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Determining the Optimum Gear Ratios to Minimize the Cost of Two-Stage Helical Gearbox with Second-stage Double Gear Sets

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ABSTRACT

This article targets to determine the optimum set of main design parameters of a two-stage gearbox with second-stage double gear sets to minimize the cost of gearbox. Ten parameters such as the total gearbox ratio, the coefficient of the face width of the first gear stage, the coefficient of wheel face width of the second gear stage, the allowable contact stress of stage 1, the allowable contact stress of stage 2, the output torque, the cost of gearbox housing, the cost of helical gears, the cost of straight gears, the cost of shafts is chosen as input parameters of optimization process. In addition, a simulation experiment is conducted to serve for finding optimum solution. It is found that the total gear ratio has the most important impact on the response. The proposed method is confirmed the reliability and can be applied to other studies.

KEYWORDS

Helical gearbox, Gear ratio, Gearbox cost, Simulation experiment, Systems Engineering.

INTRODUCTION

A Subsection Sample

The development of modern industry causes the higher requirements for machine design, in particular the optimization for parameters of components in a mechanical system. The partial and total gear ratios have strong effects on the dimensions, the mass, accurate transmission as well as the cost of the mechanical system [1-7]. Therefore, researchers in design process of gearbox or reducer have considered optimizing the gear ratios so far [8-13]. Optimization process of the partial gear ratios has been conducted by main three methods, i.e. graph method [14], practical method [15], and model method [10-13, 16]. In the mentioned method, research communities dominantly utilize the last. Vu et al. [17] optimize the splitting total gear ratio of two-stage helical reducer with first-stage double gearsets. In this study, seven main design parameters such as the total gear ratio, the gear face ratio of first step, the wheel face ratio of second step, the allowable contact stress of first step, the allowable contact stress of second step, the output torque, the gearbox housing cost, gear cost, and shaft cost are considered as input parameters of optimization process.

Moreover, the objective function for the current article is the minimal length of gearbox. The results reveal that proposed method is confirmed and showing the reliability which can be applied to other studies. The optimum set of main parameters is also found in other studies [1-3, 5, 6, 12, 16, 18, 19]. Tran et al. [20] where two-stage helical gearbox with second stage double gear sets is considered. A regression model is suggested to predict the optimum gear ratios. It is found that the total gearbox ratio has strong impact on the response. Based on the previous analysis, it is known that optimizing the partial gear ratio with the objective function of minimizing gearbox cost has poorly understood by scholars. The targets of this research are to optimize the partial gear ratio for a two-stage gearbox with second-stage double gear set. The objective function is the minimum cost of gearbox. The plan of tests is conducted by using Taguchi method thanks to Minitab@19. Furthermore, regression model is also suggested to predict the cost of gearbox.

METHODOLOGY

Gearbox volume analysis

The development of modern industry causes the higher requirements for machine design, in particular the optimization for parameters of components in a mechanical system. The partial and total gear ratios have strong effects on the dimensions, the There is the fact that the costs of bearings, gears, shafts, casing have strongly influence on the cost of a given gearbox. In this study, the cost of bearings will be ignored because of complex cost determination. Consequently, the cost of a two-stage helical gearbox, C_{gb}, can be determined as:

$$C_{gb} = C_{hg} + C_{sg} + C_{gh} + C_{s} \tag{1}$$

Where, C_{hg}, C_{sg} , C_{gh} and C_s are presentative for the helical gear cost, the straight gear cost, the gearbox housing cost, and the shaft cost, respectively.

It should be noticed that the cost of a gear contains the cost of used materials, machining process, heat treatment, operators, etc. These costs construct the final price of a gear. In term of commerce, the price of a gear can be usually determined by unit price per kilogram which regularly changes according to markets. In the current study, the cost of gears will be considered as variables and calculated by below equation (2).

The cost of the helical gears and the straight gears are determined by:

$$C_{hg} = c_{hg,m} \cdot m_{hg} \tag{2}$$

$$C_{sg} = c_{sg,m} \cdot m_{sg} \tag{3}$$

Where, $C_{hm,g}$, $C_{sm,g}$ are the cost per a kilogram of helical and straight gears (USD/kg); m_{hg} , m_{sg} are representative for the mass of the helical and the straight gears in the gearbox (kg).

In addition, the gearbox housing cost and the shaft cost can be determined by:

$$C_{gh} = c_{gh,m} \cdot m_{gh} \tag{4}$$

$$C_{s} = c_{s,m} \cdot m_{s} \tag{5}$$

In which, $c_{g,m}$, $c_{s,m}$ (USD/kg) are the cost per kilogram of gearbox housing and shaft (USD/kg) respectively, and m_{gh} , m_s (kg) are denoted orderly to the mass of gearbox housing, and all shafts.

According to early analysis, it is realized that the cost of gearbox (C_{gb}) powerfully depends on the component mass, e.g. the mass of gears, the gearbox housing, and shafts. This is due to the calculation of the cost per kilogram of gear, gearbox housing and shaft is far compared to the objective of this study.

The determination of gearbox housing mass

The gearbox housing mass (m_{gh}) can be simply calculated as following:

$$m_{gh} = \rho_{gh} \cdot V_{gh} \tag{6}$$

Where: ρ_{gh} is the weight density of gearbox housing material (kg/m^3); with gearbox housing material is cast iron, $\rho_{gh} = 7.2$ [21]; V_{gh} is the volume of the gearbox housing (m^3). It is observed that the form of gearbox housing is constituted by various component volumes. It means that:

$$V_{gh} = 2 \cdot V_b + 2 \cdot V_{A1} + 2 \cdot V_{A2} \tag{7}$$

Where, V_b is the volume of bottom housing (kg); V_{A1} and V_{A2} are the volume of side A1 and side V_{A2} (kg) (Figure 1):

$$V_b = L \cdot B \cdot S_G \tag{8}$$

$$V_{A1} = L \cdot \left(H - 2 \cdot S_G\right) \cdot S_G \tag{9}$$

$$V_{A2} = \left(H - 2 \cdot S_G\right) \cdot B_1 \cdot S_G \tag{10}$$

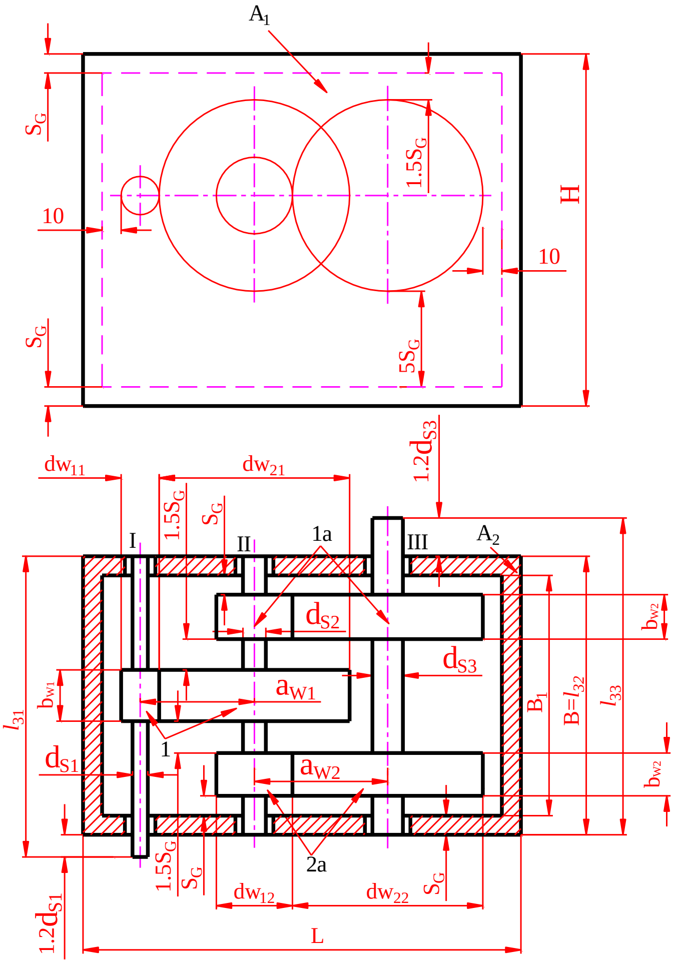

Figure 1. Calculated schema

In which, L, H, B$_1$ and S$_G$ can be determined by:

$$L = d_{w11}/2 + a_{w1} + a_{w2} + d_{w22}/2 + 20 \tag{11}$$

$$H = \max\left(d_{w21}, d_{w22}\right) + 8.5 \cdot S_G \tag{12}$$

$$B_1 = 2 \cdot b_{w1} + b_{w2} + 5 \cdot S_G \tag{13}$$

$$B = B_1 + 2 \cdot S_G = b_{w1} + 2 \cdot b_{w2} + 7 \cdot S_G \tag{14}$$

$$S_G = 0.005 \cdot L + 4.5 \tag{15}$$

The parameters d_{w11}, d_{w21}, d_{w22} in Eqs. (11) and (12) are gear pitch diameters of the first and second stages. They can be calculated by [3]:

$$d_{w11} = 2\, a_{w1}/(u_1 + 1) \tag{16}$$

$$d_{w21} = 2\, a_{w1}u_1\, /(u_1 + 1) \tag{17}$$

$$d_{w22} = 2\, a_{w2}u_2\, /(u_2 + 1) \tag{18}$$

Where, a_{w1} and a_{w2} are the center distances of the first and the second stages. These parameters can be calculated by [23]:

$$a_{w1} = k_a(u_1 + 1)\sqrt[3]{T_{11}k_{H\beta1}/([\sigma_{H1}]^2u_1X_{ba1})} \tag{19}$$

$$a_{w2} = k_a(u_2 + 1)\sqrt[3]{T_{12}k_{H\beta2}/([\sigma_{H2}]^2u_2X_{ba2})} \tag{20}$$

All parameters in Eqs. (19) and (20) are described as the following:

k_a is the material coefficient, k_a= 43 with steel material.

$k_{H\beta1}$ and $k_{H\beta2}$arerespectively the contacting load ratio for pitting resistance; for the first stage of a two-stage helical reducer with second stage double gear-sets $k_{H\beta1} = 1.0 \div 1.06$ [23];for the second stage, $k_{H\beta1} = 1.02 \div 1.28$ [23]. Therefore, $k_{H\beta1}$ and $k_{H\beta2}$can be respectively chosen $k_{H\beta1} = 1.03$ and $k_{H\beta2} = 1.15$.

$[\sigma_{H1}]$and $[\sigma_{H2}]$ are the allowable contact stress of the first and second stages (MPa), respectively;

u_1 and u_2 are, respectively, the transmission ratio of the first and second stages.

T_{11}, and $T_{12}$are respectively the torque on the first and second shafts of the gearbox (Nmm). They can be computed as:

$$T_{11} = T_{out}/(u_t\, \cdot \eta_{hg} \cdot \eta_{hg} \cdot \eta_{be}^3) \tag{21}$$

$$T_{12} = T_{out}/\left(2\, u_2 \cdot\, \eta_{hg} \cdot \eta_{be}^2\right) \tag{22}$$

Where, $u_t$is the total ratio of the gearbox; η_{sg} is the efficiency of straight gear set $\eta_{sg} = 0.96 \div 0.98$ [23]; η_{hg} is the efficiency of helical gear $\eta_{hg} = 0.96 \div 0.98$ [23]; η_{be} is the efficiency of a rolling bearing pair $\eta_{be} = 0.99 \div 9.995$ [23]. Choosing $\eta_{sg} = \eta_{hg} = 0.97$ and $\eta_{be} = 0.992$ we have

$$T_{11} = 1.0887\ \cdot T_{out}/u_t \tag{23}$$

$$T_{12} = 0.5238 \cdot T_{out}/u_2 \tag{24}$$

Gear mass calculations

The mass of straight gears in the gearbox can be found by:

$$m_{sg} = \rho_g \cdot (\pi \cdot e_1.d_{w11}^2 \cdot b_{w1}/4 + \pi \cdot e_2.d_{w21}^2 \cdot b_{w1}/4) \tag{25}$$

Where, ρ_g is the weight density of gear material (kg/m^3); with gear material is steel, $\rho_g = 7.82$ [21]; $e_1$is volume coefficient of the drive gear of the first stage; $e_2$is volume coefficient of the driven gear of the first stage; In practice, we can chose $e_1 = 1$ and $e_2 = 0.6$ [23]; b_{w1} is the gear width of the first stage (mm); $b_{w1} = X_{ba1} \cdot\ a_{w1}$ (mm);

Also, the mass of the helical gears in the gearbox can be calculated as:

$$m_{hg} = 2 \cdot \rho_g \cdot (\pi \cdot e_1.d_{w12}^2 \cdot b_{w2}/4 + \pi \cdot e_2.d_{w22}^2 \cdot b_{w2}/4) \tag{26}$$

Where, $b_{w2}$is the gear width of the second stage (mm); $b_{w2} = X_{ba2} \cdot a_{w2}$ (mm);

Shaft mass calculation

For using the same way as the gear mass calculation, the total mass of all shafts of the gearbox as shown in Figure 1 can be computed as:

$$m_s = m_{s1} + m_{s2} + m_{s3} \tag{27}$$

Where m_{s1}, $m_{s2}$and $m_{s3}$are respectively the mass of shaft 1, 2 and 3 of the gearbox (kg). They can be expressed as:

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1}/4 \tag{28}$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2}/4 \tag{29}$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3}/4 \tag{30}$$

In Eqs. (17), (18), and (19), ρ_s is the weight density of shaft material (kg/m3); l_{s1}, $l_{s2}$and l_{s3} are respectively the length of shaft 1, 2 and 3 of the gearbox as shown in Figure 1. They can be written as:

$$l_{s1} = B + 1.2 \cdot d_{s1} \tag{31}$$

$$l_{s2} = B \tag{32}$$

$$l_{s3} = B + 1.2 \cdot d_{s3} \tag{33}$$

In Eqs. (26), (27), and (28), preliminary diameters of the first, second and third shafts i.e., d_{s1}, d_{s2}, and d_{s3} are described as:

$$\mathrm{d}_{s1} = [\mathrm{T}_{11}/(0{,}2 \cdot [\tau])]^{1/3} \tag{34}$$

$$\mathrm{d}_{s2} = [\mathrm{T}_{12}/(0{,}2 \cdot [\tau])]^{1/3} \tag{35}$$

$$\mathrm{d}_{s3} = [\mathrm{T}_{13}/(0{,}2 \cdot [\tau])]^{1/3} \tag{36}$$

Where $[\tau]$ is the allowable shear stress (MPa), $[\tau] = 15 \div 20$ (MPa)[3]; it can be chosen as $[\tau] = 17$ (Mpa); T_{11} and T_{12} are respectively presented in Eqs. (23) and (24); T_{13} can be expressed as:

$$\mathrm{T}_{13} = \mathrm{T}_{\mathrm{out}} \tag{37}$$

Optimization problem

As known, to minimize the cost of the gearbox the objective function of C_{gb} should be satisfied by described problem:

$$Minimize C_{gb} \tag{38}$$

With the following constraints:

$$1 \leq u_1 \leq 9 \; ; 1 \leq u_2 \leq 9 \tag{39}$$

Nevertheless, it is known that $u_t = u_1.u_2$ is the relation between total gearbox ratio and partial ratios. Hence the optimization of u_1 is sufficient, while the optimum ratio of u_2 can be obtained by the expression: $u_2 = u_t/u_1$.

EXPERIMENTAL DESIGN

In order to estimate the impacts of main design parameters, input ones. on the partial gear ratio u_1 to obtain minimum gearbox cost, a simulation experiment is designed with ten input parameters and their investigating levels as listed in Table 1. The potential tests will be reduced but still keeping the quality of experiments, e.g. effects of input parameters on the response, minimum gearbox cost herein. Accordingly, the orthogonal array of (2^{k-p}) 2^{10-3}=128 will be adopted. It means that only 128 tests are conducted. This design is at the configuration of 5 in which no main factors or interactions are coincided with others. Minitab@18 will be utilized to develop the testing matrix with design of 2^{10-3} as previously mentioned. The testing matrixes are presented in Table 2.

Table 1. Input parameters and their investigating levels

No.	Parameters	Symbol	Unit	Level	
				Low	High
1	Total gearbox ratio	u_g	-	7	35
2	Coefficient of the face width of the first gear stage	X_{ba1}	-	0.3	0.35
3	Coefficient of wheel face width of the second gear stage	X_{ba2}	-	0.3	0.35
4	Allowable contact stress of stage 1	AS_1	MPa	350	420
5	Allowable contact stress of stage 2	AS_2	MPa	350	420
6	Output torque	T_{out}	Nm	1000	10000
7	Cost of gearbox housing	C_{gh}	USD/kg	1	5
8	Cost of helical gears	C_{hg}	USD/kg	3	7
9	Cost of straight gears	C_{sg}	USD/kg	5	9
10	Cost of shafts	C_s	USD/kg	1.5	5

RESULTS AND ANALYSIS

The influences of each input parameter are presented in Figure 2. It is observed that Total gearbox ration, u_g, is the most influential factor having impact on the response of u_1. An increase in u_g leads to an increase in u_1. Moreover, Allowable contact stress of stage 1 (AS_1) and cost of straight gear (C_{sg}) have similarly influential trend on u_1. Inversely, u_1 decreases with increasing Allowable contact stress of stage 2 (AS_2), Cost of helical gears (C_{hg}), and Cost of shafts (C_s). Meanwhile, cost of gearbox housing (C_{gh}), Coefficient of the face width of the first gear stage (X_{ba1}), Coefficient of wheel face width of the second gear stage (X_{ba2}) and Output torque (T_{out}) have little impacts on u1.

Table 2. Testing matrixes and the value of response u_1

Std Order	Run Order	CenterPt	Blocks	ug	Xba1	Xba2	AS1	AS2	Tout	Cgh	Chg	Csg	Cs	u1
1	1	1	1	7	0.30	0.30	350	350	1000	1	7	9	5.0	2.09
8	2	1	1	35	0.35	0.35	350	350	1000	1	3	9	5.0	8.46
70	3	1	1	35	0.30	0.35	350	350	1000	5	3	5	5.0	7.54
115	4	1	1	7	0.35	0.30	350	420	10000	5	7	9	1.5	2.19
108	5	1	1	35	0.35	0.30	420	350	10000	5	3	9	1.5	9.00
99	6	1	1	7	0.35	0.30	350	350	10000	5	7	5	1.5	2.09
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
18	127	1	1	35	0.30	0.30	350	420	1000	1	3	5	1.5	7.42
82	128	1	1	35	0.30	0.30	350	420	1000	5	7	5	1.5	6.32

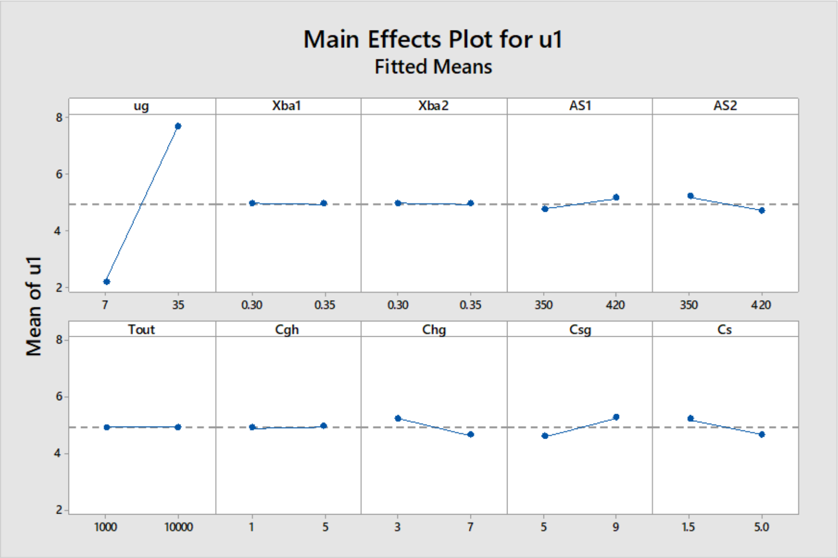

Figure 2. Influences of main factors on the first partial gear ratio, u_1.

The influences of input parameters can be also displayed in Figure 3. It is observed that Total gearbox ratio most significantly affects on u_1, shown by the first horizontal line assigned by A. Furthermore, the interactions between input parameters are exhibited. The length of each horizontal line presents the influential degree. The factors having the length over red reference line are ones that critically affect u_1 with significant level of 0.05. Specifically, others having significant effects are D (AS_1), E (AS_2), H (C_{hg}), J (C_{sg}), K (C_s) and the interactions, e.g. AD (u_g*AS_1), AE (u_g*AS_2), AH (u_g*C_{hg}), AJ (u_g*C_{sg}), AK (u_g*C_s), BC (X_{ba1}*X_{ba2}), GJ (C_{gh}*C_{sg}), HJ (C_{hg}*C_{sg}), HK (C_{hg}*C_s). The interactions of input parameters are also visualized in Figure 4 where one more time we can clearly observe the dominantly influence of u_g when this parameter is combined with the remaining ones.

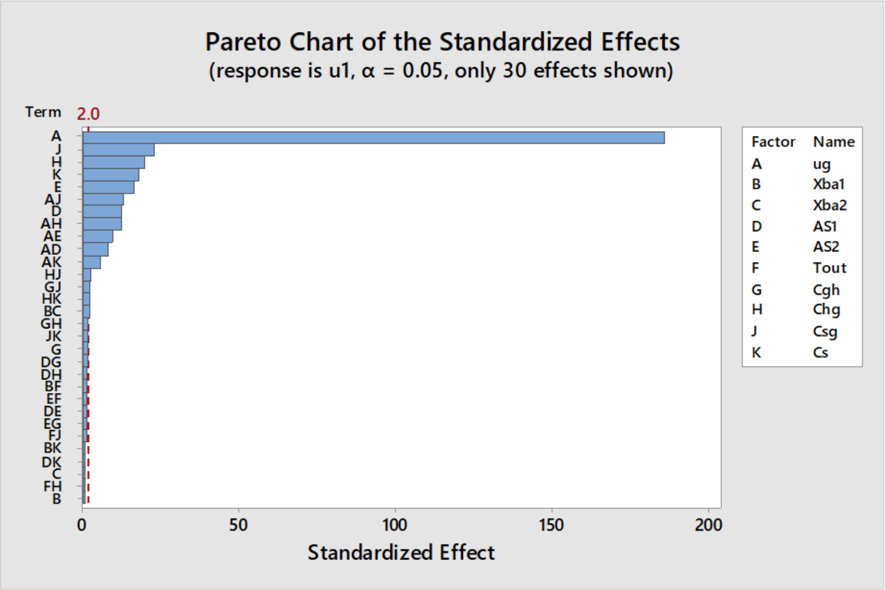

Figure 3. Pareto chart of the standardized effects on u_1.

In order to more clearly identify the tendency and influential degree of input parameters and interactions, it can be based on the normal plot of the standardized effects as displayed in Figure 5. It is observed that the factors denoted by red are the parameters having significant impacts on u_1. These parameters are far away from the reference line. The factors of A (u_g), D (AS_1), J (C_{sg}), and the interactions of AD (u_g*AS_1), AJ (u_g*C_{sg}), BC (X_{ba1}*X_{ba2}), HJ (C_{hg}*C_{sg}), HK (C_{hg}*C_s) in the right of reference line have positive influences on u_1, while the factor of E (AS_2), H (C_{hg}), K (C_s) and the interactions of AE (u_g*AS_2), AH (u_g*C_{hg}), AK (u_g*C_s), GJ (C_{gh}*C_{sg}) in the left of reference line have negative impacts on u_1. A regression model is proposed to predict the partial gear ratio of the first stage, u_1. The factors having weak impacts on the response are not included into the proposed model. The regression equation of u_1 is determined by using Minitab@19, and the estimated coefficients of the initial factors and interactions are listed in Table 3.

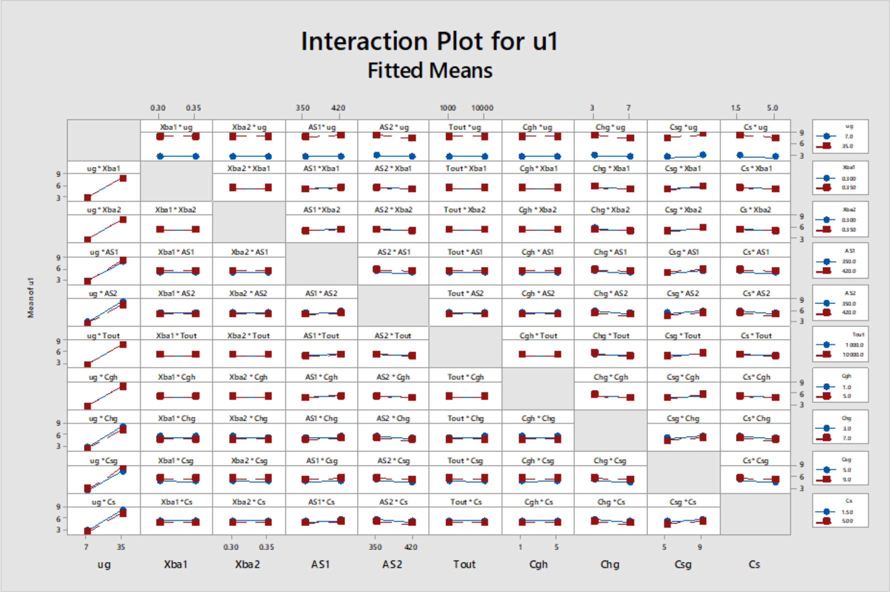

Figure 4. Interactions of input parameters on the response of u_1.

Figure 5. Normal plot of the standardized effects of input parameters and interactions on u_1.

Table 3. Estimated coefficients of the regression model

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.9305	0.0139	355.98	0.000	
ug	5.4938	2.7469	0.0139	198.32	0.000	1.00
Xba1	-0.0244	-0.0122	0.0139	-0.88	0.381	1.00
Xba2	-0.0250	-0.0125	0.0139	-0.90	0.369	1.00
AS1	0.3716	0.1858	0.0139	13.41	0.000	1.00
AS2	-0.4844	-0.2422	0.0139	-17.49	0.000	1.00
Cgh	0.0459	0.0230	0.0139	1.66	0.100	1.00

Chg	-0.5856	-0.2928	0.0139	-21.14	0.000	1.00
Csg	0.6788	0.3394	0.0139	24.50	0.000	1.00
Cs	-0.5359	-0.2680	0.0139	-19.35	0.000	1.00
ug*AS1	0.2456	0.1228	0.0139	8.87	0.000	1.00
ug*AS2	-0.2884	-0.1442	0.0139	-10.41	0.000	1.00
ug*Chg	-0.3672	-0.1836	0.0139	-13.26	0.000	1.00
ug*Csg	0.3803	0.1902	0.0139	13.73	0.000	1.00
ug*Cs	-0.1687	-0.0844	0.0139	-6.09	0.000	1.00
Xba1*Xba2	0.0616	0.0308	0.0139	2.22	0.028	1.00
Cgh*Csg	-0.0631	-0.0316	0.0139	-2.28	0.025	1.00
Chg*Csg	0.0847	0.0423	0.0139	3.06	0.003	1.00
Chg*Cs	0.0631	0.0316	0.0139	2.28	0.025	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.156700	99.74%	99.70%	99.64%

The regression model is presented as follows:

$$u_1 = 7.03 + 0.2095\,u_g - 16.49\,X_{ba1} - 16.51\,X_{ba2} + 0.000045\,AS_1 - 0.000739\,AS_2 + 0.0667\,C_{gh} - 0.1121\,C_{hg} - 0.0022\,C_{sg} - 0.1259\,C_s + 0.000251\,u_g{*}AS_1 - 0.000294\,u_g{*}AS_2 - 0.006557\,u_g{*}C_{hg} + 0.006791\,u_g{*}C_{sg} - 0.003444\,u_g{*}C_s + 49.3\,X_{ba1}{*}X_{ba2} - 0.00789\,C_{gh}{*}C_{sg} + 0.01059\,C_{hg}{*}C_{sg} + 0.00902\,C_{hg}{*}C_s$$

The suitability of the proposed model is estimated by the chart of residual evaluation distribution to identify the deviation between experiments and predictions (c.f. Figure 6). Based on the results given in Figure 6, it is observed that on the chart showing error distribution (displayed by blue points) comparing with a normal distribution (solid line), most errors are close to the normal distribution except for two points far away from the reference line. Additionally, regarding the appearing frequency of errors on the charts, the numbers approximate 0 are dominant and strength two points have the least appearing frequency. Observing the charts showing error relation and the corresponding values of the regression model (versus fit) reveals that the points randomly distribute. This result shows that u1 is not dependent on other parameters except input parameters mentioned earlier. Similarly, in the chart of observation order, the data points are arbitrarily distributed. This also means that u1 is not affected by the time. Based on the previous analysis of estimating errors of the charts and value of R-square which is adjusted approximately 99% (c.f. Table 3), it can be said that the proposed model is reliability and can be applied in practice and other studies.

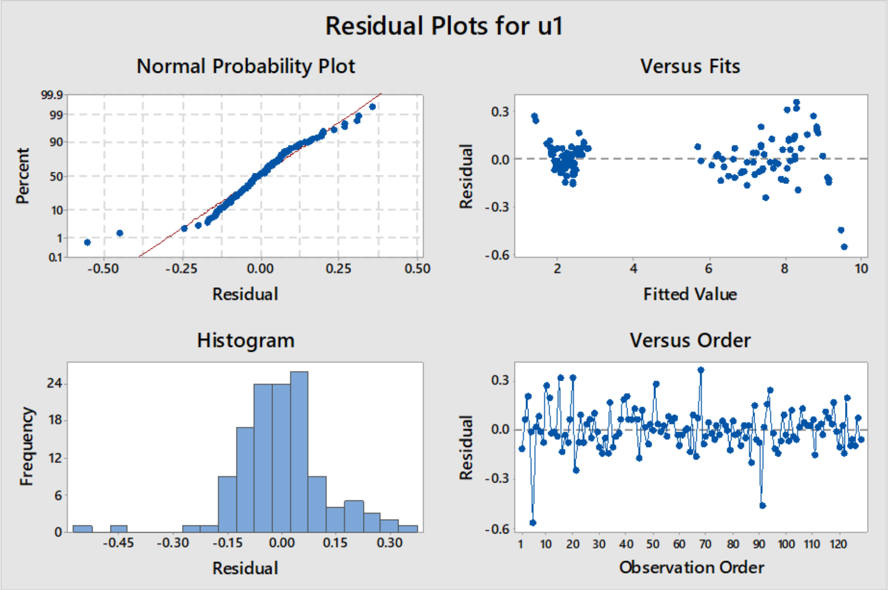

Figure 6. The charts of residual evaluation distribution

CONCLUSION

This study presents the optimization protocol of minimizing gearbox cost of a two-stage gearbox with second-stage double gear sets. The main design parameters of Total gearbox ratio, Coefficient of the face width of the first gear stage, Coefficient of wheel face width of the second gear stage, Allowable contact stress of stage 1, Allowable contact stress of stage 2, Output torque, cost of gearbox housing, Cost of helical gears, cost of straight gears, cost of shafts are selected as input parameters of optimization process. The following conclusions can be made:

- It is found that Total gear ratio has the most important impact on the response, while other parameters have little impacts;
- Interactions of input parameters have also strong influences on the response of u_1;
- The proposed method showing the R-square value of 99% is validated exhibit's high reliability and can be applied to other studies.

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