

Static Buckling Behavior of FGM Timoshenko Beam Theory Resting on Winkler Elastic Foundation

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ABSTRACT

Basing on the Timoshenko beam theory, the static buckling behavior of functionally graded materials (FGM) beam exposed to axial force resting on Winkler elastic base is investigated. Using a power-law distribution, mechanical properties of the FGM beam such as shear modulus and Young's modulus are supposed to differ in the thickness direction. Also, the Poisson ratio value can be considered constant in the thickness direction. This term, by applying the total potential principle, the governing equations are derived. According to the Navier-type solution manner the load of critical buckling is employed for simply supported beam. The effects of spring constant and power-law exponent of FGM on the critical buckling load are discussed. The results obtained by numerical methods show an excellent agreement through comparison with the results available in the previous studies. Furthermore, the critical load of buckling decreases as the slenderness ratio and the power-law index values increase.

KEYWORDS

FGM, Winkler, elastic foundation, Timoshenko theory, load of buckling critical

INTRODUCTION

Functionally graded materials are those smart materials having a location-specific gradual change in properties induced through changes in micro-and/or macrostructure or chemical composition. The traditional approach to material selection in terms of design and manufacture of components relies on the classification of the engineering materials present. In FGM, a reverse design is adopted whereby the selection of basic material and material processes to form the graded structures and components for demanding applications. This material can be generated by modifying the content of two or more materials in a percentage that contains in a spatial sense the desired property gradient. Armağan Karamanl [1] investigated the governing equations based on Reddy-Bickford beam, Timoshenko, and Euler- Bernoulli theories and using total potential energy principle for FG beams in two-directional exposed to a distributed transverse load. The SSPH method offers adequate predicted results for the problems investigated, according to the findings. Noori et al. [2] described the bending analysis of FG beams exposed to transverse distributed load utilizing Timoshenko beam theory (TBT). The total potential energy theory is used to find the governing equations. Chen et al. [3] studied the Timoshenko beam theory to examine the buckling and static bending behavior. According to Hamilton principle, the governing equations of the functionally graded porous beam are found to be exposed to compressive axial force and to uniformly distributed load. Li and Batra [4] derived the governing equations by using FGM Timoshenko and homogeneous Euler-Bernoulli beam theories. The shooting method is utilizing to measure the load of buckling of a FGM beam under different boundary conditions which is subjected to compressive axial force. Chikh [5] investigated the uniformly distributed load of the FG beam to calculate the equations of motion depend on hyperbolic shear deformation theory and Hamilton's principle. With simple supported boundary conditions, the transverse deflection is calculated using the Navier-type solution technique. Kahya and Turan [6] studied the shear deformation theory for FGM beam under an axial compressive force. In the case of FG, boundary conditions of simply supported beam, numerical analysis is used to calculate both normal frequency and buckling load. Akbaş [7] presented post-buckling analysis for FG beam subjected to an compressive axial force. In the case of cantilever beam, the finite element method is utilize to measure the load of critical buckling. Katili et al. [8] studied the static bending analysis of FGM beam by using Timoshenko beam theory. Also, it is used to

determine both maximum transverse deflections and natural frequencies for a set of different boundary conditions. Daneshmehra et al. [9] studied the buckling analysis depend on many theories such as Timoshenko beam theory(TBT), Euler-Bernoulli beam theory(EBT), and some higher-order shear deformation beam theories for FG beam. Almitani [10] illustrated the buckling behavior of the FG Euler-Bernoulli beam. Numerical analysis is employed to calculate the load of critical buckling using the finite element approach for simply supported boundary conditions. Rahmani et al. [11] studied the TBT, EBT, and higher-order shear deformation beam theory upon Hamilton's principle are all used to derive the governing equations for FG beams. The numerical results show both maximum deflection, critical load of buckling, and natural frequency using the finite element method for altered boundary conditions. Fouad et al. [12] studied the FG beam static analysis using hyperbolic shear deformation beam theory. Analytical solutions using Navier's method for boundary conditions of simply supported beam are used to calculate the maximum transverse deflection. The analytical solutions show the effect of the elastic foundation springs constant type on the maximum deflection. Attia et al. [13] used the refined shear deformation theory based on Hamilton's principle for FG porous beam to derived the equations of motion. To find the maximum deflection, the Navier's form solution approach is used. In addition, natural frequency of FGM beam exposed to uniformly distributed load with simply supported boundary conditions. Hadi et al. [14] studied the elastic bending analysis based on FGM Timoshenko beam theory subjected to distributed transverse load. According to the total potential principle, the governing equations of equilibrium of FG beam are obtained. Chikh [15] investigated the isotropic homogeneous and FGM beams in order to obtained the buckling, and static bending . For the simply supported beam, the Navier's solution method is employed to measure natural frequency, maximum deflection, and critical buckling load. Kumar [16] derived the equations of motion using FGM TBT and EBT based on Hamilton's principle. The numerical results suggested that the influence of the spring constant was on the usual frequencies. Babaei et al. [17] used the Reddy-Bickford beam theory of the FG porous beam under uniformly distributed load to conduct a static bending and buckling analysis. Maximum deflection and critical load of buckling are calculated by a numerical analysis by a finite element method. Zohra et al. [18] presented free vibration FGM beam analytical solutions based on refined hyperbolic theory of shear deformation. According to Hamilton's principle the motion equations are derived. In the event that FGM simply supports beam boundary conditions, the Navier solution approach is utilized to determine the frequency. Khorramabadi and Nezamabadi [19] derived the equations of motion of equilibrium on FG using Hamilton's principle, which is submitted to axial compressive load resting on continuous elastic foundation .

The Governing Equations for FGM Beam

A simply supported FGM beam with a length L , thickness h , and width b , with a metallic top surface (Aluminum) and a ceramic bottom surface (Alumina). As shown in Figure1, the beam is exposed to axial compression force when resting on a Winkler elastic base with a spring constant .

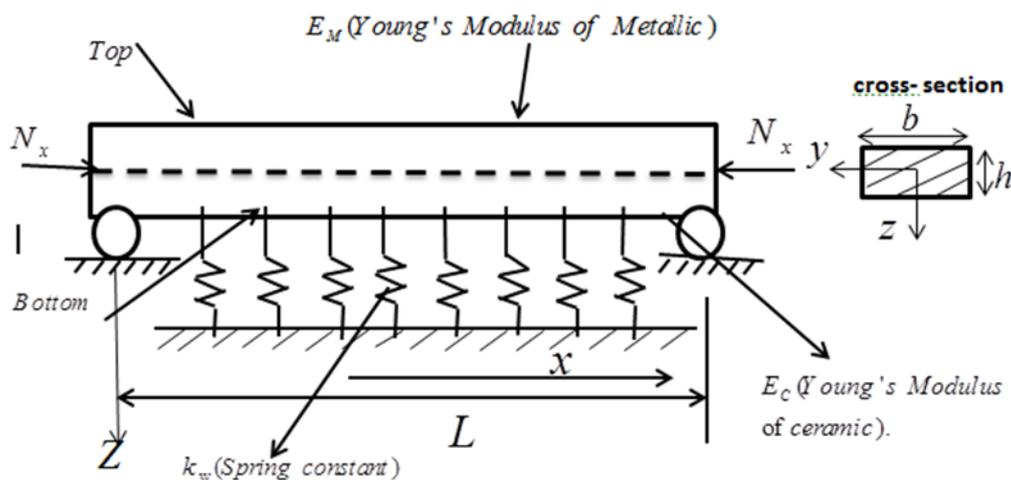


Figure 1. FGM simple supported beam under compressive axial force resting on elastic foundation.

In the thickness direction, the Young's modulus and shear modulus of elasticity of FGM beams change depending on the power-law distribution the power-law distribution provided by equations:

$$E(z) = E_m + (E_c - E_m) * (z / h + 0.5)^n \quad (1)$$

$$G(z) = G_m + (G_c - G_m) * (z / h + 0.5)^n \quad (2)$$

In top surface the shear modulus of elasticity and Young's modulus of elasticity are G_m , E_m , and in bottom surface the shear modulus of elasticity and Young's modulus of elasticity are G_n , E_n , FGM's Young's modulus variance is $E(z)$, and its power-law exponent is n . The displacement field of TBT can be given as:

$$u(x, y, z) = u_0(x) - z \varphi(x) \quad (3)$$

$$v(x, y, z) = 0 \quad (4)$$

$$w(x, y, z) = w_0(x) \quad (5)$$

Where u and w are the axial and transverse displacements of the beam in $x - z$ directions, v is the displacement of the beam in the y -direction w_0 is the maximum deflection of the beam, $\varphi(x)$ is the rotation on the neutral axis at any point of the cross-section. By using Eqs. (3) & (5) can be found axial and shear strains of Timoshenko beam theory (TBT) as following:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z \frac{\partial \varphi}{\partial x} \quad (6)$$

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} - \varphi(x) \quad (7)$$

Where, ε_{xx} and γ_{xz} are axial and shear strains. Can be written the strain energy (potential energy) as following:

$$U = \frac{1}{2} \int_V \sigma_{ij} \varepsilon_{ij} dV \quad (8)$$

$$\delta U = \int_V \sigma_{ij} \delta \varepsilon_{ij} dV \quad (9)$$

By using Eqs. (6), (7) & (9) the strain energy (potential energy) of TBT can be written as:

$$\delta U = \int_0^L \int_A (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{xz} \delta \gamma_{xz}) dA dx \quad (10)$$

Where, σ_{xx} and σ_{xz} are axial and shear stresses. By substituting Eqs. (6) & (7) into Eq.(10) and using Eqs. (12) can be find final strain energy (potential energy) by using variational method for FGM Timoshenko beam theory (TBT) as following:

$$\delta U = \int_0^L \left(-\frac{\partial N_x}{\partial x} \delta u_0 + \frac{\partial M_x}{\partial x} \delta \varphi - \frac{\partial Q_x}{\partial x} \delta w_0 - Q_x \delta \varphi \right) dx \quad (11)$$

Where:

$$\begin{aligned}
 N_x &= \int_A \sigma_{xx} dA \\
 M_x &= \int_A \sigma_{xx} z dA \\
 Q_x &= k_s \int_A \sigma_{xz} dA
 \end{aligned} \tag{12}$$

Where, M_x is the bending moment, N_x axial normal force, Q_x shear force, and K_s is the shear correction factor. The external work is calculated applying the axial force for FGM Timoshenko beam theory as following:

$$W_{ext} = \frac{-1}{2} \int_0^L \int_{-b/2}^{b/2} (F_{ext} w_0) dy dx \tag{13}$$

$$\delta W_{ext} = -b \int_0^L (F_{ext} \delta w_0) dx$$

$$F_{ext} = F_{elastic\ foundation} + F_{buckling} \left(\frac{\partial^2 w}{\partial x^2} \right)$$

$$F_{elastic\ foundation} = -k_w w$$

$$F_{buckling} = N_{x0} \frac{\partial^2 w}{\partial x^2} \tag{14}$$

Substituting Eqs. (14.3) &(14.4) into Eq. (14.1) can be find final external work done of FGM Timoshenko beam theory (TBT) is as:

$$\delta W_{ext} = -b \int_0^L \left[\left(-k_w w_0 + N_{x0} \frac{\partial^2 w}{\partial x^2} \right) \delta w_0 \right] dx \tag{15}$$

The governing equations for FGM Timoshenko beam theory (TBT) based on the total potential energy principle are as follows:

$$\begin{aligned}
 \Pi &= (U + W_{ext}) \\
 \delta \Pi &= 0 \\
 \delta U + \delta W_{ext} &= 0
 \end{aligned} \tag{16}$$

Substituting Eqs. (11) & (15) into (16.3) and Set the parameters of δu_0 , δw_0 and $\delta \varphi$ to zero. The final equations of equilibrium can be inscribed as follows:

$$\begin{aligned}
 \delta u_0 : -\frac{\partial N_x}{\partial x} &= 0 \\
 \delta w_0 : -\frac{\partial Q_x}{\partial x} + b k_w w_0 - b N_{x0} \frac{\partial^2 w}{\partial x^2} &= 0 \\
 \delta \varphi : \frac{\partial M_x}{\partial x} - Q_x &= 0
 \end{aligned} \tag{17}$$

Axial and shear stress law of Hooke.

$$\sigma_{xx} = E(z) * \epsilon_{xx} \tag{18}$$

$$\tau_{xz} = G(z) * \gamma_{xz} \tag{19}$$

By using Eqs. (12.1),(12.2) & (12.3) with using Axial and shear stress law of Hooke Eqs.(18) & (19) and by substituting into Eqs. (17.1) , (17.2) &(17.3) the governing equations of equilibrium of FGM Timoshenko beam theory (TBT) can be derived as following:

$$\begin{aligned}
 A_{xx} \frac{\partial^2 u_0}{\partial x^2} - B_{xx} \frac{\partial^2 \varphi}{\partial x^2} &= 0 \\
 A_{xz} k_s \left(\frac{\partial^2 w_0}{\partial x^2} - \frac{\partial \varphi}{\partial x} \right) - k_w w_0 + N_{x0} \frac{\partial^2 w}{\partial x^2} &= 0 \\
 -B_{xx} \frac{\partial^2 u_0}{\partial x^2} + D_{xx} \frac{\partial^2 \varphi}{\partial x^2} + A_{xz} k_s \left(\frac{\partial w_0}{\partial x} - \varphi \right) &= 0
 \end{aligned} \tag{20}$$

Where:

$$\begin{aligned}
 (A_{xx}, B_{xx}, D_{xx}) &= \int_{-h/2}^{h/2} E(z) * (1, z, z^2) dz \\
 A_{xz} &= \int_{-h/2}^{h/2} G(z) dz
 \end{aligned} \tag{21}$$

Where, A_{xx} , B_{xx} , D_{xx} and A_{xz} are stiffness coefficients of FGM beam.

Analytical solutions of buckling of FGM simply supported beam using Navier-type solution method

Can be solving the governing equations depend on FGM Timoshenko beam theory (TBT) and using Navier-type solution method. The beam is subjected to axial compressive force with simply-supported FGM beam is given as:

$$\begin{aligned}
 x = 0 \Rightarrow u = 0, \varphi = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial \varphi}{\partial x} = 0 \\
 x = L \Rightarrow u = 0, \varphi = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial \varphi}{\partial x} = 0
 \end{aligned} \tag{22}$$

Depending on the Navier-type solution method, the simple-supported FGM TBT can be resolved and the variables $u_{(x)}$, $w_{(x)}$ and $\varphi_{(x)}$ is stated as follows:

$$\begin{aligned}
 u(x, y, z) &= \sum_{m=1,2,3}^{\infty} U_m \cos\left(\frac{m\pi x}{L}\right) \\
 w(x, y, z) &= \sum_{m=1,2,3}^{\infty} w_m \sin\left(\frac{m\pi x}{L}\right) \\
 \varphi(x, y, z) &= \sum_{m=1,2,3}^{\infty} \varphi_m \cos\left(\frac{m\pi x}{L}\right)
 \end{aligned} \tag{23}$$

Where the Fourier coefficients are unknown , U_m , W_m and φ_m .

Substituting Eqs.(23.1) - (23.2) & (23.2) into Eqs.(20.1) -(20.2) & (20.3) can be obtained as:

$$\begin{aligned} & \left[-A_{xx} \left(\frac{m\pi}{L} \right)^2 \right] U_m + \left[B_{xx} \left(\frac{m\pi}{L} \right)^2 \right] \varphi_m = 0 \\ & \left[-A_{xz} k_s \left(\frac{m\pi}{L} \right)^2 - N_{x0} \left(\frac{m\pi}{L} \right)^2 - k_w \right] W_m + \left[A_{xz} k_s \left(\frac{m\pi}{L} \right) \right] \varphi_m = 0 \\ & \left[B_{xx} \left(\frac{m\pi}{L} \right)^2 \right] U_m + \left[A_{xz} k_s \left(\frac{m\pi}{L} \right) \right] W_m + \left[-D_{xx} \left(\frac{m\pi}{L} \right)^2 - A_{xz} k_s \right] \varphi_m = 0 \end{aligned} \tag{24}$$

Using Eqs. (36.1)-(36.2) & (36.3) can be find final matrix form of FGM Timoshenko beam theory (TBT) as following:

$$\begin{bmatrix} A_{xx} \left(\frac{m\pi}{L} \right)^2 & 0 & -B_{xx} \left(\frac{m\pi}{L} \right)^2 \\ 0 & A_{xz} k_s \left(\frac{m\pi}{L} \right)^2 + N_{x0} \left(\frac{m\pi}{L} \right)^2 + k_w & -A_{xz} k_s \left(\frac{m\pi}{L} \right) \\ -B_{xx} \left(\frac{m\pi}{L} \right)^2 & -A_{xz} k_s \left(\frac{m\pi}{L} \right) & D_{xx} \left(\frac{m\pi}{L} \right)^2 + A_{xz} k_s \end{bmatrix} \begin{bmatrix} U_m \\ W_m \\ \varphi_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

NUMERICAL RESULTS AND DISCUSSIONS

Using Navier's method, the critical load of buckling for simply supported FGM Timoshenko beam theory is calculated. The FGM beam is made up of two materials: metallic (aluminum) on top $E_m=70 \text{ GPa}$, and ceramic (alumina) on the bottom $E_c=380 \text{ GPa}$. Assuming that the beam with Length $L=1\text{m}$, thickness $h=0.1\text{m}$ and width $b=0.1\text{m}$. The beam is exposed to the N_{x0} compressive axial force. The Poisson's ratio for metallic (Aluminum) and ceramic (Alumina) is similar $\nu_c=\nu_m=0.23$. The factor of shear correction is obtained as $k_s=5/6$ for Timoshenko beam theory (TBT) and longitudinal wave number $m=1$. Dimensionless critical load of buckling is defined as:

$$P_{cr} = \frac{p^* L^2}{E_m^* I} \tag{26}$$

Tables 1&2 show the critical load of buckling P_{cr} with different value of power-law index n of the FGM theory of Timoshenko beam (TBT) for constant values of slenderness ratio $L/h=5$ and $L/h=10$, respectively. From these tables are noted with increases in values of the n index, the P_{cr} decreases. Thus, the numerical results show that the Li and Batra solution is in excellent agreement [4].

Table 1. The critical loads of buckling dimensionless of FGM Timoshenko beam theory (TBT) with the n index and constant ratio of slenderness $L/h=5$ have varying values.

n	Li and Batra [4] (Pcr) for TBT	Present work (Pcr) for TBT
0.0	48.835	48.835
0.5	31.967	31.967
1	24.687	24.687
2	19.245	19.245
5	16.024	16.024
7	15.265	15.265

10	14.427	14.427
100	10.020	10.020
$10^{11}(\infty)$	8.9959	8.9959

Table 2. FGM Timoshenko beam theory (TBT) dimensionless critical loads of buckling with different values of the n index and constant ratio of slenderness $L/h=10$.

n	Li and Batra [4] (Pcr) for TBT	Present work (Pcr) for TBT
0.0	48.835	48.835
0.5	31.967	31.967
1	24.687	24.687
2	19.245	19.245
5	16.024	16.024
7	15.265	15.265
10	14.427	14.427
100	10.020	10.020
$10^{11}(\infty)$	8.9959	8.9959

Figures 2&3 show the variation of the P_{cr} of FGM versus slenderness ratio for three different values of the Winkler elastic foundation with spring constant by using Timoshenko beam theory TBT. Figures 2 & 3 show that the P_{cr} increases as increasing in the values of the spring constant K_w because with the increase in the values of the K_w the beam becomes high stiffer. It is also apparent the P_{cr} decreases as the values of the slenderness ratio L/h increase as shown in Figure 2&3.

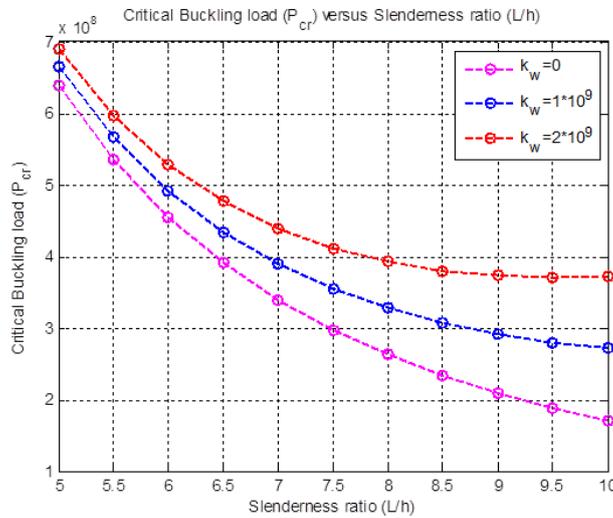


Figure 2. Spring constant influences the critical load of buckling of FGM Timoshenko beam theory with ratio of slenderness L/h .

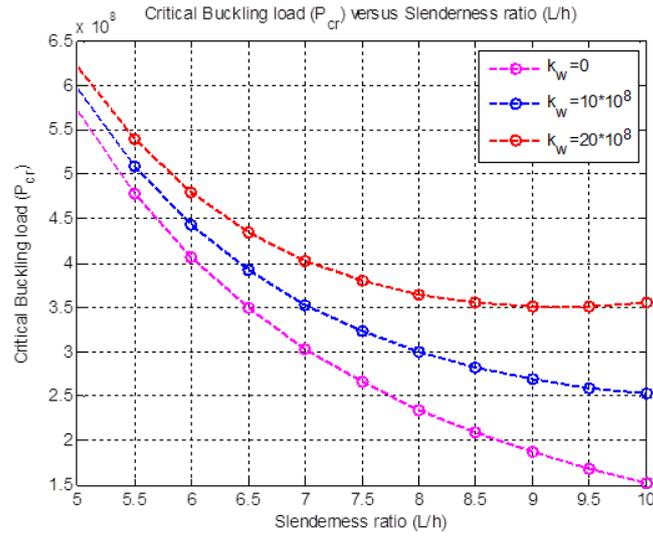


Figure 3. Spring constant influences the critical load of buckling of FGM Timoshenko beam theory with ratio of slenderness L/h .

Figs. 4 & 5 show the influence of longitudinal wave number m and n index on load of critical buckling with ratio of slenderness L/h of simply supported boundary conditions FGM based on TBT. It is seen from Figs. 4 & 5 that the P_{cr} decreases as increases in the values of the L/h slenderness ratio and decreases as increases in the values of the n index this is because increases in the values of the n index make the beam more flexible and so close to full aluminum. Also, it is can be seen from Figs. 4 & 5 the critical load of buckling increases as the longitudinal wave number m values increases.

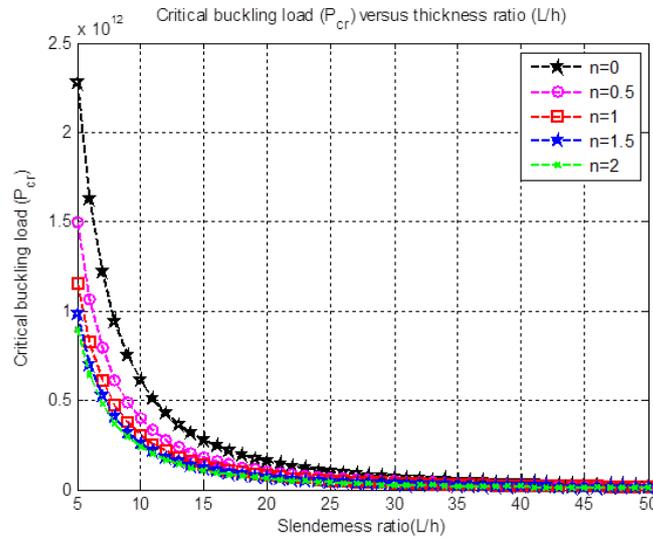


Figure 4. The n index influence on the P_{cr} of the FGM TBT with a L/h ratio of slenderness.

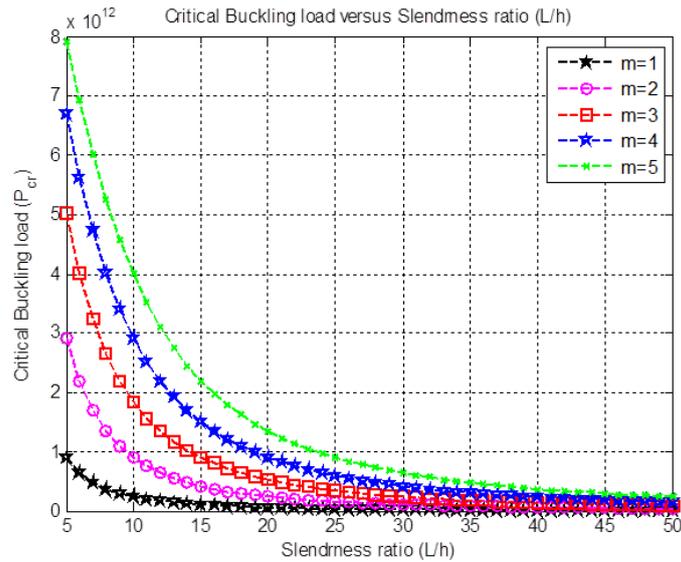


Figure 5. Longitudinal wave number m influence on the critical load of buckling of FGM upon TBT with ratio of slenderness L/h .

CONCLUSION

Since the beam becomes stiffer as the spring constant values increase, the critical load of buckling of the FGM upon TBT increases. As the longitudinal wavenumber increases, the critical load of buckling of FGM upon TBT increases. The numerical results show that n index has an impact on the the critical load of buckling of FGM upon TBT. It can be seen with increases in the values of the n index the P_{cr} decreasing because with increases in the values of the power-law index n the FGM beam gets more flexible so the FGM beam close to full aluminum. The critical load of buckling by utilize FGM TBM decreases as increasing in the values of the L/h slenderness ratio.

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