force for each ratio of hemi-elliptical was recorded, as shown “in Figures 3, 4 and 5”, and the algorithm of the weighted least-squares curve fitting was used for the purpose of fitting the experimental data to obtain the constants of the power-law equation \( \beta \) and \( C_e \) for each type of silicone that listed in “Table 1”. To be consistent with the assumption of \( \delta << R \), the data points with vertical deformation such that \( \left( \delta/R \right) < 30\% \) were used. The dimensionless parameter \( \kappa \), which presented the influence of curvature ratio \( (R_x/R_y) \) on the vertical depression, can be obtained from “Equation (6)” \( \left( \kappa = C_s/R \right) \) and listed in “Table 1”. Where, \( C_s \) is a proportional constant for the hemispherical soft fingertips \( (R_x/R_y = 1) \).

**Figure 3.** The experimental data weighted least-squares best fitting (LSBF) for the hemi-elliptical soft fingertips (G-828) with \( \left( \delta/R \right) < 30\% \).
Figure 4. The experimental data and the weighted least-squares best fitting (LSBF) for the hemi-elliptical soft fingertips (G-815) with $(\delta/R) < 30\%$

Figure 5. The experimental data and weighted least-squares best fitting (LSBF) for the hemi-elliptical soft fingertips (G-801) with $(\delta/R) < 30\%$

Table 1. The experimental results for exponent $\beta$ and constant $C_\nu$, $\kappa$ for the hemi-elliptical soft fingertip with different curvature radius ratios $(R_x/R_y)$

<table>
<thead>
<tr>
<th>Type of silicone</th>
<th>$R_x/R_y=1$</th>
<th>$R_x/R_y=1.5$</th>
<th>$R_x/R_y=2$</th>
<th>$R_x/R_y=3$</th>
<th>$R_x/R_y=4$</th>
<th>$R_x/R_y=5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-828 $\beta=0.6067$</td>
<td>Proportional constant $C_\nu$</td>
<td>0.1828</td>
<td>0.1795</td>
<td>0.1773</td>
<td>0.1698</td>
<td>0.1612</td>
</tr>
<tr>
<td>Parameter factor ($\kappa$)</td>
<td>1</td>
<td>0.982</td>
<td>0.970</td>
<td>0.929</td>
<td>0.887</td>
<td>0.840</td>
</tr>
<tr>
<td>G-815 $\beta=0.5302$</td>
<td>Proportional constant $C_\nu$</td>
<td>0.4274</td>
<td>0.4222</td>
<td>0.4167</td>
<td>0.3992</td>
<td>0.3850</td>
</tr>
<tr>
<td>Parameter factor ($\kappa$)</td>
<td>1</td>
<td>0.988</td>
<td>0.975</td>
<td>0.940</td>
<td>0.901</td>
<td>0.857</td>
</tr>
<tr>
<td>G-801 $\beta=0.4545$</td>
<td>Proportional constant $C_\nu$</td>
<td>0.8187</td>
<td>0.8105</td>
<td>0.8015</td>
<td>0.7818</td>
<td>0.7491</td>
</tr>
<tr>
<td>Parameter factor ($\kappa$)</td>
<td>1</td>
<td>0.990</td>
<td>0.979</td>
<td>0.955</td>
<td>0.915</td>
<td>0.870</td>
</tr>
</tbody>
</table>
The data in “Table 1” was used to plot the parameter factor $\kappa$ for three types of hemi-elliptical soft fingertip with respect to different curvature radius ratios $\left( \frac{R_x}{R_y} \right)$, as shown “in Figure 6”.

**Figure 6.** Chart for the determination parameter factor $\kappa$ for three different hemi-elliptical soft fingertips with different curvature radius ratios $\left( \frac{R_x}{R_y} \right)$

**NONLINEAR CONTACT STIFFNESS**

The nonlinear contact stiffness of soft fingertips can be defined as the ratio of the change in normal load related to the change in vertical approach at the contact. Based on the proportional relationship in “Equation (8)”, obtained,

$$N = (\kappa C_s)^{w_\beta} \delta^{u_\beta}$$  \hspace{1cm} (10)

Differentiating “Equation (10)” with respect to deflection, the contact stiffness of soft fingers $K$ is:

$$K = \frac{\delta N}{\delta \delta} = (\kappa C_s)^{w_\beta} \frac{1}{\beta} \delta^{u_\beta-1}$$  \hspace{1cm} (11)

Substituting “Equation (10)” into “Equation (11)” leads:

$$K = \frac{1}{\beta} \left( \frac{N}{\delta} \right)$$  \hspace{1cm} (12)

Also, the contact stiffness can be derived as a function of normal force by substituting “Equation (8)” into “Equation (12)” as:

$$K = \frac{1}{\kappa C_s} \frac{1}{\beta} N^{1-\beta}$$  \hspace{1cm} (13)

Equation (12) indicates that the contacts stiffness of the soft fingers always increases due to the increase of the ratio $N/\delta$ with the constant $\beta$. That means the soft finger contact becomes stiffer with a larger depression and force. The change in contact stiffness related to normal force can be derived as:

$$\frac{\partial K}{\partial N} = \left( \frac{1}{\beta} - 1 \right) / \delta$$  \hspace{1cm} (14)

Equation (14) refers that $\frac{\partial K}{\partial N} > 0$ because $1/\beta > 1.5$ and that means the contact stiffness $K$ increases with the normal load. Furthermore, the increasing rate in the contact stiffness is inversely related to the vertical deformation $\delta$. This asserts that the change of increasing the rate of stiffness is gradually diminishing with increase of vertical approach. For the sake of discussion and comparison, the contact stiffness of the hemi-elliptical soft fingertips as a function of vertical approach is plotted “in Figures 7, 8 and 9” for three different materials.
DISCUSSION

After estimating the experimental results, the parametric relationship ($C_e$ and $\beta$) between the vertical approach and the normal force was determined using the weighted least-square algorithm. The exponent $\beta$ was found dependent on the material properties and its value was within the range $(0 - 2/3)$. Where, the harder materials tend to have higher values for the exponent, these results are perfectly in line with the models presented by Kao and Yang [19]. It is also concluded that the deformation of hemi-elliptical fingertips in the linear elastic Hertzian
theory in “Equation (2)” is a special case of the general model which presented in this work by “Equation (6)” with exponent of $\beta = 2/3$.

From close observations in “Table 1”, it can be concluded that the value of proportional constant $C_t$ tends to have smaller values with increasing the ratio of curvature radii $R_y/R$, and harder grade fingertips. For analyzing the geometrical effect, a dimensionless parameter factor $\kappa$ is introduced. Where, the value of the parameter $\kappa$ can be obtained experimentally and was found within the range $0 \leq \kappa \leq 1$ that depends on the ratio of curvature as well as materials ($\kappa = 1$ for the hemispherical with curvature ratio $R_y/R = 1$ and $\kappa = 0$ for the curvature ratio $R_y/R = \infty$). That indicates that the parameter factor $\kappa$ is inversely proportional to the curative ratio $R_y/R$. Thus, a smaller deformation of contact is obtained for fingertips having a higher ratio of curvature radii $R_y/R$ under the same magnitude of normal force. This physical phenomenon is a derivative of changes in the apparent stiffness of fingertip due to the change of the curvature radii ratio.

In addition, the experiments that produced the results listed in “Table 1” are used to draw the chart of parameter factor $\kappa$ for three different silicones, as illustrated “in Figures 4, 5 and 6”. It is evident that the softer grade fingertip has a higher magnitude parameter factor $\kappa$ under the same curvature radii ratio. This indicated that the influence of the curvature radii ratio $(R_y/R_s)$ becomes lower with softer fingertip. The results also exhibited that the softer materials under the same curvature ratio have a relatively smaller value of contact stiffness, which supports the derived equations [20]. Also, “Figures 7, 8 and 9” elucidate the contact stiffness increase with the increase of the curvature radii ratio under the same material. This is reasonable due to the additional constraints of the hemi-elliptical fingertip at different radius ratios. Furthermore, the results manifested that the rate of change in the stiffness contact gradually decreases as the vertical depression and normal load increase.

In general, the accurate model presented in this study with the power-law equation can be employed for the soft fingertip materials, such as silicone or soft rubber or visco-elastic fingertip. Once the value of exponent $\beta$ is determined experimentally and using the proposed chart in “Figure 4” to find parameter factor $\kappa$, this model can be employed for the design and application of a hemi-elliptical soft fingertip that involves computing the deformation and contact stiffness using the preceding analysis presented in this paper.

CONCLUSION

In this paper, a hemi-elliptical shaped soft fingertip for contact problem was analyzed, and a nonlinear vertical approach related to the normal load was derived analytically and validated through experimental investigations. The equations that describe the nonlinear contacts stiffness behavior were presented in different forms. The new dimensionless parameter ($\kappa$) has been introduced as a factor to study the influence of the hemi-elliptical geometry on the vertical approach equation and stiffness, which will serve as a key factor in further developments. It is found that the Hertzian theory of linear contact is a special case of the general model which offered in this paper with exponent of $\beta = 2/3$. The weighted least-squares algorithm was employed to analyze the experimental data. Also, it can be seen that the experimental results supported the theoretical analysis very well, and it is clearly indicated that the nonlinear contact stiffness increases with the increase of curvature ratio $(R_y/R_s)$. The new modeling with the power-law equation that proposed in this paper is very important for the application that involves studying the soft contact factors for the grasping stability. Hence, the analysis of the fingertips with various kinds of shapes will improve the design and development of the anthropomorphic robotic fingertips.

REFERENCES


2015.


