Mathematical Model of Suspension Filtration in a Dual-Zone Porous Medium Taking Into Account the "Charging" Effect in Both Zones

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ABSTRACT

In the paper a mathematical model of the suspension filtration process in a porous medium, consisting of active and passive zones, taking into account the "charging" effect is compiled. The kinetic equations of particle deposition are generalized taking into account the "charging" effect in both zones. Based on this model, the filtration problem was formulated and numerically solved using finite difference method. The effect of changes in the kinetics of particle deposition on filtration characteristics was analyzed. The role of the "charging" effect on the general process of suspended particles transport has been established. It is shown that in profiles of concentration two zones are formed: one is before "charging" and the other is after. In these zones, filtration characteristics have different rates of change.

KEYWORDS

Deep bed filtration; finite-difference scheme; multistage deposition kinetics; "charging" effect.

INTRODUCTION

During filtration of a suspension in a porous medium, suspended particles are deposited in the pore space, which leads to a significant change in the filtration-capacity characteristics of the medium [1]. Currently, various theoretical and experimental approaches to the study of the flow of a suspension through a porous medium, mechanisms of particle capture, changes in the permeability and structure of the pore space are being actively developed [2-5]. The complexity and multiparametry of the process cause great difficulties in the experimental modeling of this process and the interpretation of the experimental results. There are four types of approaches used in literature to develop models of the process: empirical (phenomenological) models [6], network models [7], models for analyzing particle trajectories [8] and stochastic models [9]. For the theoretical description of the dynamics of the deposition of particles of the dispersed phase in the pore space, phenomenological models based on the mass balance equation and the kinetic equations of particle deposition are used [6, 10-14].

The relationship between permeability and porosity is also described by phenomenological relationships [6]. A review of the literature shows that several varieties of phenomenological relations have been obtained to describe the flow of various kinds of disperse systems in porous media [15]. But, the question of the limits of applicability of the obtained regularities has not yet been fully determined. The filtration process can be described by the equation for changing the concentration, which expresses the conservation law of the mass of the suspension, while the deposition in the pore volume of the medium should be described by the kinetic equation, which expresses the intensity of the change in the volume of deposition over time [16-19]. In [20], the filtration process is considered in the following two main stages: the first stage is the one in which deposition occurs mainly due to the direct adhesion of individual particles to the filter grains. A consequence of this deposition regime is the formation of a relatively smooth layer of deposits on the filter grains.
This first stage will continue until the local deposition reaches a certain transient value; At the second stage of the deposition process, due to the formation of particle aggregates, the re-entrainment of some of these aggregates and their re-deposition into pores prevail. Thus, the consequence of the processes prevailing in this second deposition stage is blocking of certain parts of the filter bed. It should be noted that a few research has been carried out on the basis of this multistage process of particle deposition. In [21], a review of microscopic and macroscopic models of filtration with particle deposition is given. As noted in the work, there are several stages in the filtration of inhomogeneous fluids. In the first stage, called the "initial stage", the pore space is clean and the deposition of particles occurs on the surface of the grain of the medium. After the first stage, all other stages are called transitional. Here, the deposition of particles occurs on the surface of the granules of the porous medium, partially already covered with particles of the suspension, deposited in the first stage of the process. During the transition period, particle deposition can either improve or degrade the filter's efficiency for cleaning the suspension. It depends on the chemical conditions of the particles and the porous medium.

Usually, in the one-dimensional case, the mass balance equation for particles in a porous medium is written in the form [22,23]

\[
\frac{\partial (mc)}{\partial t} + \frac{\partial \rho}{\partial t} + v \frac{\partial c}{\partial x} = 0, \tag{1}
\]

where \( c \) is the concentration of the suspension, \( v \) is the filtration velocity, \( m \) is the porosity of the medium, \( \rho \) is the concentration of deposition, \( t \) is the time, \( x \) is coordinate of space.

In Equation (1), the term \( \frac{\partial \rho}{\partial t} \) characterizes the rate of deposition and release of particles (kinetics of the process). It is clear that it depends on the number of particles concentration \( c \), as well as from the concentration of deposited particles, i.e. \( \rho \). Consequently, the kinetic equation of particle deposition can be written in the form [24,25]

\[
\frac{\partial \rho}{\partial t} = f(\alpha, c, \rho), \tag{2}
\]

where \( \alpha \) is a vector of parameters, \( f \) is some known function.

In this work, a model of suspension filtration in a porous medium, consisting of active and passive zones, was generalized to the case of multistage kinetics of deposition of suspension particles in pores. For this model, the problem of suspension filtration has been formulated and numerically solved. The influence of various parameters of the model on the characteristics of particle transport is established.

PROBLEM FORMULATION

We will consider a semi-infinite homogeneous porous media with initial porosity \( m_0 \), filled with a homogeneous fluid. The deposition in this pore volume has two forms, respectively, the filter zones are called active and passive [26]. Active zones washed by the flow and form a reversible deposition with a concentration \( \rho_a \), passive zones, which are stagnant, form a irreversible deposition with a concentration \( \rho_p \). Let us denote the full capacity of the filter through \( \rho_0 \). From the above it follows

\[
\rho_0 = \rho_{a0} + \rho_{p0},
\]

where \( \rho_{a0} \) and \( \rho_{p0} \) are capacities of active and passive zones, respectively.

At the point \( x = 0 \), starting from \( t > 0 \), a suspension with a concentration \( c_0 \) enters to the porous media with filtration velocity \( v(t) = v_0 = \text{const} \).

The system of equations for suspension filtration with a given flow velocity regime consists of the mass balance equation and kinetic equations for each zone, which in the one-dimensional case is represented in the form [27]
\[ m_0 \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} + \frac{\partial \rho_a}{\partial t} + \frac{\partial \rho_p}{\partial t} = 0, \]  
(3)

\[ \frac{\partial \rho_a}{\partial t} = \beta_a \left( c - \frac{\rho_a}{\rho_{a0}} c_0 \right), \]  
(4)

\[ \frac{\partial \rho_p}{\partial t} = \beta_p (\rho_p) c, \]  
(5)

where \( m_0 \) is the porosity of the medium, \( \beta_a \) is the kinetic coefficient for the active zone, \( \beta_p \) is the coefficient associated with the effect of compaction (aging) of the deposition and \( \beta_p = \alpha \left( \rho_p \right) \beta_{p0} \),

\[
\alpha(\rho_p) = \begin{cases} 
1, & \rho_p \leq \rho_{p1}, \\
\rho_{p1} / \rho_p, & \rho_{p1} < \rho_p < \rho_{p0}, \\
0, & \rho_p = \rho_{p0}.
\end{cases}
\]  
(6)

Hence, at the beginning of deposition formation \( \alpha = 1 \). Starting from a certain concentration \( \rho_{p1} \), the value of \( \alpha \) becomes less than 1, and a further decrease is inversely proportional to the amount of deposition \( \rho_p \).

Finally, when the concentration of the deposition is close to saturation, it decreases more intensively. This area approximated by a step [26].

Here we modify the kinetic equation for the active zone of the formation Equation (4) based on the following physical representation. Suspension filtration takes place in a clean filter, initially free of particle deposition. With the initial accumulation of deposition, there is an increase in the specific surface area and, accordingly, in the kinetic coefficients. The influence of this effect, called "charging" of the filter, is significant at the initial stages of filtration [26]. "Charging" lasts until the surface of the grains of the medium becomes covered with a monolayer of particles. As soon as the particles begin to interact primarily with the previously deposited layer, the deposition of the particles and the filtration cycle enters the next stage. Here, both the deposition of suspended particles and their release by the flow occur in parallel [6]. The transition of deposition from irreversible to reversible occurs when the deposition concentration reaches a predetermined value \( \rho_{ar} \). The volume of particles that can be captured in the active zone of the filter is finite. In the final stage, the deposition of particles reaches its maximum possible value, i.e. \( \rho_{ar0} \) the active zone of the filter is completely saturated with deposition or the deposition concentration does not change over time.

Taking into account the above assumptions, here instead of Equation (4) we use the following kinetic equation in the form:

\[
\frac{\partial \rho_a}{\partial t} = \begin{cases} 
\beta_a c, & 0 < \rho_a \leq \rho_{ar}, \\
\beta_a c - \beta_a \rho_a, & \rho_{ar} < \rho_a \leq \rho_{a0}, \\
0, & \rho_a = \rho_{a0}.
\end{cases}
\]  
(7)

where \( \beta_a \) is the kinetic coefficient associated with the "charging" effect, \( \beta_a \) and \( \beta_{a1} \) are the coefficients of deposition and release of solid particles in the active zone, respectively.

Here, instead of Equation (5), an equation is used taking into account the filtration velocity and the "charging" effect in the passive zone too. The "charging" effect manifests itself during the initial accumulation of deposition, when a monolayer is formed on the loading surface and the effect of aging can be neglected [27]. Therefore, the kinetic properties change relatively quickly and this change in the kinetic coefficient \( \beta_{p0} \) in the area \( 0 \leq \rho_p \leq \rho_{p1} \) can be approximated by a step function [26].
\[ \beta(p_p) = \begin{cases} \beta_{p^*}, & 0 < p_p < p_{p^*}, \\ \beta_{p_0^*}, & p_{p^*} \leq p_p < p_{p_1}. \end{cases} \] (8)

When \( p_p \geq p_{p_1} \) the effect of deposition aging is taken into account in accordance with Equation (6) and Equation (5), taking into account (8), is expressed as

\[ \frac{\partial \rho_p}{\partial t} = \begin{cases} \beta_{p_1} p_{p_1} + \rho_{p_1} / \rho_p, & p_{p_1} \leq p_p < p_{p_0}, \\ 0, & p_p = p_{p_0}. \end{cases} \] (9)

We solve system Equations (3), (7), (9) with the following initial and boundary conditions

\[ \rho_a(x,0) = \rho_p(x,0) = 0, \]
\[ c(x,0) = 0, \quad c(0,t) = c_0 = \text{const} \] (10)

Problem Solution

To solve problem Equations (3), (7), (9), (10), we apply the finite difference method [28, 29]. In the area \( D = \{0 \leq x < \infty, 0 \leq t \leq T\} \) we introduce a net, where \( T \) is the maximum time during which the process is investigated. To do this, divide the interval \( [0,\infty) \) with a step \( h \), and divide \( [0,T] \) into \( J \) parts with a step \( \tau \).

As a result, we have a grid

\[ \omega_{h\tau} = \{(x_i,t_j), \ x_i = ih, \ i = 0,1,..., \ t_j = j\tau, \ j = 0,1,...,J, \ \tau = T/J\} \]

Instead of functions \( c(t,x), \ \rho_a(t,x), \ \rho_p(t,x) \) we will consider grid functions, the values of which at the nodes will be denoted by \( c_i, \ \rho_{a,i}, \ \rho_{p,i}. \)

Equation (3) is approximated on the grid \( \omega_{h\tau} \) in the following form

\[ m_i c_i^{i+1} - c_i^i + \frac{c_i^{i+1} - c_i^i}{h} + \frac{\rho_{a,i}^{i+1} - \rho_{a,i}^i + \rho_{p,i}^{i+1} - \rho_{p,i}^i}{\tau} = 0 \] (11)

The difference scheme for Equation (7) will have the form

\[ \frac{\rho_{a,i}^{i+1} - \rho_{a,i}^i}{\tau} = \begin{cases} \beta_a c_i^i, & 0 < \rho_{a,i}^i \leq \rho_{a,u}, \\ \beta_a c_i^i - \beta_a \rho_{a,i}^i, & \rho_{a,u} < \rho_{a,i}^i \leq \rho_{a,0}, \\ 0, & \rho_{a,i}^i = \rho_{a,0}. \end{cases} \] (12)

Difference scheme for Equation (9)

\[ \frac{\rho_{p,i}^{i+1} - \rho_{p,i}^i}{\tau} = \begin{cases} \beta_{p^*} c_i^i, & \rho_{p,i}^i \leq \rho_{p^*}, \\ \beta_{p^*} c_i^i - \beta_{p^*} \rho_{p,i}^i, & \rho_{p^*} < \rho_{p,i}^i \leq \rho_{p,1}, \\ \beta_{p^*} p_{p_1} / \rho_{p,i}^i c_i^i, & \rho_{p_1} < \rho_{p,i}^i \leq \rho_{p,0}, \\ 0, & \rho_{p,i}^i = \rho_{p,0}. \end{cases} \] (13)
The initial and boundary conditions (10) are also represented in the net form

\[ \rho_{a,i}^0 = 0, \; \rho_{p,i}^0 = 0, \; c_i^0 = 0, \; c_i^j = c_0, \; i = 0,1, \; j = 0, J, \]

where \( I \) is a sufficiently is a sufficiently large number for which the equation is \( c_i^j = 0 \) approximately satisfied.

Transforming the difference schemes (11) - (13), we obtain

\[ c_{i+1}^{j+1} = \frac{h}{v} \left( \frac{\rho_{c,i}^{j+1} + m \rho_{c,i}^j}{h} - \left( \frac{\rho_{a,i}^{j+1} - \rho_{a,i}^j + \rho_{p,i}^{j+1} - \rho_{p,i}^j}{} \right) \right) \]

(15)

\[ \rho_{a,i}^{j+1} = \begin{cases} \rho_{a,i}^j + \beta v c_i^j, & 0 < \rho_{a,i}^j \leq \rho_{a0}, \\ \rho_{a,i}^j + (\beta v c_i^j - \beta a \rho_{a,i}^j), & \rho_{a,i}^j < \rho_{a,i}^j \leq \rho_{a0}, \\ \rho_{a,i}^j, & \rho_{a,i}^j = \rho_{a0}. \end{cases} \]

(16)

\[ \rho_{p,i}^{j+1} = \begin{cases} \rho_{p,i}^j + \beta p \rho_{c,i}^j, & \rho_{p,i}^j \leq \rho_{p0}, \\ \rho_{p,i}^j + \beta p \rho_{c,i}^j \rho_{p,i}^j / \rho_{p,i}^j, & \rho_{p,i}^j < \rho_{p,i}^j \leq \rho_{p0}, \\ \rho_{p,i}^j, & \rho_{p,i}^j = \rho_{p0}. \end{cases} \]

(17)

Stability of difference schemes has been proved. System (15) - (17) is solved with known initial and boundary conditions (14). Calculations are carried out in the following sequence. According to (16) and (17), the values of \( \rho_{a,i}^{j+1} \) and \( \rho_{p,i}^{j+1} \) are determined through the known quantities \( \rho_{a,i}^j \), \( \rho_{p,i}^j \) and \( c_i^j \) of the lower layer at the corresponding points, from Equation (15) we find \( c_i^{j+1} \).

RESULTS AND DISCUSSION

We will take the following numerical values as initial parameters \([23, 26] \): \( c_0 = 0.05, \; m_0 = 0.3, \; v_0 = 10^{-4} \) m/s, \( \rho_0 = 0.1, \; \rho_{a0} = 0.03, \; \rho_{p0} = 0.07 \).

In order to analyse the model numerical experiment carried out with program developed by the authors in Python. In the results of numerical experiments, three fields are determined: the current concentration of particles in the fluid \( c \), the concentration of deposited particles in the active \( \rho_a \) and passive zones \( \rho_p \). With an increase in the current time, these fields move deeper into the porous media, while their values increase (Fig. 1-2).

Comparing Fig.1.b. with Fig.2.b it can be noted that the effect of "charging" in the active zone significantly affects the changing profiles of concentration \( \rho_a \). With an increase in the parameter \( \rho_{a1} \), two zones are formed: one with \( \rho_a < \rho_{a1} \) and the other with \( \rho_a > \rho_{a1} \). In these zones, filtration characteristics have different rates of changing (Fig. 2.b.). When \( \rho_{a1} = 0.01 \) formation of two zones is clearly visible (Fig. 3). This statement is also can be seen in Fig.4.b. The rate of change in the concentration of deposition in the active zone \( \rho_a \) also affects the characteristics of the distribution of concentrations of suspended particles \( c \) and deposition in the passive zone \( \rho_p \) (Fig. 4).)

In Fig. 5, it is shown that the effect of "charging" in the active zone also significantly affects the dynamics of changes in concentration \( \rho_a \). Here also, as the parameter \( \rho_{a1} \) increases, two zones are formed: as in Fig.2. When \( \rho_{a1} = 0.005 \) “charging” at a point \( x = 0.04 \) ends approximately with \( t = 400 \) s, and when \( \rho_{a1} = 0.01 \) - with \( t = 500 \) s, (Fig. 5).
Analysis of the graphical results shows that the “charging” effect in the passive zone significantly affects the filtration characteristics of the suspension (Fig. 6). It is shown that here also, two zones are formed: one with $\rho_p < \rho_{pr}$ and the other with $\rho_p > \rho_{pr}$. In these zones, filtration characteristics have different rates of change (Fig. 6). With an increase in the parameter $\rho_{pr}$, the formation of two zones is clearly visible. This statement can also be seen in Fig. 7, which shows the dynamics at a given point. From Fig. 7, can be seen that in the beginning of the process dynamics of changes in suspension deposition are same, than when the “charging” effect is finished the dynamics are different for various $\rho_{pr}$.

Figure 1. Profiles of $c/c_0$ (a), $\rho_a$ (b), $\rho_p$ (c), at $\rho_{pl}=0.012$, $\rho_{pr}=0.0$, $\rho_{w}=0.001$, $\beta_a=30$ m$^{-1}$, $\beta_f=10$ m$^{-1}$, $\beta_d=0.005$ s$^{-1}$, $\beta_{p0}=50$ m$^{-1}$.
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Figure 2. Profiles of $c / c_0$ (a), $\rho_a$ (b), $\rho_p$ (c), at $\rho_{p1} = 0.012$, $\rho_{pp} = 0.0$, $\rho_{ar} = 0.005$, $\beta_a = 30$ m$^{-1}$, $\beta_r = 10$ m$^{-1}$, $\beta_d = 0.005$ s$^{-1}$, $\beta_{pd} = 50$ m$^{-1}$. 
Figure 3. Profiles of $c / c_0$ (a), $\rho_a$ (b), $\rho_p$ (c), at $\rho_{p1} = 0.012$, $\rho_p = 0.0$, $\rho_w = 0.01$, $\beta_a = 30$ m$^{-1}$, $\beta_s = 10$ m$^{-1}$, $\beta_d = 0.005$ s$^{-1}$, $\beta_{p0} = 50$ m$^{-1}$.

Figure 4. Profiles of $c / c_0$ (a), $\rho_a$ (b), $\rho_p$ (c), at $t = 1350$ s, $\rho_p = 0.0$, $\rho_w = 0.01$, $\beta_a = 30$ m$^{-1}$, $\beta_s = 10$ m$^{-1}$, $\beta_d = 0.005$ s$^{-1}$, $\beta_{p0} = 50$ m$^{-1}$.

Figure 5. Dynamics of $\rho_a$ at the point $x = 0.04$ m at different values of $\rho_{a1}$.
Figure 6. Profiles of $c / c_0$ (a), $\rho_a$ (b), $\rho_p$ (c), at $\rho_{p1} = 0.012$, $\rho_{p2} = 0.004$, $\rho_{a} = 0.001$, $\beta_a = 30$ m$^{-1}$, $\beta_r = 10$ m$^{-1}$, $\beta_d = 0.005$ s$^{-1}$, $\beta_{p0} = 50$ m$^{-1}$.

Figure 7. Dynamics of $\rho_a$ (a) and $\rho_p$ (a) at the point $x = 0.02$ m at different values of $\rho_{p2}$. 

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CONCLUSION

In this work, a generalized model of suspension filtration in a porous medium is compiled, taking into account the multistage kinetics of particle deposition in both active and passive zones of the medium. The problem of suspension filtration in a porous medium was formulated and numerically solved, taking into account this multistage deposition kinetics. The porous medium consists of active and passive zones, which differ significantly in the nature of particle deposition. The kinetic equation of deposition and release of particles in both zones of the medium was modified, taking into account the effect of “charging” the filter, and it is shown that this is essential in the initial stages of filtration. To solve the problem, the finite difference method has been used. Based on the numerical results, the profiles of changes in the concentration of suspended particles in the fluid, deposition in the active and passive zones have been determined. In these zones, filtration characteristics have different rates of change. It is shown that in profiles of concentration two zones are formed: one is before “charging” and the other is after. In these zones, filtration characteristics have different rates of change.

REFERENCES


