Determining of Optimum Gear Ratios for Minimum Gearbox Volume of Two-stage Helical Gearbox with Second-stage Double Gear Sets

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ABSTRACT

In this paper, a study aims to determine the optimal gear ratio of a two-stage gearbox with second-stage double gear sets to target the minimum gearbox volume. To solve this problem, a simulation experiment is designed and performed. Eight main design parameters, including the total gearbox ratio, the coefficient of the face width of the first gear stage, the coefficient of wheel face width of the second gear stage, the allowable contact stress of stage 1, the allowable contact stress of stage 2, and the output torque were selected as the input parameters of this experiment. The influence of input parameters on the optimal gear ratio was investigated. In addition, a regression formula to determine the optimal gear ratio has been proposed.

KEYWORDS

Helical gearbox, Gear ratio, Gearbox volume, Simulation experiment, Systems Engineering

INTRODUCTION

In various industries, mechanical transmission systems are widely used. The reason is because it has a simple structure, works with high reliability, easy to operate and low cost. A mechanical transmission system usually consists of an electric motor, a gearbox, and an operating machine. This system may have one or two additional external drives and is usually a V-belt or a chain drive. Among the elements of a mechanical transmission system, the gearbox is the most important. It is responsible for reducing the motor speed to a reasonable speed of the working machine and increasing the torque. Therefore, optimal design of a gearbox is an important task for the researchers. In the optimal design of the gearbox, the problem of determining the optimal gear ratios is the most meaningful work.

This is because the gear ratios of a gearbox are a decisive parameter to the mass, the size and the cost of the gearbox. Therefore, up to now, there have been quite a few studies to determine the optimal gear ratios of the gearbox. Studies on calculating optimal gear ratios have been carried out for different types of gearboxes such as helical gearboxes [1-4], worm gearboxes [5, 6], bevel-helical gearboxes [7-9], etc. The problem of determining the optimal gear ratios was also done for gearboxes with different stage numbers such as 2-stage [2, 9], 3-stage [1, 3, 7, 8, 10], and 4-stage [4] gearboxes. Various objective functions are also of interest to researchers such as the minimum gear mass [4, 10], the minimum gearbox mass [9], the minimum gearbox length [2, 8, 10], the minimum gearbox cross-sectional area [3, 7, 10], and the minimum gearbox cost [1].

METHODOLOGY

Gearbox volume analysis
From the calculation schema in Figure 1, the volume of the gearbox is calculated according to the following formula:

\[ V_{gb} = H \cdot L \cdot B \]

(1)

Where, \( H \), \( L \), and \( B \) can be determined as follows (Figure 1):

\[ H = \max (d_{w21}, d_{w22}) + 6.5 \cdot S_G \]

(2)

\[ L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} + 20 \]

(3)

\[ B = b_{w1} + 2 \cdot b_{w2} + 7 \cdot S_G \]

(4)

In which, \( S_G \) can be found by [11]:

\[ S_G = 0.005 \cdot L + 4.5 \]

(5)

Also, \( d_{w11}, d_{w21}, d_{w22} \) in the above equations are gear pitch diameters of the first and second stages. They can be calculated by [12]:

\[ d_{w11} = 2 \cdot a_{w1}/(u_1 + 1) \]

(6)

\[ d_{w21} = 2 \cdot a_{w1} \cdot u_1/(u_1 + 1) \]

(7)

\[ d_{w22} = 2 \cdot a_{w2} \cdot u_2/(u_2 + 1) \]

(8)

Wherein, \( a_{w1} \) and \( a_{w2} \) are the center distances of the first and the second stages which can be calculated by [12]:

\[ a_{w1} = k_a \cdot (u_1 + 1) \sqrt[3]{T_{11} \cdot k_{H1}/([\sigma_H1]^2 \cdot u_1 \cdot X_{ba1})} \]

(9)

\[ a_{w2} = k_a \cdot (u_2 + 1) \sqrt[3]{T_{12} \cdot k_{H2}/([\sigma_H2]^2 \cdot u_2 \cdot X_{ba2})} \]

(10)
In which, $k_a$ is the material coefficient, $k_a = 43$ with steel material [12]; $k_{H1}$ and $k_{H2}$ are contacting load ratio for pitting resistance of the first and the second stages. For this gearbox, $k_{H1} = 1.0 \div 1.06$ and $k_{H2} = 1.02 \div 1.28$ [12] and we can choose $k_{H1} = 1.03$ and $k_{H2} = 1.15$. $[\sigma_{H1}]$ and $[\sigma_{H2}]$ are the allowable contact stress of the first and second stages (MPa); $u_1$ and $u_2$ are, respectively, the gear ratio of the first and second stages; $T_{11}$, and $T_{12}$ are respectively the torque on the first and second shafts of the gearbox (Nnm). They are determined by:

$$T_{11} = T_{out} / (u_1 \cdot \eta_{hg} \cdot \eta_{hg} \cdot \eta_{bE})$$

$$T_{12} = T_{out} / (2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{bE}^2)$$

Where, $u_t$ is the total gearbox ratio; $\eta_{sg}$ is the straight gear efficiency and $\eta_{sg} = 0.96 + 0.98$ [12]; $\eta_{hg}$ is the helical gear efficiency and $\eta_{hg} = 0.96 + 0.98$ [12]; $\eta_{be}$ is the efficiency of a pair of rolling bearing and $\eta_{be} = 0.99 \div 9.995$ [12]. Choosing $\eta_{sg} = \eta_{hg} = 0.97$ and $\eta_{be} = 0.992$ we have

$$T_{11} = 1.0887 \cdot T_{out} / u_1$$

$$T_{12} = 0.5238 \cdot T_{out} / u_2$$

Optimization problem

From the analysis in section 2.1, the optimization problem to determine the optimum gear ratio for the smallest gearbox volume can be described as follows:

$$\text{Minimize} V_{gb}$$

With the following constraints:

$$1 \leq u_1 \leq 9 : 1 \leq u_2 \leq 9$$

SIMULATION EXPERIMENT

To evaluate the influence of the input parameters on the gear ratio $u_1$ of the gearbox to achieve the smallest gearbox volume, a simulation experiment is designed with 6 input parameters. The experimental plan and the input factor levels are shown in Table 1. With the design of $2^k$ full factorial design with $k = 6$, the number of experiments is $2^6 = 64$. This experimental design aims to evaluate the influence of the main factors and the interactions between the factors without being duplicated. To build the experimental matrix, the Minitab 18 software was used with experimental design $2^k$. The experimental planning matrix and the response (gear ratio $u_1$) are described in Table 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Symbol</th>
<th>Unit</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Total gearbox ratio</td>
<td>$u_g$</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Coefficient of the face width of the first gear stage</td>
<td>$X_{ba1}$</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>Coefficient of wheel face width of the second gear stage</td>
<td>$X_{ba2}$</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>Allowable contact stress of stage 1</td>
<td>$AS_1$</td>
<td>MPa</td>
<td>360</td>
</tr>
<tr>
<td>5</td>
<td>Allowable contact stress of stage 2</td>
<td>$AS_2$</td>
<td>MPa</td>
<td>360</td>
</tr>
<tr>
<td>6</td>
<td>Output torque</td>
<td>$T_{out}$</td>
<td>Nm</td>
<td>1000</td>
</tr>
</tbody>
</table>

RESULTS AND ANALYSIS

Figure 2 shows the influence of the main design parameters on the optimum gear ratio $u_1$. From this graph it can be seen that when $u_g, X_{ba1}, AS_1$ increase, $u_1$ increases. On the contrary, when $X_{ba2}, AS_2$ increases, $u_1$ decreases. Besides, $T_{out}$ has a negligible effect on $u_1$. The Pareto plot (Figure 3) shows the influence of input parameters and their interactions on $u_1$. The influence of the parameters is shown in the length of the blue column. Here, the
parameters whose magnitude exceeds the red reference line are those that have a significant influence on \( u_1 \) with the significance level \( \alpha = 0.05 \). Among them, the influence of \( A(u_g) \) on \( u_1 \) is the largest. Other parameters that have a significant influence on \( u_1 \) include their interactions, namely: \( B (X_{ba1}) \), \( C (X_{ba2}) \), \( D (A_1) \), \( E (A_2) \) and \( AB \) interactions \( (u_g \cdot X_{ba1}) \), \( AC \ (u_g \cdot X_{ba2}) \), \( AD \ (u_g \cdot A_1) \), \( AE \ (u_g \cdot A_2) \), \( DE \ (A_1 \cdot A_2) \). The effect of the interactions is also clearly shown in Figure 4.

**Table 2.** Experimental matrix and the response \( u_1 \)

<table>
<thead>
<tr>
<th>Run Order</th>
<th>( u_g )</th>
<th>( X_{ba1} )</th>
<th>( X_{ba2} )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( T_{out} )</th>
<th>( u_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>420</td>
<td>420</td>
<td>10000</td>
<td>8.38</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>420</td>
<td>360</td>
<td>1000</td>
<td>9.00</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>360</td>
<td>420</td>
<td>10000</td>
<td>7.96</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>360</td>
<td>360</td>
<td>10000</td>
<td>8.38</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>360</td>
<td>360</td>
<td>1000</td>
<td>8.82</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>420</td>
<td>360</td>
<td>10000</td>
<td>9.00</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>35</td>
<td>0.35</td>
<td>0.35</td>
<td>420</td>
<td>420</td>
<td>10000</td>
<td>7.96</td>
</tr>
<tr>
<td>64</td>
<td>5</td>
<td>0.35</td>
<td>0.35</td>
<td>360</td>
<td>360</td>
<td>10000</td>
<td>2.41</td>
</tr>
</tbody>
</table>

**Figure 2.** Influences of main design parameters on \( u_1 \)

The influence of the input parameters can be seen more clearly in Figure 3 that shows the Pareto chart of the standardized effects. For the response model, the parameters are statistically significant at the 0.05 level. The magnitude of the effect of the input parameters on the optimal transmission ratio is arranged from the lowest value to the highest value. The strongest influence on the optimum transmission ratio \( u_2 \) is the total gearbox ratio \( u_1 \) (see factor A in Figure 3). The effect is gradually reduced in the sequence of the remaining parameters i.e., the allowable contact stress of the second stages \( A_2 \) (factor E), the cost of gearbox housing \( c_{g_h} \) (factor G), the allowable contact stress of the first stages \( A_1 \) (factor D), the cost of shaft \( c_s \) (factor J), the output torque \( T_{out} \) (factor F), the coefficient of wheel face width of the first stage \( X_{ba1} \) (factor B), the cost of gears \( c_g \) (factor H), the coefficient of wheel face width of the second stage \( X_{ba2} \) (factor C).
Determination of Optimum Gear Ratios for Minimum Gearbox Volume of Two-stage Helical Gearbox with Second-stage Double Gear Sets

Figure 4. Interactions of input factors on $u_1$

The normal distribution chart (Figure 5) aims to better define the trend and influence of input parameters and interactions on $u_1$. From this graph, the parameters marked in red are those that have a significant influence on $u_1$. The parameter that is further away from the standard curve is the parameter with the greater influence on $u_1$. The parameters to the right of the calibration curve are those that have a proportional effect on $u_1$. These parameters include: $A$ ($u_g$), $B$ ($X_{ba1}$), $D$ ($AS_1$) and the interactions $AB$ ($u_g \cdot X_{ba1}$), $AD$ ($u_g \cdot AS_1$), $DE$ ($AS_1 \cdot AS_2$). The parameters on the left side of the standard curve are those that have an inverse effect on $u_1$, including: $C$ ($X_{ba2}$), $E$ ($AS_2$) and interactions $AC$ ($u_g \cdot X_{ba2}$), $AE$ ($u_g \cdot AS_2$).

Determine the regression equation to calculate $u_1$:

To calculate $u_1$, a regression equation was built. This equation is obtained after removing parameters that have little or no effect on $u_1$. The estimated coefficients of the input parameters and their interactions after removal are described in Table 3.

\[ u_1 = 5.99 + 0.2009 \cdot u_g + 1.654 \cdot X_{ba1} - 1.654 \cdot X_{ba2} - 0.00939 \cdot AS_1 - 0.01472 \cdot AS_2 + 0.1442 \cdot u_g \cdot X_{ba1} - 0.1442 \cdot u_g \cdot X_{ba2} + 0.000255 \cdot u_g \cdot AS_1 - 0.000255 \cdot u_g \cdot AS_2 + 0.000031 \cdot AS_1 \cdot AS_2 \]  

(17)

Since $u_1 = u_1 \cdot u_2$, after having $u_1$, the gear ratio $u_2$ is easily determined by:

\[ u_2 = u_1 / u_1 \]  

(18)

From Table 3, the minimum value of the correlation coefficients (R-sq= 99.93, R-sq(adj)= 99.92 and R-sq(pred) = 99.90) is also 99.90. That shows that formula (17) fits the experimental data very well.
Table 3. Estimated coefficients of regression model for $u_1$

<table>
<thead>
<tr>
<th>Term</th>
<th>Effect</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T-Value</th>
<th>P-Value</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td></td>
<td>5.3097</td>
<td>0.0111</td>
<td>477.44</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>$u_g$</td>
<td></td>
<td>6.0256</td>
<td>3.0128</td>
<td>0.0111</td>
<td>270.91</td>
<td>0.000</td>
</tr>
<tr>
<td>Xba1</td>
<td></td>
<td>0.2269</td>
<td>0.1134</td>
<td>0.0111</td>
<td>10.20</td>
<td>0.000</td>
</tr>
<tr>
<td>Xba2</td>
<td></td>
<td>-0.2269</td>
<td>-0.1134</td>
<td>0.0111</td>
<td>-10.20</td>
<td>0.000</td>
</tr>
<tr>
<td>AS1</td>
<td></td>
<td>0.4656</td>
<td>0.2328</td>
<td>0.0111</td>
<td>20.93</td>
<td>0.000</td>
</tr>
<tr>
<td>AS2</td>
<td></td>
<td>-0.4656</td>
<td>-0.2328</td>
<td>0.0111</td>
<td>-20.93</td>
<td>0.000</td>
</tr>
<tr>
<td>$u_g^*Xba1$</td>
<td></td>
<td>0.1081</td>
<td>0.0541</td>
<td>0.0111</td>
<td>4.86</td>
<td>0.000</td>
</tr>
<tr>
<td>$u_g^*Xba2$</td>
<td></td>
<td>-0.1081</td>
<td>-0.0541</td>
<td>0.0111</td>
<td>-4.86</td>
<td>0.000</td>
</tr>
<tr>
<td>$u_g^*AS1$</td>
<td></td>
<td>0.2294</td>
<td>0.1147</td>
<td>0.0111</td>
<td>10.31</td>
<td>0.000</td>
</tr>
<tr>
<td>$u_g^*AS2$</td>
<td></td>
<td>-0.2294</td>
<td>-0.1147</td>
<td>0.0111</td>
<td>-10.31</td>
<td>0.000</td>
</tr>
<tr>
<td>AS1*AS2</td>
<td></td>
<td>0.0556</td>
<td>0.0270</td>
<td>0.0111</td>
<td>2.50</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Model Summary

<table>
<thead>
<tr>
<th>S</th>
<th>R-sq</th>
<th>R-sq(adj)</th>
<th>R-sq(pred)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0889701</td>
<td>99.93%</td>
<td>99.92%</td>
<td>99.90%</td>
</tr>
</tbody>
</table>

Figure 6. Residual plots for $u_1$

The suitability of the proposed model is evaluated through the residual evaluation distribution chart (Figure 6). This graph is intended to determine the difference between the experiment and the formula for calculating $u_1$. From Figure 6, it can be seen that on the Normal Probability Graph, most of the errors (the blue dots) are very close with the normal distribution (the red solid line). On the graph of the frequency of errors, the errors near 0 from -0.0005 to 0.0005 account for the majority. In addition, in the Versus Fits graph, the errors and the calculated values are randomly distributed points. It proves that $u_1$ is not significantly affected other than the input parameters. Similarly, the dots representing the relationship between the error and the order of the data points (the Versus Order graph) are also randomly distributed. It shows that $u_1$ independent of the time factor. Through the error evaluations in the charts above, it can be concluded that the proposed model is applicable usable.

CONCLUSION

In this paper, the results of an optimization problem to determine the optimum gear ratios of a two-stage helical gearbox with second stage double gear-set for getting the minimum gearbox volume. In this work, a simulation experiment was designed and conducted. Also, the effect of main design factors on the optimum gear ratios for the gearbox was investigated. Moreover, a regression model to find the optimum gear ratios have been suggested. It was also noted that the proposed model is very consistent with the experimental data.

ACKNOWLEDGEMENTS

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REFERENCES


